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Guide

MATH WORKSHOP

CURRICULUM

VISED EDITION

Robert W. Wirtz ▪ Morton Botel
Max Beberman ▪ W. W. Sawyer

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level A

Teacher's Guide

MATH WORKSHOP

REVISED EDITION

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I. A PERSPECTIVE

Loose talk about reform in the teaching of elementary mathematics has spread confusion and shattered the confidence of many teachers. Uneasiness and fear have developed. This mood has damaging effects in the classroom.

Teachers have been told they are teaching horse-and-buggy mathematics in a space age. New and esoteric language has begun to flood the literature and to be introduced into discussions. Some innovators have pointed the finger of scorn at traditional subject matter, pronouncing it "bad mathematics."

Relax! The mathematics our pupils are learning in traditional classrooms is good, sound, useful mathematics; it is good enough to play a central role in any "moon shot." Traditional content in elementary school arithmetic is sound; it is at the core of man's advance, including his recent bursts into space.

Our real concern is with finding solutions for two interrelated problems. How can we help pupils (a) to learn *more* mathematics, and (b) to develop a *deep appreciation* for this increasingly important part of our culture? All proposals and discussions should be evaluated in terms of their contributions to these fundamental problems.

The traditional content taught to pupils in elementary schools is completely valid—they simply aren't mastering it. Many pupils have too weak a grasp on basic facts such as $7 + 2 = 9$ and $8 \times 9 = 72$. (Most of these pupils will drop mathematics as soon as they can, losing contact with a tool that is essential in coping with the problems of the time.) Pupils will be able to master more of the traditional content if they *understand* what they learn and if they have ample opportunity to *use* what they learn.

The job of teaching more mathematics will become easier to the extent that pupils develop an appreciation for the subject. This appreciation can develop only when mathematics is learned with understanding—it will not take place when mathematics is learned by rote. Mathematics is a world of endless fascination. It is a world brimming with surprises, patterns, and enticing vistas. To see a pattern emerge from a conglomeration of facts; to organize seemingly disconnected ideas—these are some of the joys of learning mathematics. It is this sense of excitement in mathematics which pupils ought to be experiencing in elementary schools.

Giant steps are currently being taken to alter the traditional approach to teaching mathematics in order to get more mathematics into the elementary and secondary curricula. To this end, brilliant pioneering work is being done at the Universities of Illinois, Maryland, Minnesota, Syracuse, Stanford, and elsewhere. Generous financial support has been given by the National Science Foundation, the Carnegie Corporation of New York, and under the National Defense Education Act for research projects, workshops, and institutes. The National Council of Teachers of Mathematics (NCTM) in its 24th Yearbook, *The Growth of Mathematical Ideas, Grades K-12,**

**The Growth of Mathematical Ideas Grades K-12.* (The National Council of Teachers of Mathematics, Washington, D. C., 1959).

has prepared a comprehensive statement of what the aim of mathematics instruction should be from Kindergarten to Grade 12 and beyond.

Fundamentally, the recommendations of the various groups which are striving to improve the teaching of elementary school mathematics can be summarized as follows:

Pupils must learn that arithmetic is not just a collection of fragments, but rather is a mathematical system organized according to definite principles. For pupils to understand this concept, methods of instruction must be developed which place the emphasis on mathematical problems that reveal these principles and require their use.

MATH WORKSHOP is a program that tries to reflect the thinking of leading mathematicians and educators on what needs to be done to improve the teaching of elementary school mathematics. It is a program that has evolved for over ten years. Its ideas and techniques have been field-tested with thousands of teachers and tens of thousands of pupils over this period.

II. CONTENT AND TEACHING APPROACH

The authors of MATH WORKSHOP believe that a sound mathematics program can be made ineffective by methods of teaching which rely mainly on rote learning. We also believe that even inspired teaching of isolated fragments of mathematics will fail to produce pupils who understand and can use what they have learned. The MATH WORKSHOP puts its *major subject matter emphasis* on basic principles and unifying ideas which tie together the facts and manipulative procedures of arithmetic. This emphasis simplifies the subject, makes it understandable, insures recall, and increases interest. Our *major pedagogical emphasis* is that of teaching for discovery. We present content to pupils so that they are challenged to think about relationships, to look for patterns and clues, and to draw logical conclusions for themselves.

A. The *What* of Mathematics

The Spiral Curriculum

The *24th Yearbook* of the NCTM talks about "the basic mathematical understandings which should be continuously developed and extended throughout the entire mathematics curriculum, grades K-12 and beyond." This concept of the spiral curriculum has been used in designing MATH WORKSHOP. At each level of the MATH WORKSHOP the main ideas of mathematics have been introduced. Each successive year's activities deal with these unifying ideas with increasing sophistication.

However, pupils are not expected to master such ideas as, say, multiplication of fractions or addition of positive and negative numbers the first time they meet them. They will encounter such ideas again and again, building always more layers of understanding.

The spiral curirculum rejects the "building block" approach to arithmetic which has been used in the past. Rather, the spiral curriculum calls for the early introduction of fundamental and unifying ideas with repeated reconsideration at later times. It calls for continual nurturing of important concepts year after year as understanding gradually develops.

The hope for mastery must be limited to the basic facts of arithmetic. Here, mastery can and must be achieved. But where big ideas are involved, mastery does not come in this way. It develops slowly and is based completely on understanding.

The Unifying Ideas or Strands

MATH WORKSHOP emphasizes nine unifying ideas in mathematics:

1. Structure
2. Sets
3. Number and counting
4. Numeration
5. Addition and subtraction
6. Multiplication and division
7. Functions and relations
8. Geometry
9. Measurement

The coverage of these ideas on the pupil's pages is summarized in the chart on page 28. Each teacher's guide page lists the unifying ideas considered on the corresponding pupil's page and contains many suggestions for extending these ideas beyond what is called for on the pupil's page.

B. *How* to Teach Mathematics

It's easy to ask a question that is so difficult that almost no one can answer it. It's easy to ask a question so easy that almost anyone can answer it. And it's easy to dig up trick questions—there are books full of them.

But challenging questions that encourage independent research and lead to discovery must be neither too hard nor too easy, and must not rely on clever tricks. Rather, challenging questions are less-than-obvious but will yield answers if the student applies thoughtfully what he already knows.

Such questions are hard to come by. They are worth their weight in gold. Often they lurk in very simple situations.

Practically every page and activity in MATH WORKSHOP poses such less-than-obvious, challenging questions. MATH WORKSHOP presents these questions through a wide assortment of techniques, many of which were invented by the authors of the program. Challenging questions are posed through the game of "What's My Rule?"; Cross Number Puzzles; activities concerned with "Numbers Have Many Names"; and through problem solving. Let us examine examples of these four vehicles we use for asking challenging questions.

What's My Rule?

In first grade, Miss Atkins tells the children that she has a secret rule and she will help them read her mind.

"Call out a number and I will answer with a number."

"Five," a child calls out.

"Four," Miss Atkins answers. "Someone call out another number."

"One," a child calls out.

"Zero," the teacher replies.

"Seven," another child calls out.

"Six," the teacher answers. "Have you guessed my rule?"

"Yes," Bill is quick to reply.

"Don't give my secret away. Just call out a number and tell me what you think I will reply."

"If I say 9, you are going to say 8."

"That's right. I think that you know my secret."

Thus the children have learned a way to demonstrate their discovery.

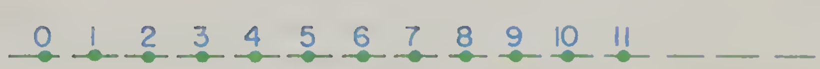
Many children are not so quick as Bill. So Miss Atkins may put on the board a record of the development thus far, first listing the number called out and then the reply:

$5 \rightarrow 4$
$1 \rightarrow 0$
$7 \rightarrow 6$
$9 \rightarrow 8$

The last item was Bill's contribution. Others might be:

$6 \rightarrow 5$
$2 \rightarrow 1$
$4 \rightarrow 3$

When the game is new or the rule difficult, a number line on the board will help:



"Let's act out the game we have played. When someone called out 5, I answered 4. Point to 5 on the number line. Keep your finger on that point. I answered by saying 4. Point to 4 on the line."

Each step is similarly acted out.

"Now, if you call out 8, put your finger at that point on the number line."

$8 \rightarrow$ ____

"What do you think I will reply?"

"Seven, of course!"

In some such way the idea can be made clear even to the slowest children. Care was taken not to talk about the teacher's rule itself—only about the results of applying it. Thus, every child must make the discovery for himself—some with minimum clues, others only after several different kinds of clues. There is a bit of artistry in giving clues only when needed and finding ways to reserve at least a little for even the slowest child to find out for himself.

Usually some child will note a curious fact about this rule.

"Suppose I call out zero. What will you answer?" a child may ask.

"Point to zero on the number line," the teacher suggests. "You can see that I would have to ask you to pick some other number." (Unless the teacher would like to talk about positive and negative numbers and zero.)

A new game begins with another secret rule. Some child may want to take over the teacher's role and invent a rule of his own.

Rules can become more complicated.

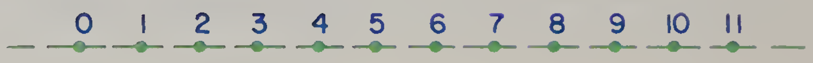
"I have a new rule. Call out a pair of numbers and I will reply by saying a number." Here is a record of a simple rule:

$2, 3 \rightarrow 5$
$4, 0 \rightarrow 4$
$3, 1 \rightarrow 4$
$1, 2 \rightarrow 3$
$1, 1 \rightarrow 2$
$3, 1 \rightarrow \underline{\quad}$

A more difficult rule is behind the following record:

$2, 4 \rightarrow 3$
$0, 2 \rightarrow 1$
$4, 0 \rightarrow 2$
$5, 9 \rightarrow 7$

These last two items may puzzle the children. Perhaps a number line can help shed light on the rule.



"If someone called out 2 and 4, I would reply with 3. Point to the numbers called out (2 and 4). I would reply with 3.

"If someone called out 4 and 0, I would reply with 2. Point to 4 and 0 on the number line. I would call out 2.

"Point to 5 and 9—and remember that I would answer 7. What would I answer if someone called out 1 and 7?"

$1, 7 \rightarrow \underline{\quad}$

"Four," Bill might answer, and quickly ask: "But what would you do if I called out 5 and 6 or if I called out 3 and 0?"

"I would have to ask you to pick another pair of numbers." (Unless the teacher is ready to discuss fractions.)

An unlimited number of rules can be devised on any level of sophistication. Further, there are countless ways to suggest different rules, such as:

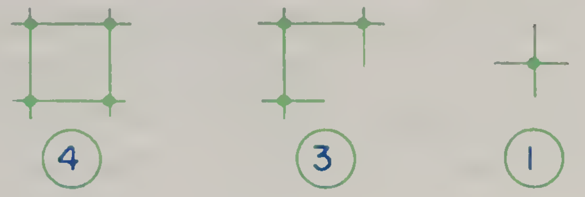
a. 1, 4, 7, 10, 13, ___, ___, ___

b.

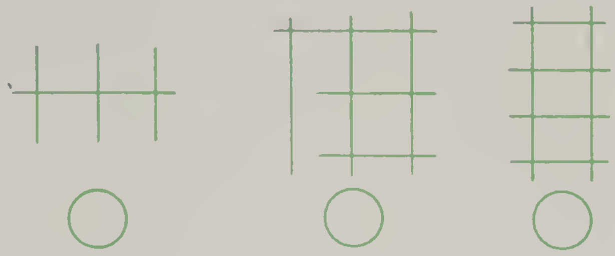
3	1	4	10	0	9	6
7	3	9	21	1		

c.

7	4	3	1
9	9	0	9
11	6	5	1
	8	6	
6			2

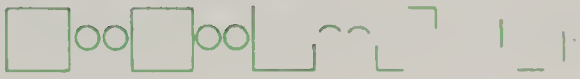


d.



In each case, a certain pattern is given that can be extended. Such patterns need not involve numbers:

e.



f. B , D , F , H , J , ___ , ___ , ___

Mathematics is essentially a study of patterns. It has its own fascinating patterns and it is often employed to help reveal patterns in the physical world. When a scientist or any observer is searching for a better understanding of what he notices, he usually tries to set down a description of events. Whenever possible he will try to write down numbers that in some way help him record relationships. He might record the following:

	1st	2nd	3rd	4th	5th
minutes	1	3	4	5	10
miles	1	9	16		

In his first three observations he notes time and distance. A pattern is strongly suggested. He may check his guess by making further checks for 5 minutes and 10 minutes. If his results are 25 miles and 100 miles, respectively, he will feel that he can formulate a rule or law that will suggest a pattern in certain real events.

In mathematics we study patterns because they suggest basic principles of the subject itself and because mathematics helps record and explore patterns of the world we live in.

Thus, this exercise or game of "What's My Rule?" has three important dimensions: (1) It can be used to motivate drill in the basic combinations of arithmetic. (2) It poses problems that children can think through to a solution. (3) It uses a method fundamental to the experimental sciences.

Numbers Have Many Names

In any arithmetic program there must be adequate provision for practice in the basic combinations and operations. Children must have many opportunities to answer questions such as the following:

2 + 1 = ___
0 + 4 = ___
3 + 0 = ___
2 + 2 = ___
4 + 0 = ___

1 + 3 = ___
1 + 2 = ___
3 + 1 = ___
0 + 3 = ___

However, let's see if we can provide motivation that will lead to more efficient learning and to a more thoughtful approach to our content.

Consider this question: Can you use just the counting numbers and 0 to fill the blanks below to make true statements which are different from each other?

___ + ___ = 3
___ + ___ = 3
___ + ___ = 3
___ + ___ = 3
___ + ___ = 3
___ + ___ = 3

___ + ___ = 4
___ + ___ = 4
___ + ___ = 4
___ + ___ = 4
___ + ___ = 4
___ + ___ = 4

Were you able to make six different true statements by using only the counting numbers and 0? The first four were easy:

2 + 1 = 3
0 + 3 = 3
1 + 2 = 3
3 + 0 = 3

Then it becomes more difficult. Perhaps there are only four ways.

In the second part of the exercise you probably found five ways to answer the question without much trouble:

2 + 2 = 4
0 + 4 = 4
1 + 3 = 4
3 + 1 = 4
4 + 0 = 4

And again the search becomes difficult.

Notice that, in finding these nine statements, you have used all the facts included in the original drill exercise. But now the search has become a game—"How many can we find?"—and it is not at all obvious to children that the number is limited.

"Have you any reason to think that there are no other statements (when limited to the counting numbers and 0)? Perhaps arranging them in some kind of order will help. Consider the following:

0 + 3 = 3
1 + 2 = 3
2 + 1 = 3
3 + 0 = 3

0 + 4 = 4
1 + 3 = 4
2 + 2 = 4
3 + 1 = 4
4 + 0 = 4

This arrangement exhibits a pattern that supports our belief that we have exhausted the possibilities.

How many different true statements can you get, using only the counting numbers and 0?

___ + ___ = 0
___ + ___ = 1
___ + ___ = 2

How many different true statements can you get in the following:

___ + ___ = 10

Thus an activity that is packed full of drill can be as interesting as a puzzle. It can be something to think about. While memory was involved, that was not all. A pattern emerged. We are led to a generalization. And this is mathematics!

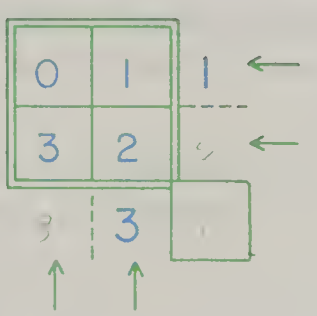
An interesting way to talk about this and similar activities is to refer to the fact that numbers have many names. If it is understood that we are limited to the counting numbers and 0, we ask:

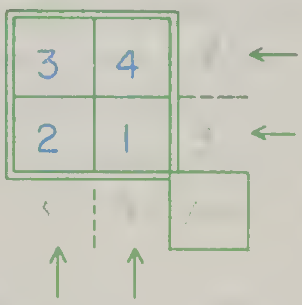
"How many names for 3 can you find in the form ___ + ___ ?" We can find four such names.

Cross Number Puzzles

Too few of us realize that even six- and seven-year-olds can carry out research problems in arithmetic. Do you doubt it? Let's leave speculation and get to work.

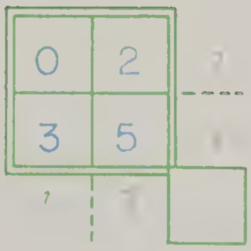
In the spaces indicated by arrows, write the sum of the two numbers listed in each row and in each column:

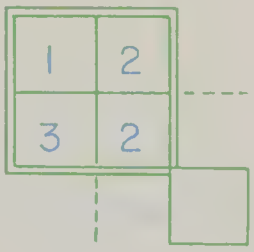




Do you have a desire to fill in the lower right-hand corner of each diagram above? Notice the two numbers listed above the corner and the two numbers listed to the left of the corner: 1 and 5, 3 and 3 in the first example; 7 and 3, 5 and 5 in the second example.

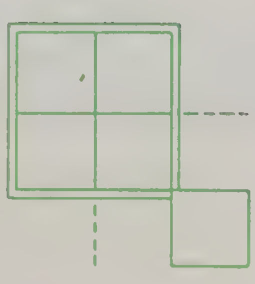
Most people would feel an urge to write 6 in the lower right-hand corner of the first example, and 10 in that corner of the second example. Try two more:

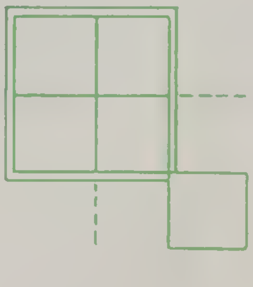




Again, a similar result. This is the kind of surprise that is the spice of arithmetic!

Can you select four numbers to list in the cells inside the double lines so that some surprise happens? Try your hand at two examples:

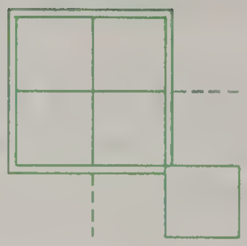


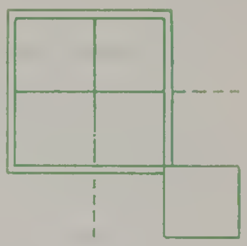


Did you succeed?

This problem is relatively simple. Even first-graders can find examples that "work." They are putting their arithmetic to use, practicing the basic facts of addition.

Now try another problem. Find four numbers to list in the four cells inside the double lines—numbers that will not work in the same way. Try two examples:





Do you find that this problem is easier? Or is it more difficult?

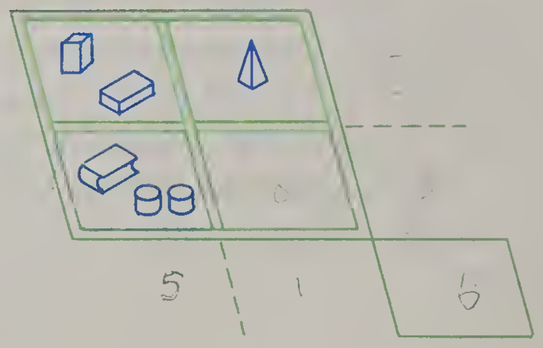
When children tackle this problem they sometimes claim their numbers won't work. But when the arithmetic involved is checked, mistakes turn up—and the examples work after all.

Some children jump to the conclusion that it's impossible to find exceptions. Most will extend the search, trying the largest numbers they can handle. But the search will not yield an exception.

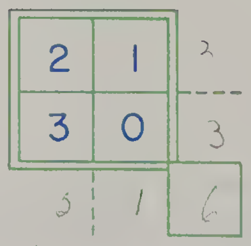
This certainly will be a surprising result to most children (and adults). Yet all children who have at least a few facts of addition at their command can discover this principle.

An understanding of the principle can be illustrated in an activity designed for children who are just learning to count and to record their results.

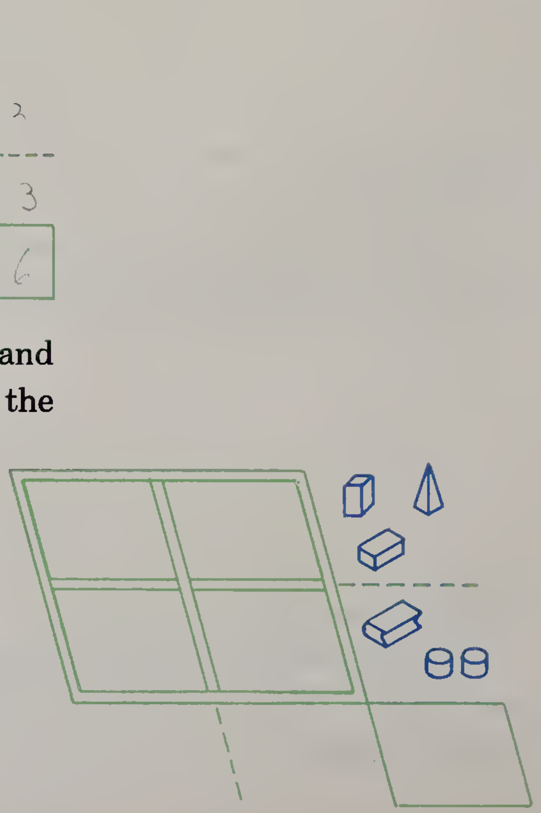
Suppose we create a small model on a table with masking tape—or on the floor of the classroom. We place objects in the four boxes:



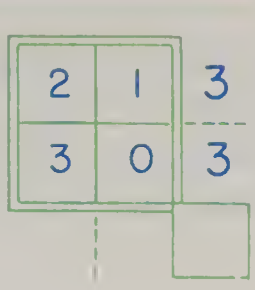
On the chalkboard a record is made for later use:



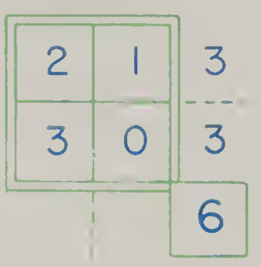
We now sweep the objects in the top row of boxes to the right and count them. There are 3 (2 plus 1). Next we sweep the objects in the bottom row to the right and count them. There are 3 (3 plus 0).



Include these results in the record on the board:

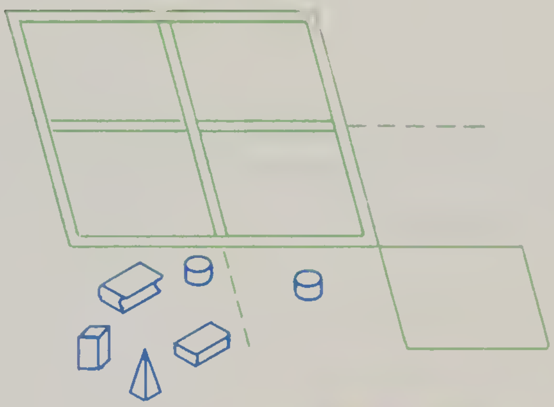


Now sweep the objects into the box in the lower right-hand corner and count them. There are 6 (3 plus 3). Include this in the record:

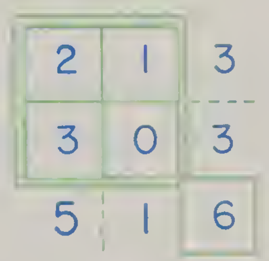


Can we put the objects back into the original four boxes so that each has the same number of objects as it had in the beginning? We have an adequate record on the board. It does not tell us which objects were in each box, but it does tell us the number of objects—and we make it plain that we are interested only in the number of objects.

After the objects have been replaced so that they conform with the original record, we sweep the objects in the boxes on the left down to the bottom and count them. There are 5 (2 plus 3). Similarly, we sweep the other object down and count it. There is 1 (1 plus 0).



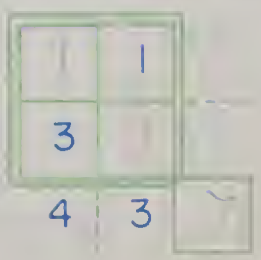
Include these results in the record on the board:



We sweep all the objects into the bottom right-hand corner and count them. There are 6 (5 plus 1).

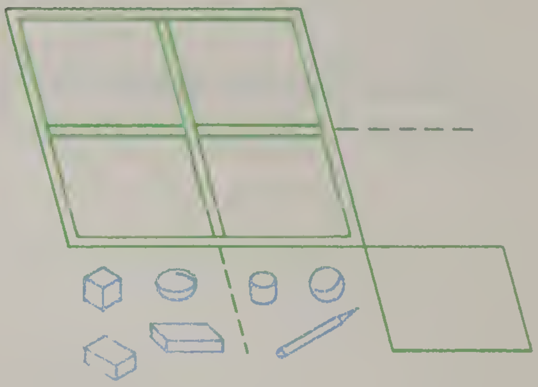
We find that we do not need to record this result, since we already noted that 6 objects were in that corner when we swept them there by a different route. Are you surprised? Certainly not. In fact, if any other result had occurred, you would be understandably disturbed.

Reconstructing the puzzle. Now suppose we start this activity a second time by putting a partly completed record on the board:



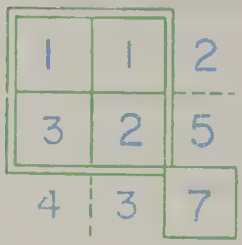
Can we find out the number of objects that were originally placed in each of the four main squares?

If we place four objects in the space beneath the two boxes on the left, and three objects in the space beneath the other two boxes, the result might look like this:

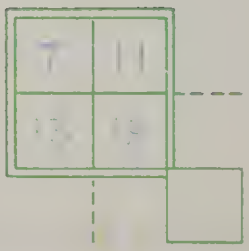


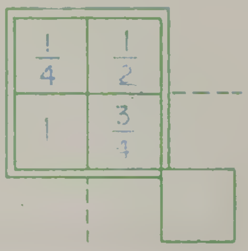
Since the record tells us that 3 of the 4 objects on the left were in one of the boxes, place three of them there—and there is one left for the other box on the left.

In a similar fashion, we can think and work our way through to a completion of the record. It would look like this when completed:

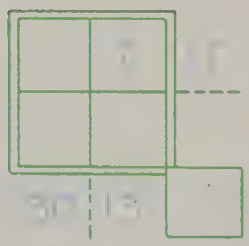


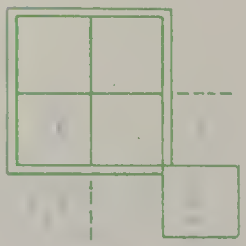
Although the simple activity just described is for beginners, the underlying principle is the same when the initiated are faced with such Cross Number Puzzles as:





Similar but more sophisticated reasoning is required for reconstructing the full record from such fragments as:





The Cross Number Puzzle requires considerable practice of the basic facts of arithmetic; it produces surprises; it helps to build a conviction that the outcome in addition is not influenced by the order or the grouping of the addends.

These exercises will introduce the idea that addition is *commutative* and *associative*. The commutative principle is indicated by the algebraic sentence: $a + b = b + a$. The associative principle is indicated by the algebraic sentence: $(a + b) + c = a + (b + c)$. Later this idea is extended to include multiplication. The idea is important; the names of the principles can be given at the time the pupil is ready.

The Cross Number Puzzle activity appears in various forms in MATH WORKSHOP at every grade level. This activity does at least three things for pupils:

1. *Motivation:* There is an element of puzzle and surprise which children find attractive.

2. *Drill:* Completion of a puzzle requires many acts of recall in addition, subtraction, multiplication, or division. This drill is effective because of the high degree of motivation behind it. Children are using their knowledge of basic combinations as they do mathematical research.

3. *Development of Algorisms:* Algorisms are the formal procedures we learn to follow automatically in doing such things as column addition, long multiplication, and long division. The Cross Number Puzzle activity furnishes us with many opportunities to develop an understanding of the principles which are the bases for these procedures.

Problem Solving

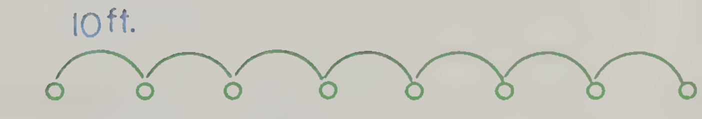
Story Problems. Over the years, "story problems" or "word problems" have been a source of vexation to many teachers of arithmetic, to many children—and to many parents. This activity, which properly seeks to relate the abstractions of mathematics to the world of events, often ends without a sense of accomplishment for anyone.

Story or word problems are usually reserved for the end of a development, as a sort of test or exercise of the pupil's ability to apply an acquired knowledge of arithmetic.

Examples vs. problems. A critical analysis of the questions usually asked in traditional arithmetic programs reveals that they constitute primarily a test of a child's reading ability, and that if the child can read, the questions are trivial and constitute *examples* rather than *problems.*

Example: Mr. Jones bought 8 fence posts. He paid $10 apiece. What was his total expense? (If you can read, this constitutes an example of the fact that $8 \times 10 = 80$.)

Problem: Mr. Jones bought 8 fence posts. He set them out in a straight line, 10 feet apart. What is the distance from the center of one end post to the center of the other end post? (If you can read, this is a problem that requires careful consideration of the situation presented; perhaps a sketch to show that there are 7 spaces between posts.)



The first situation is an example; the second is more of a problem. If a child clears the reading hurdle, then the example taxes little more than his memory, while the problem depends on a less-than-obvious question that makes greater demands on the mind.

Examples and problems play different roles in helping children understand mathematics—and both are important.

Headline stories. When we notice a pattern repeating itself in the limited world around us—the world we can manipulate—we often use an expression in the language of arithmetic that points to that pattern, such as:

$2 + 3 = 5$

At this point we ought to direct children's attention to an application of this idea to the wide world of fact and fancy that they live in.

"Suppose this is a headline."

$\boxed{2 + 3 = 5}$

"Tell a story that might fit such a title or headline," the teacher urges.

"There are five children in our family, two boys and three girls."

"Fine. Who has another story that would fit?" the teacher asks.

"I have three cookies out of the five I started with. I ate two of them."

"It's the second of December and my birthday is on the fifth. My birthday is in three days."

"On my hand I have a thumb and a little finger. There are three fingers in between."

"Out of my allowance of five cents I have two cents left."

"Three hours from now will be five o'clock." (Check the clock to determine whether or not this is a true sentence.)

Each of these situations has within it the conditions we point to in the headline. Each of the story-tellers reveals that he is beginning to grasp the relationship between arithmetic and the real world.

Whenever we abstract a bit of mathematics out of the world at hand and write it down in the language of arithmetic, good classroom technique should include the next logical step—to find examples from the world that reflect this idea.

A handy device is the Headline Story. Box in the mathematical expression and ask for fitting stories. Here are some early examples of such headlines:

$\boxed{1}$

"Who can tell me a story about this headline?"

"I have *one* bicycle."

"I see *one* clock."

"Here is *one* finger up."

"I am *one* girl."

"Last night the good fairy left me *one* quarter."

$\boxed{2}$

"Can you think of a headline story for two?"

"We all have *two* eyes (ears, arms, etc.)."

"My bicycle has *two* wheels."

"My mother and father are *two* persons."

"Soon it will be *two* o'clock."

Having pupils tell stories about a number is a very important activity since it accomplishes many important purposes:

1. It helps pupils to relate arithmetic continually to the real world from which it came.
2. It permits pupils to see the tremendous power of a number in representing an unlimited number of experiences involving things that can be matched with fingers or with points on the number line.
3. It provides for individual differences both in the quantity and quality of responses. More able pupils will think of more interesting sentences and stories suggested by the numbers as well as more examples.
4. It develops the language power of children.

You might write each headline story on the board after the name of the pupil who told it. Ask each pupil to read his own story.

Headline stories will elicit such ideas as:

For 3: wheels on a tricycle; knife, fork, spoon; 2 brothers and 1 sister (many stories which have the idea of addition), etc.

For 4: wheels on a car; my hands and feet (addition again); number of people in my family; legs on a chair, etc.

For 5: fingers on one hand; toes on one foot; a nickel, etc.

When pupils are fluent with one-number headline stories, ask them to tell stories about headlines with two numbers:

2, 3

"I have two brothers and three sisters."

"A bicycle has two wheels and a tricycle has three wheels."

"In two days, my dad will give me three cents."

When pupils are ready, we might introduce the headline that has the idea of addition:

2, 3   5

"I saved two cents and I found three cents. Now I have five cents."

"We have two boys and three girls in our family. That's five children."

"Here are two fingers and three fingers—five in all."

Further examples of headlines in different forms include:

"Tell a story with 1 in it."

"Tell a story suggested by the picture ☆ ☆ ☆ ."

"Tell a story including the idea that 5 is more than 3."

"Complete the headline and tell a story that fits ☐ + 4 = 7 ."

"Harry had one more book than Ruth. Complete the story."

"There were 5 children in a family. If ____ are boys, then ____ are girls."

"Can you have exactly 23¢ with 5 coins? 6 coins? 14 coins?"

On every page throughout MATH WORKSHOP, problem-solving experiences are presented. This includes headline stories, problems in the real world, and problems in the realm of pure mathematics. Every teacher's guide page includes a "headline story of the day." This is a signal that at least part of the discussion in each lesson should remind pupils that mankind has developed the language of mathematics to help report events that happen in the world of things.

Teaching for Discovery

Each page in MATH WORKSHOP and the activities suggested in the guide pages are designed so that there are several levels on which children can participate—including opportunities for success for even the slowest learners.

The activities we present are truly open ended, with challenge for even the most advanced and able children in the class. While some questions will have been settled, more will have been left open. Some children will want to explore these open questions under their own steam. Such children ought to learn to work independently, sparing the teacher's time for those who need it more. Actually, what the more able students need most is encouragement and opportunity to develop a taste for independent study.

The highest level of learning has been reached when a child begins to pose questions of his own and to work toward their solution.

There is in MATH WORKSHOP provision for introducing or practicing the basic facts of arithmetic. Usually such facts must be recalled and organized so that a pattern emerges. As patterns appear, they provide answers to less-than-obvious questions.

Discovering patterns is one of the most rewarding experiences in mathematics. It is called the "Aha!" or the "Gee-Whiz!" experience—a gasp; the light dawns! "I see it!" These are events teachers bend their best efforts to help bring about. One of the most disheartening facts teachers must face is that some children simply "don't see it." Yet this is also a challenge—it calls for exercising skill in teaching. Slow learners need more clues—more obvious clues; but the finest teacher will reserve a wee bit for each child to "see" for himself. If the teacher's best efforts fail, he may have to give away part of the secret and try again next time. This is an art that does not come easily, but the eventual payoff is deeply rewarding.

When the results of work in arithmetic exhibit an unusual regularity, the inquiring mind looks for an explanation. One problem may be solved but others are suggested. This is an exciting characteristic of mathematics.

When children discover a relationship, they are excited and anxious to stay with that discovery. They will eagerly put this new idea to work, showing more examples of *it*. They come alive, often with an "Oo-o-o! I've got it!" or with a burst of industry using a newly-discovered relationship to reel off answers with speed and confidence.

What is the *it* they have glimpsed? Well, if we distract their attention with a request for them to *tell about it* or *put it in words*, we have selected a method that could not be better calculated to mutilate the experience.

That a child can move ahead, putting *it* to work successfully is certainly all the evidence we need in order to know that he *has it*. Our only justification for veering off in the direction of a word-pulling exercise is a feeling that *it* is a generalization that ought to be verbalized in order to be usable. But research does not support this as a valid reason.

Transfer to future experiences is not increased by attempting to *verbalize a generalization*. Sub-vocal or nonverbal awareness, the new-found ability to move ahead more speedily, more directly, more confidently, is our main goal.

When children experience an insight—when they leap ahead because they have discovered a pattern—we ought to be very careful about letting words into the act too soon. The children probably do not have the language needed to express their discovery in words. If they try, we must, of course, help them. But too often such help and explanation degenerates into hair-splitting, an activity far removed from the act of discovery.

Gertrude Hendrix of the University of Illinois Committee on School Mathematics, who has done much research on discovery in the teaching of mathematics, warns that imprecise verbalization may "mutilate" discovery.

On the other hand, when some children (and teachers) make discoveries, they want others to know about it. But herein lies a danger. Whenever a person reveals his secret to one who has not been as fortunate, the latter is deprived of the delight of discovering it for himself.

So we invent ways in which children can share their discovery without revealing the secret too widely. We have them whisper their new idea in the teacher's ear; we encourage them to make up additional examples which will provide further clues for those who haven't found a pattern that underlies several examples. We name the new idea after its discoverer—Bill's Rule or Mary's Law or John's Pattern. But we keep urging children to keep the secret—for a while at least.

III. REPRESENTATIONAL MATERIALS

When mathematics loses contact with the world of things and with the events out of which it came, the subject becomes mysterious to most children. If they do not understand that we are discussing the way things happen as we manipulate objects, they must resort to rote memory.

MATH WORKSHOP activities stress the fact that arithmetic provides a means of recording or reporting what happens in the physical world.

Many kinds of manipulative or representational materials are suggested and discussed in this Teacher's Guide. These include various types of counting materials, geometric shapes, Cuisenaire© Rods, number lines, etc. See section G on page 22 of the introduction.

Perhaps the most accessible source of representational materials are the pupil's pages themselves. We believe that all of elementary mathematics can be abstracted out of experiences with the physical world. The very word WORKSHOP in the title of our program indicates our belief in the value of learning mathematics by working with physical objects. In some cases, the pupil's pages are the workshop—pupils work with the two-dimensional pictorial representations of physical objects and apparatus (Cross Number Puzzles, number lines, colored strips) and use the forms provided to keep records. Frequently the teacher will decide that pupils need to work with three-dimensional equipment. In those cases, the teacher may provide equipment just like that pictured on the pupil's pages or like that suggested in the guide pages.

IV. MASTERY OF BASIC FACTS AND ALGORISMS

Mathematics provides a language to help report events that happen in the world of things. Counting is at its core. Because this process becomes tedious when many things must be counted, shortcuts to counting have been invented and improved. These shortcuts depend on two characteristics—memory and patterns. Thus, elementary school mathematics is essentially concerned with counting and with shortcuts to counting.

At first glance, MATH WORKSHOP activities may appear to be highly motivated exercises to provide practice in the basic facts and combinations and processes of arithmetic. In the vernacular, they wear the garb of "sugar-coated drill." Actually, they are valid educational devices, because they help establish that automatic recall of "fundamentals" without which children are mathematical stutterers.

Every teacher knows that many children are mathematical stutterers because they can't easily recall the basic facts. They hesitate and flounder; the discussion bogs down; they lose the argument; they miss the feeling of movement toward a solution; they cannot enjoy mathematics.

This area of trouble is so universally known that it is surprising so many are only casually concerned! There is much hue and cry to bring new and unusual content into the elementary school curriculum—numeration in bases other than ten, sets, properties of numbers, and other concepts—yet there seems to be silence about such a crucial matter as helping children master the fundamental combinations without which mastery their progress in understanding mathematical ideas is very painful.

While we have emphasized the scope of our program in terms of nine unifying ideas, *the basic facts of arithmetic and algorisms are taught and retaught throughout the program.*

V. THREE WAYS TO USE PUPIL'S PAGES

There are at least three generally successful ways in which individual pages or groups of pages can be used in the classroom:

1) Use the page to conclude a development.
2) Use the page as the basis for discussion.
3) Let children work independently.

1) In those areas in which the teacher has many ideas of his own, there is no substitute for direct teaching—with liberal use of manipulative material, the blackboard, and class discussion. The page is then used late in the development to tie the idea down, or simply to record the progress that has been made toward deeper understanding.

There are limitless ways to initiate lively activities around important and useful concepts. The MATH WORKSHOP provides many such activities for you to choose from.

After handling manipulative materials, playing number games, and other such preliminary work, pupils are ready to move through the page in the final few minutes of the arithmetic period.

2) In contrast, some pages might be gone through very carefully with an entire class. Such a page or sequence of pages offers many opportunities to dig deeply into "What's going on here?"

3) A third approach might be called *Do-it-all-by-yourself.* Begin by saying, "Open your book to (name the page). Look it over. Think about it. See if you can decide for yourself what is to be done. Wait a while before you ask for help. And let's make today a 'quiet day.' "

Often, the first question the page poses to children is, "What am I supposed to do?" There are almost always sufficient clues provided so that, with a bit of study, children can discover the intent of the exercises. Traditional materials have provided too little incentive for children to "figure out for themselves" the nature of the business at hand.

Now you feign reluctance to help but are, of course, watching carefully for those who haven't sufficient self-confidence to get started on their own. If several children are unable to get started, suggest that they work in pairs, helping each other quietly. Enlarge these helper combinations if necessary. Grouping like this will usually get the activity under way for everyone. Let time run out on the session if you wish, suggesting that those who haven't finished may want to take it home with them to finish.

Although every page of MATH WORKSHOP involves problem-solving situations, pages 50, 51, 60, 61, 70, 71, 80, 81, 90, 91, 100, and 101 are of particular interest to teachers. The authors designed these pages so that they can serve as multipurpose tests. You will note on the answer pages that each contains 10 test items singled out by red dots.

A careful analysis of each test page will reveal the following features:

a) several items are useful for testing achievement of mastery of basic computational skills in addition and subtraction;
b) several items are useful for detecting those pupils who need remedial help;
c) several power-test items are useful for detecting those talented pupils who need more varied enrichment experiences.

"The Lesson" on the teacher's page accompanying each test page contains many suggestions as to how teachers can obtain maximum mileage from the tests.

VI. TEACHING AIDS

A. In the Teacher's Guide

1. The Introduction presents a point of view and a rationale for our approach to arithmetic. You will find this useful in interpreting many of the things we have included in the pupil's pages. You will also find it useful in explaining the program to parents.

2. Each pupil's page is reproduced in the Teacher's Guide in full-size and in full-color with answers printed in red.

3. Every guide page presents a mathematical headline so that pupils may create stories of their own of real-world events or imaginative stories that have within them the same conditions expressed in the headline. (See the section *Headline Stories* on pages 13-15.)

4. Each pupil's page is accompanied by a guide page which is divided into several sections. Let's consider each of these sections:

Tasks. This section contains a very brief description of what the pupil will be doing as he works on the pupil's page. We suggest that you study the pupil's page in a pupil's book and work on it before you read our description and the answers we give.

Purpose. Here we describe what we think the pupil will learn as he works on the pupil's page or as he carries out some of the activities described in The Lesson. (Naturally, our description contains a conservative estimate of what might happen. Mathematics is full of surprises and so are pupils.) If you are aware of our purpose, you will find it easier to help students achieve it.

Unifying Ideas. In this section we list those of the nine unifying ideas mentioned on page 18 to which the exercises on the pupil's page can be expected to contribute. Please regard the list as a minimum statement, and be prepared for pupils who may wish to go beyond what we have described for the page.

Materials for Teacher. This is a description of the equipment you would need if you followed all of the suggestions made in The Lesson. Some teachers will be able to look at this description and then plan their own lessons without reading further.

Materials for Pupils. This section describes the equipment needed by individual pupils in carrying out the tasks or in pursuing the activities suggested in the lesson.

The Lesson. In this section we describe one or more of the lessons we might teach after studying the pupil's page and reading the sections *Purpose* and *Unifying Ideas.*

Practically all of our lessons would involve representational materials. We might decide to use a workshop and laboratory approach using the equipment directly suggested by the pupil's page (see section III on page 17). In that case, the pupil's page would be used as review material.

We might use equipment quite unlike that suggested by the pupil's page, but in keeping with the purpose and unifying ideas. In some cases, the pupil's page might be worked on at the beginning of the lesson, so that the laboratory would then provide an application of what was learned on the pupil's page. In other cases, we might reverse the order and make the pupil's page the application of what was learned in the laboratory.

We would assign some pupil's pages to be worked on with very little explanation from us (see section V on page 18). For other pages, we would do lots of question-asking. Hopefully, we would find some means of projecting such pages on the chalkboard so that we could talk about them easily (see section B, below).

No teacher could possibly find the time to use all our suggestions in any school year.

Choose the ones which best suit your style of teaching, and vary your choices from year to year.

B. Supplementary Activities

A package of supplementary activity worksheets is available at every level, A through F. Each package includes 32 copies of 64 different practice pages. These can be used independently by pupils for seatwork and for homework. Able pupils should be given supplementary activity material at advanced levels. Each set of supplementary activities includes a Teacher's Guide with brief suggestions for each page and an answer key.

C. Supplementary Work Forms

Many of the interesting and versatile page-forms developed in MATH WORKSHOP texts have been extracted and made available as Ditto and Multilith masters called *Work Forms.* In a few minutes, a teacher can convert one of these forms with pen or pencil into a rich supplementary activity for the whole class or for a group within the class. Providing, as they do, significant independent learning activities for creative seatwork and meaningful homework, and taking into consideration, as they do, the wide range of individual abilities within any class, the *Work Forms* should prove very useful to busy teachers.

The Supplementary Work Forms Manual itself suggests many ways in which these forms can be used to help develop important ideas (and, at the same time, provide more drill in the fundamental facts). Teachers can create similar activities by adding whatever numbers they want in a matter of minutes.

Countless other uses for the *Work Forms* can be found in the pupil's texts and the teacher's guides to the other levels of the MATH WORKSHOP. The ideas in the particular adaptations found there are easily applied to any level of development.

The *Work Forms* provide the teacher with a flexibility and versatility never before available in an organized and integrated program. While they are not essential to the use of the MATH WORKSHOP texts, they are an invaluable supplement, an effective means of meeting individual needs, and a help to every teacher in expressing his own individual approach to helping children understand arithmetic.

D. Mastery Tests of the 390 Basic Facts of Arithmetic

These tests are contained in a booklet including a battery of six tests, each of which presents all the basic facts of addition, subtraction, multiplication, and division. An answer sheet is provided so that children can check their own or their neighbor's work. In addition, an individual record sheet enables each child to chart his own progress in terms of accuracy and time required to recall all the basic facts.

E. "Discovery in Elementary School Mathematics"

This is a professional development textbook which suggests to teachers many ideas for teaching mathematics at every grade level. This book is divided into two major sections—Multilevel Research Activities and Multilevel Work-Sheet Activities.

Multilevel Research Activities present 27 exploratory experiences which can be introduced on the chalkboard and with simple, teacher-made manipulative materials at any grade level. Each activity starts simply and spirals upward with extensions and elaborations which will appeal to the more mature and sophisticated mind. Each activity focuses on mathematical structure in a learning context of discovery.

Multilevel Work-Sheet Activities present 24 units, each of which explores some of the unifying ideas of mathematics. Each unit begins with a brief introductory statement regarding mathematical structure and discovery. Each unit includes six activity work-sheets, scaled in difficulty so that the teacher can present the ideas discussed in the unit to pupils at any level. These pages may be copied or modified by teachers for classroom use.

F. "Helping Children Discover Arithmetic"

This is a brief guide to help parents understand the ideas behind MATH WORKSHOP. Specifically, it helps parents see:

1. How we can get children to think creatively in arithmetic by posing challenging questions.
2. How we help children master the basic facts of arithmetic.
3. How parents can help children learn arithmetic.
4. How multiplication, one of the central ideas of arithmetic, is taught from the first through sixth grades in a spiral curriculum.

G. Games and Enrichment Activities

We have prepared a manual on the use of representational materials, such as Cuisenaire® Rods and Geo-Boards. These materials can be used as the basis for games and other enrichment activities—many of which are mentioned or introduced on the guide pages. They contain many unusual applications of the concepts considered in the pupil's pages.

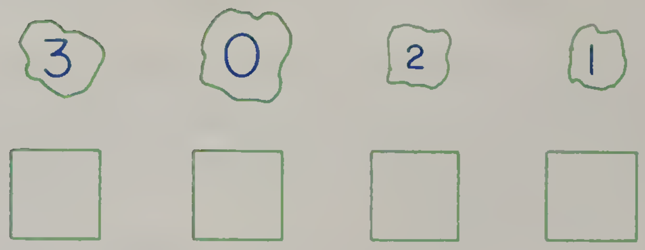
VII. SOME FINAL THOUGHTS

A. Number or Numeral—What Distinctions Shall We Make?

Suppose that you were conducting the following activity with your class and the class was being observed by a visitor who knows nothing about numbers:

"Let's cut out numbers and paste them on a piece of paper in some sensible order. In what order shall we paste them?"

"Suppose that these are my cut-out numbers: (Put a sketch on the board.)



How should I paste them down?"

A discussion may ensue. "Let's put the smallest number on the left. Which one is that? Zero, of course. Then, the one, the two, and the three."

The pupils then work alone or in groups, cutting out the various shapes and pasting them on a sheet of paper with the smallest number on the left.

The visitor would be completely mystified by the pupils' paste-ups. Surely, the pupils must be mistaken, because the smallest cut-out is the 2, not the 0.

The children are not confused. When you tell these children to draw a tree, they know that you expect them to draw a picture of a tree. When you tell them to cut out numbers, they know that you expect them to cut out *symbols* for numbers. They know this because they have learned from activities involving sets of objects and one-to-one correspondence that numbers are abstract entities. They know that they can see 3 stars, they can eat 3 candy bars, and they can lift 3 books, but they can't see, feel, hear, smell, or taste the number 3. They can write a name for this number—the simplest name looks a little like a turned-around capital E. They can say this name; one of them has a cousin in France who makes a much different sound when he says his name for this number.

If children have a rich background of experience in handling physical objects and comparing sets of such objects, they will not confuse numbers with the symbols (called numerals) which are the names of the numbers. The reformers of secondary school mathematics education in the 1950's found it necessary to make students aware of the distinction between numbers and numerals because many of these students came out of elementary school with impoverished backgrounds in arithmetic. Such students were taught a brand of arithmetic which consisted largely of symbol-pushing. Big mathematical ideas for such students were rules about turning fractions upside down and moving decimal points. Whatever understanding they may have developed about the abstract existence of numbers through manipulation of physical objects in the first grade diminished severely in the later grades under an avalanche of symbol-pushing activities.

The authors of MATH WORKSHOP have decided to abide by common usage in which the word "number" has two meanings—the abstraction and the symbol. To help pupils build the kind of understanding which will enable them to pick the correct meaning from context, the authors urge a rich and varied series of activities which involve physical objects. (Of course, we also abide by common usage when we speak of Roman numerals and Arabic numerals.) Exercises of the "numbers have many names" variety also help in building an understanding of the distinction between numerals and numbers.

B. Achievement and Intelligence

Makers of I.Q. tests search for measures of the child's ability to think. They try to rule out questions that merely assess achievement in subject-matter areas. Therefore, they would not ask children to complete the following familiar examples from arithmetic. These questions would more likely appear in an achievement test.

$$\begin{array}{r} 3 \\ +7 \\ \hline \end{array} \qquad \begin{array}{r} 41 \\ -16 \\ \hline \end{array} \qquad \begin{array}{r} 346 \\ \times\ 27 \\ \hline \end{array} \qquad 5\overline{)135} \qquad 8 \times 3 =$$

$$\frac{1}{2} + \frac{2}{3} = \qquad\qquad 1.19 = 17\% \times \_\_$$

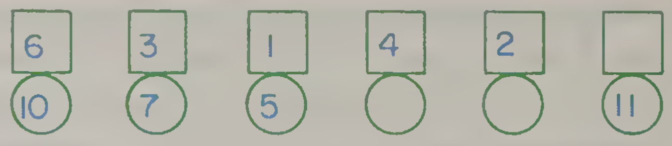
For measurement of an Intelligence Quotient the questions from mathematics would be more like those that follow:

I. Extend each of the following series:

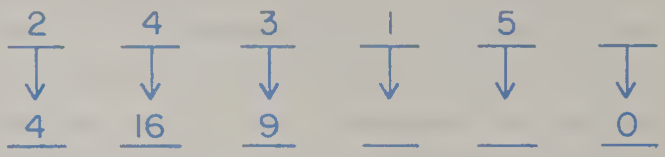
a. 2, 5, 8, 11, 14, __, __, __ . . . . .
b. 1, 2, 4, 8, 16, __, __, __ . . . . .
c. 3, 4, 6, 9, 13, __, __, __ . . . . .
d. 1, 1, 2, 3, 5, __, __, __ . . . . .

II. Fill in the blanks in the following examples:

a.



b.



c.

1 , 3 → 8
5 , 0 → 10
2 , 5 → 14

3 , 2 → 10
6 , 3 → ___
___ , 4 → 12

III. Circle the number in each row that seems most out of place:

a. 6 , 10 , 2 , 7 , 8 , 4
b. 25 , 70 , 43 , 16 , 31 , 52
c. 13 , 15 , 19 , 3 , 11 , 7

Such problems requiring a familiarity with arithmetic are held to be more appropriate as measures of the ability to think. They are designed to test intelligence rather than achievement.

Mathematics is a fertile source for I.Q. test questions because mathematics is fundamentally a study of patterns and relationships. Many of the questions selected are similar to the examples given above. Behind that choice is the test-maker's estimate that children have not already explored and investigated such questions.

What an indictment of the elementary school curriculum in math!

If arithmetic is limited to memory of number combinations and rules for manipulation, then it is a dead, dull, dreary task against which an active, eager, creative mind must surely rebel. However, if we broaden our thinking and look everywhere for patterns and relationships, the subject stimulates the mind to inquire and search out and discover!

Every page in MATH WORKSHOP probes beneath the basic facts and processes. We are always exploring the question: "Why does arithmetic work?" Each activity invites the child to ask: "What's going on here?"—and urges him to find out.

C. A Final Word

The MATH WORKSHOP has not invented a new mathematics! Nor have its authors spun novel theories of education. We have tried to bring into a whole the most universally accepted ideas of both educators and mathematicians.

Robert W. Wirtz
Morton Botel
Max Beberman
W. W. Sawyer

WORD LIST

Words To Help in Lesson Planning and Presentation

The words and phrases given below suggest concepts to be developed throughout the first two grades. You may find it useful to refer to this list as you prepare your daily lessons.

1. Structure

any other
equal
equals
greater than
is equal to
less than

2. Sets

braces
equivalent
has the same number as
match
one-to-one correspondence
picture
set (s)

3. Number and Counting

all
column (s)
counters
counting
diagonal
few
first
first through tenth
frame (s)
last
many
none
number
number line (s)
one-half
one-quarter
one-third
order
pattern (s)
placeholder (s)
record (s)
row (s)
some
table
tally
tally marks
zero through ten

4. Numeration

0-10
name
numeral
ones
tens

5. Addition and Subtraction

add
addend
difference
fact team
minus
missing addend
plus
subtract
sum
take away
undo

6. Multiplication and Division

away
compare
divide
division
factor
group
missing factor
multiplication
multiply
product

7. Functions and Relations

arrow
comma
goes to
pair of numbers

8. Measurement

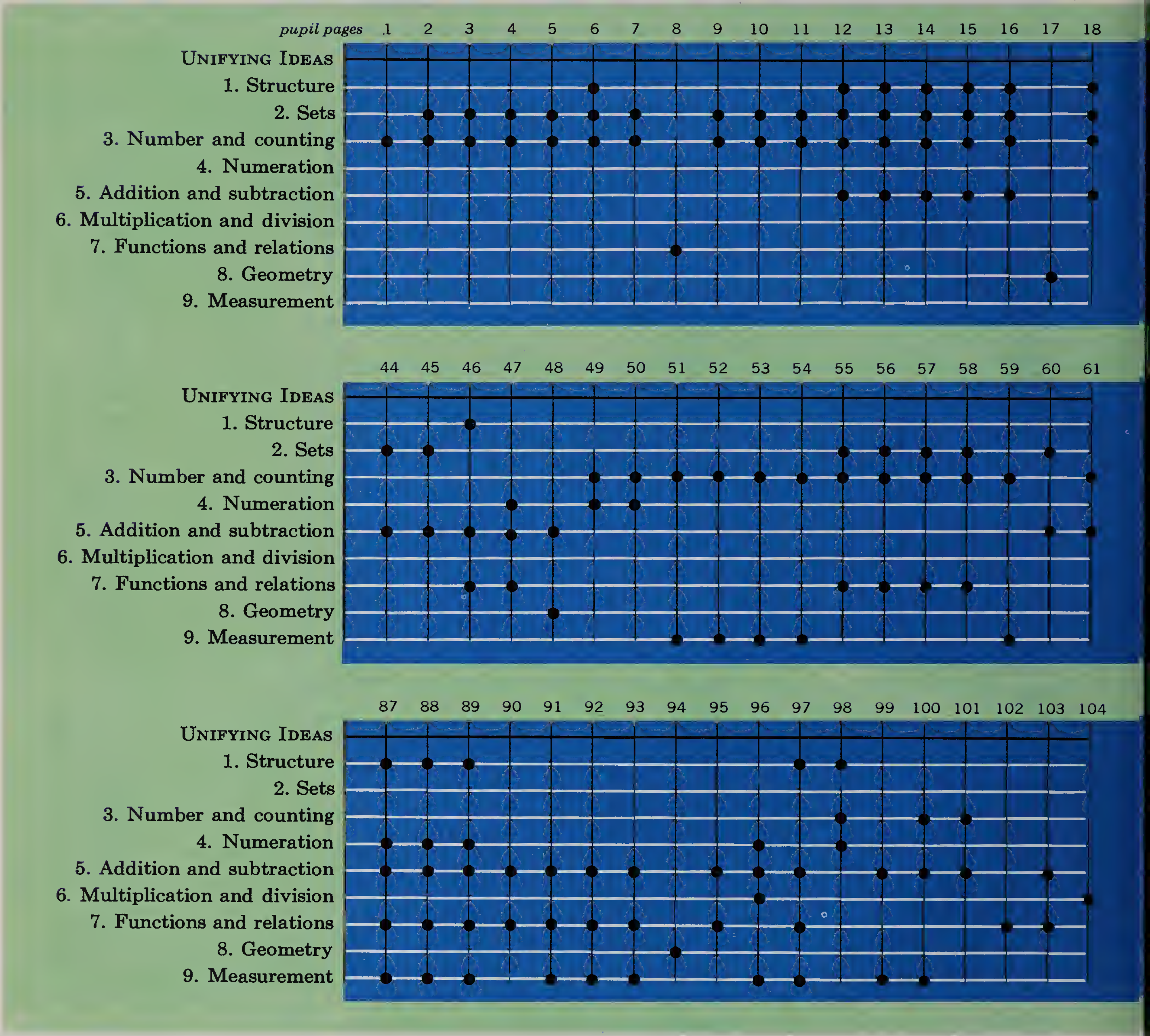
centimeter
day
decimeter
dime (s)
dozen
foot
hour
inch
inches
large (-er-est)
long (-er-est)
minute
money
nickel (s)
ounce (s)
pennies
penny
pound (s)
ruler
short (-er-est)
small (-er-est)
straightedge
time
week
weigh (s)
weight

9. Geometry

across
bar
behind
bottom
box
center
circle
cross
dot
down
end point (s)
exterior
height
in between
in front of
inside
interior
left
length
line (s)
middle
outside
over
point (s)
ray (s)
rectangle
rectangular
region (s)
right
round
segment (s)
shape
square (s)
top
triangle
triangular
under
up

SCOPE AND SEQUENCE CHART

Level A Revised



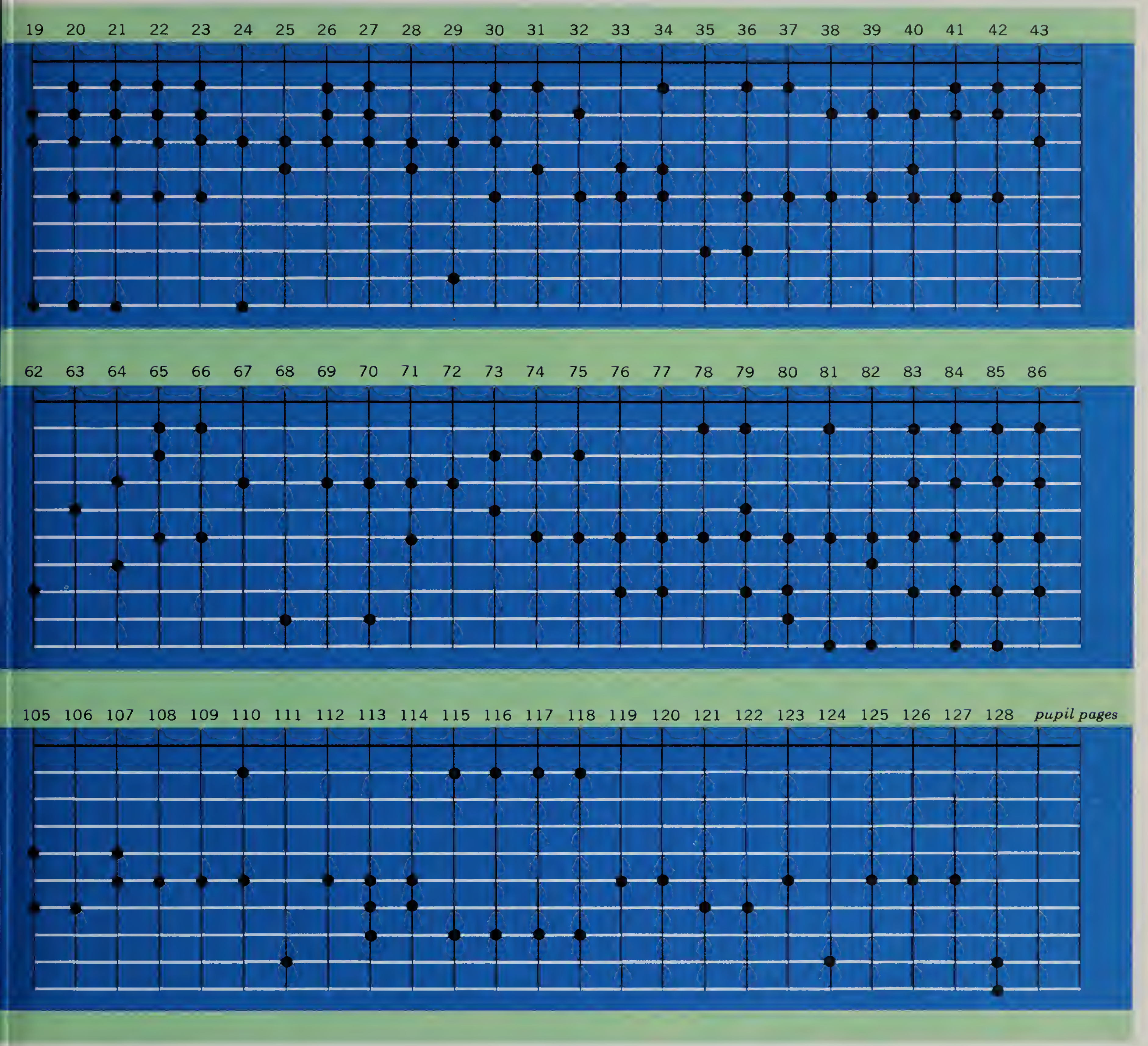
pupil pages	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
UNIFYING IDEAS																		
1. Structure						●						●	●	●	●	●		●
2. Sets		●	●	●	●	●	●		●	●	●	●	●	●	●	●		●
3. Number and counting	●	●	●	●	●	●	●		●	●	●	●	●	●	●	●		●
4. Numeration																		
5. Addition and subtraction												●	●	●	●	●		●
6. Multiplication and division																		
7. Functions and relations								●										
8. Geometry																	●	
9. Measurement																		

	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61
UNIFYING IDEAS																		
1. Structure			●															
2. Sets	●	●										●	●	●	●		●	
3. Number and counting						●	●	●	●	●	●	●	●	●	●	●		●
4. Numeration				●		●	●											
5. Addition and subtraction	●	●	●	●	●												●	●
6. Multiplication and division																		
7. Functions and relations			●	●								●	●	●	●			
8. Geometry					●													
9. Measurement								●	●	●	●					●		

	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104
UNIFYING IDEAS																		
1. Structure	●	●	●								●	●						
2. Sets																		
3. Number and counting												●		●	●			
4. Numeration	●	●	●							●		●						
5. Addition and subtraction	●	●	●	●	●	●	●		●	●	●		●	●	●		●	
6. Multiplication and division										●								●
7. Functions and relations	●	●	●	●	●	●	●		●		●					●	●	
8. Geometry								●										
9. Measurement	●	●	●		●	●	●			●	●		●	●				

Consider each unifying idea . . . on how many pages will you find it?

1. Structure	41 pages
2. Sets	40 pages
3. Number and counting	54 pages
4. Numeration	19 pages
5. Addition and subtraction	71 pages
6. Multiplication and division	10 pages
7. Functions and relations	34 pages
8. Geometry	10 pages
9. Measurement	24 pages



Consider each page . . . how many unifying ideas will you find on it?

27 pages contain just one unifying idea,
55 pages contain two unifying ideas,
25 pages contain three unifying ideas,
14 pages contain four unifying ideas, and
7 pages contain five unifying ideas.

Notes on Pupil Page 1

Tasks: *Pupils use different colors to color in the numbers (that is, number symbols) on the blue section of the page, then find matching numbers in the green section and use matching colors for them; both 0's the same color, both 6's the same color, and so on.*

Purpose: To give pupils practice in recognizing and writing numbers, and to enable the teacher to see which pupils are competent in these activities and which are not.

Unifying Ideas: Number and counting.

Materials for Teacher: Colored chalk.

Materials for Pupils: Crayons.

The Lesson:

It is essential that your pupils have had the experiences involved in the MATH WORKSHOP BEGINNER'S BOOK, or the equivalent, before they attempt the work done here. MATH WORKSHOP, LEVEL A, assumes that pupils have had experiences in counting. We also assume that they have had experience in reading and writing symbols for numbers and in associating these numbers with appropriate sets of objects. If your pupils have not had these experiences, you must provide them as they work in this book.

Page 1 is useful for diagnosis as well as for practice. Note which pupils are already reasonably competent in writing the numbers and which pupils will require a lot more help and extra practice.

In implementing the learning activities on this page and on all other pages in the pupils' books, USE ANY, ALL, OR NONE OF THE SUGGESTIONS found on the manual pages.

Numo and his sister Numa are fantasy characters invented for MATH WORKSHOP. Pupils have come to love these characters. They appear on the cover, and again on a number of the pages. You need to know something about Numo and Numa so that your pupils can talk about them, tell stories about them, and draw them.

1. They are from another world.
2. Numo has antennae instead of hair. When he gets an idea, his antennae say "ping-ping."
3. They are like other brothers and sisters—they love each other and sometimes they argue.
4. Numo loves mathematics. In fact, he wants to find out everything he can about numbers because some day he wants to be a "number explainer."

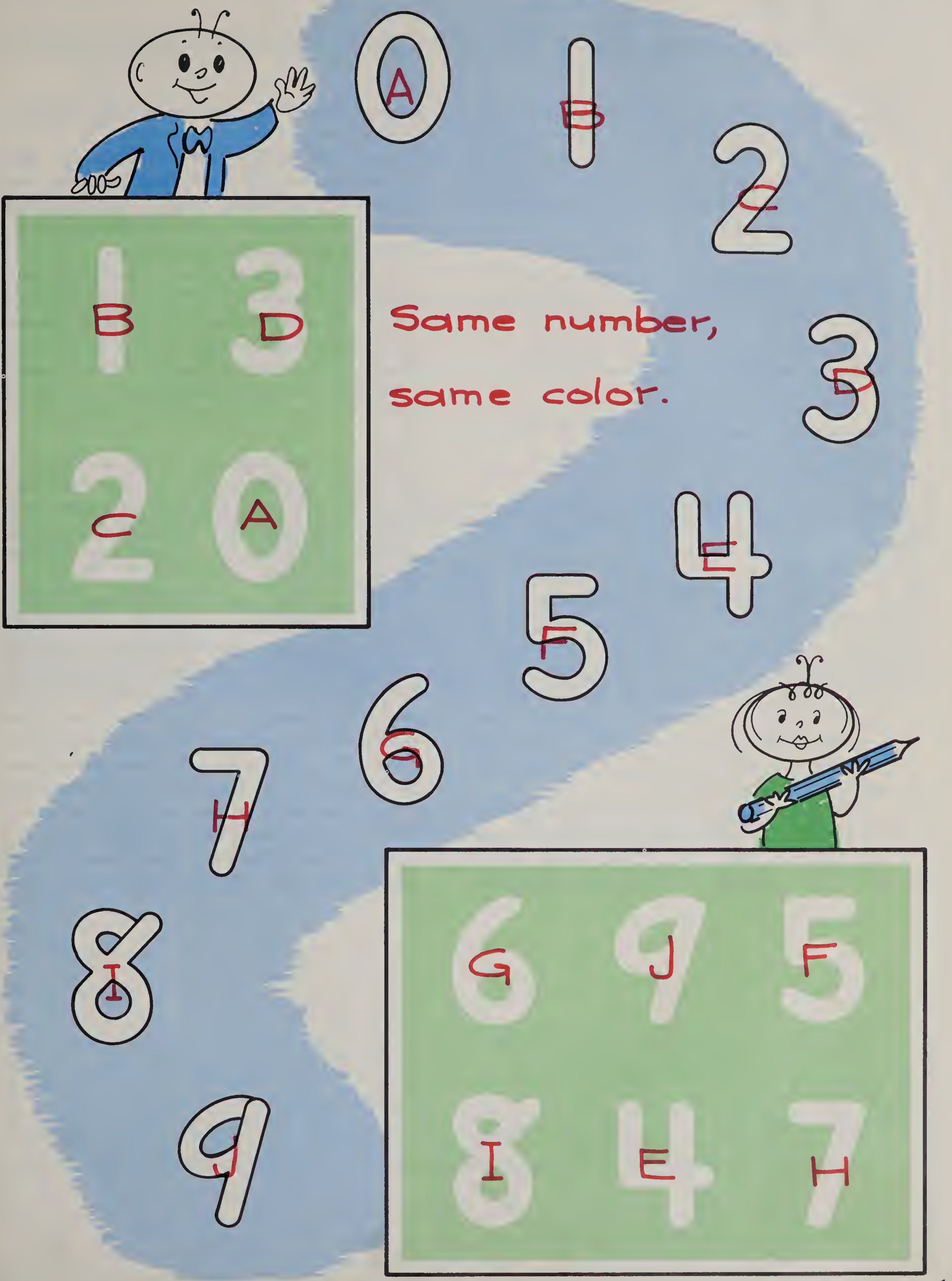
Employ some device to direct the pupils' attention to numbers, possibly that of discussing the calendar for the current month. Pupils might help you plan and make a large calendar on a poster or on the chalkboard. They could show how they would write the first several numbers in the cells.

Talk about sky writing. Have each pupil pretend he has a miniature airplane on the tip of his index finger. Have him show how he could make the little airplane write a 1, a 3, an 8, and so on. The whole class can do these exercises together, and you can watch to see that all pupils start at the top in writing the numbers and have an accurate idea of the form of each number.

Copy each of the ten digits in two places on the chalkboard. Can the pupils pick out those which are alike? A pupil might trace a selected number with colored chalk. Another pupil could trace the other copy of this number with the same color.

To provide extra practice in writing numbers, rule a Ditto master into appropriate rectangles. Write the numbers in any order you please down the left side. Have the pupils fill the boxes in each row with copies of the number in the first box in that row.

Headline Story: Tell a story with a number in it.


0 A
1 B
2 C
3 D
4 E
5 F
6 G
7 H
8 I
9 J
1 B
3 D
2 C
0 A
6 G
9 J
5 F
8 I
4 E
7 H

Same number,
same color.

Notes on Pupil Page 2

Tasks: *Pupils color the numbers on the tags at the top of the page and then use matching colors for matching numbers in the gray section.* (See the letter code on the answer page.) *They write appropriate numbers in the tags at the bottom of the page.*

Purpose: To give practice in recognizing and writing the digits 0-3; to give practice in simple counting.

Unifying Ideas: Sets; number and counting.

Materials for Teacher: Number tags like those pictured on page 2.

Materials for Pupils: Counters; crayons.

The Lesson:

A number tag ("N" for "Number") is used to tell how many things are in the box to which the tag is fastened. If the box is empty (use that word), the number on its tag should be 0. Never say that 0 is nothing; the cardinal number 0 is the number of things in an empty box. Treat 0 from the start as the thoroughly respectable number it is, make no fuss about it, and the pupils will have no difficulty with it.

Have the pupils get out their counters. Hold up one tag at a time. Ask the pupils to put this number of counters on their desks. Check to see that all pupils have used the correct number of counters. Some pupils may be able to go beyond 9. See how far they can go.

Pupils will employ different methods. Some will put counters back each time and start over. Others may add more counters or remove some counters to represent the next number you call for. Encourage the use of various methods.

Have the pupils work in teams. Supply the team leaders with number tags. The leader holds up a tag and the other members of the team represent the corresponding number with their counters. Members of the teams may take turns being leader.

Explain to the pupils what they are to do on page 2 of their books. Have them complete the page.

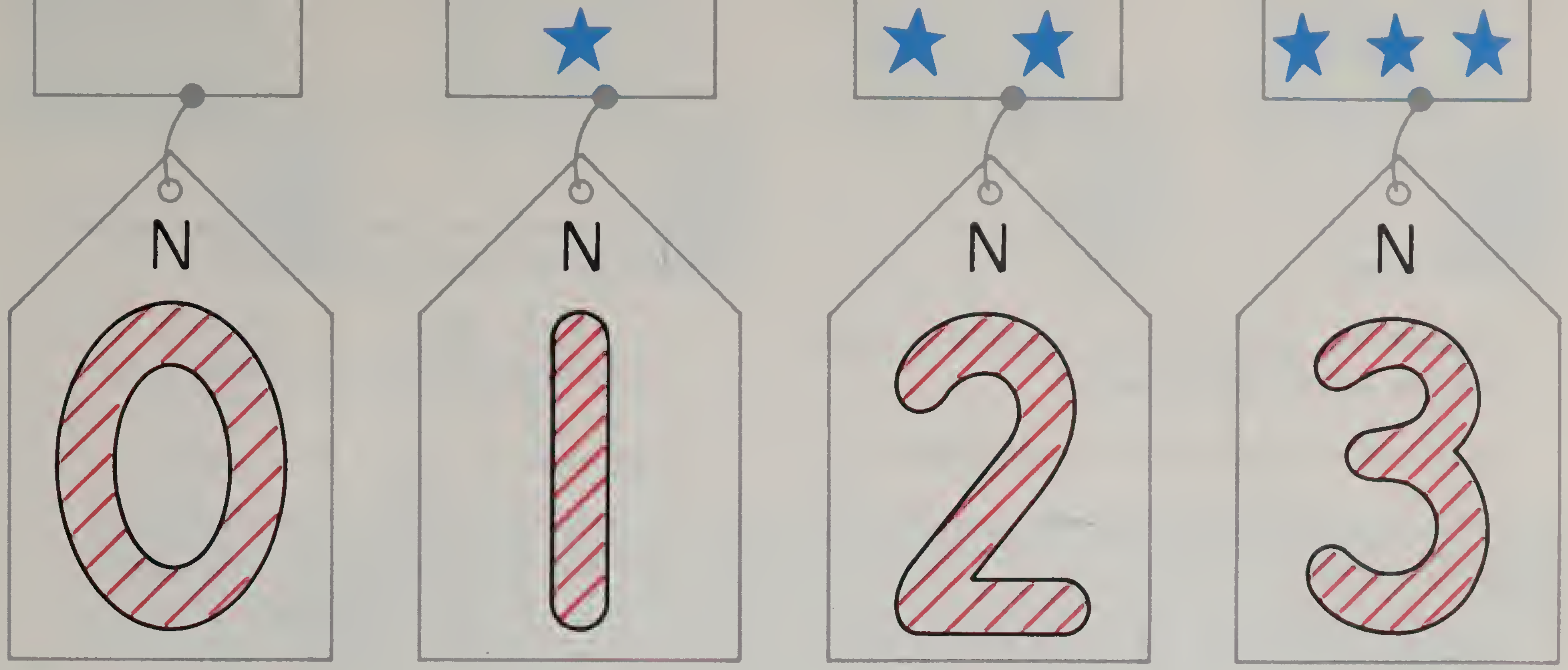
Note carefully what each pupil has written in the last four tags on the page. If a pupil drew some dots in the empty box and then wrote the correct number to go with them, commend him, and ask him what number he would have written if he had not drawn those dots.

Some pupils may have the correct idea of what the number should be but have trouble in writing it when there are no guide lines. Group such pupils for appropriate help. Worksheets in which the guide lines are gradually eliminated may help such pupils develop the desired skill. Or you may have pupils write numbers in the sand, or with finger paints.

Have the pupils look at their completed pages. Let them discuss anything they think is interesting. (Did anyone notice the symmetry in the gray section? It reads the same way from both directions.)

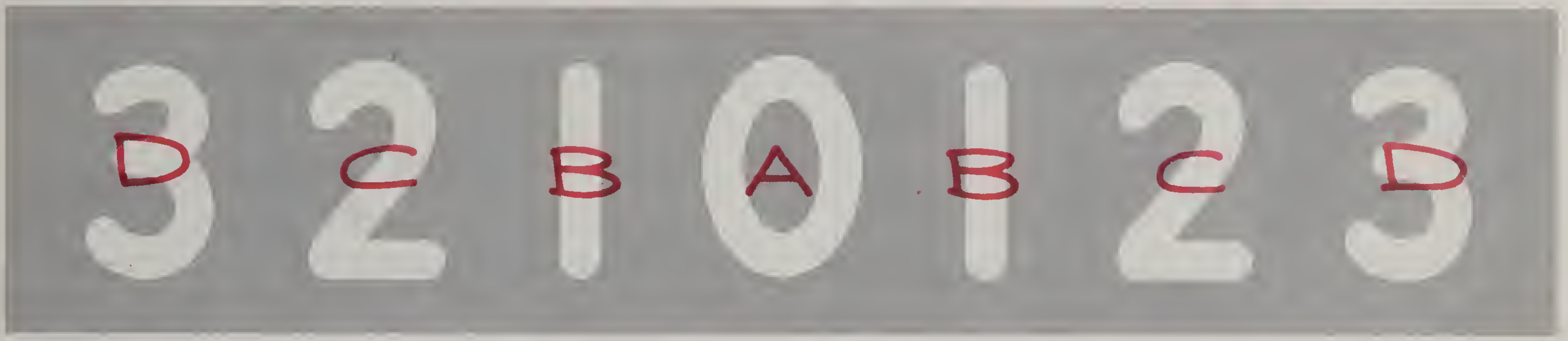
The material in the gray section presents a good opportunity for teaching the words *left* and *right*.

Headline Story: Tell a story with 1 in it.

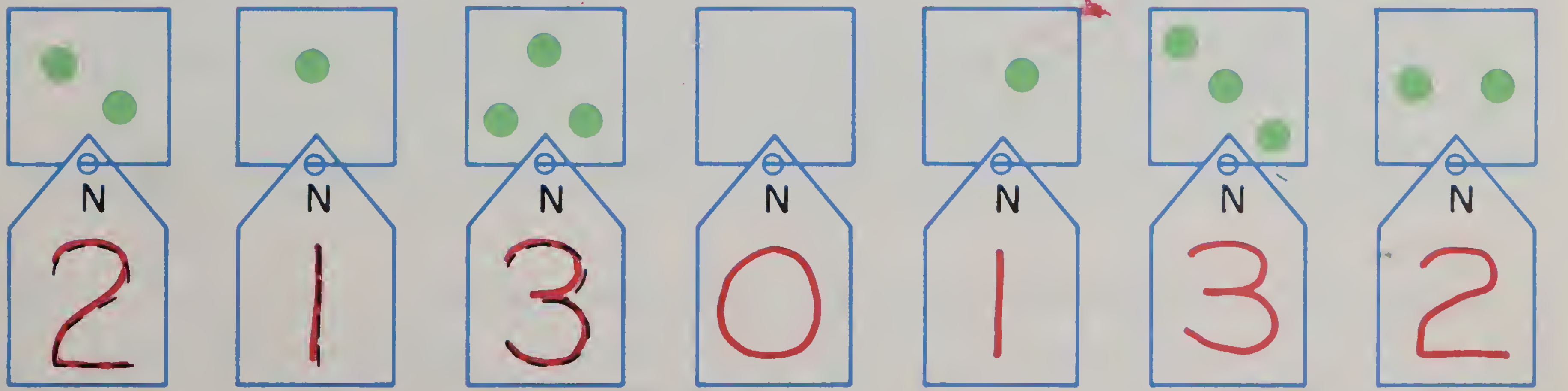


Color A B C D

Use same colors for numbers below.


3
D
2
C
1
B
0
A
1
B
2
C
3
D

3 2 1 0 2 0 3 2 1 0


N
2
N
1
N
3
N
0
N
1
N
3
N
2

Notes on Pupil Page 3

Tasks: *Pupils count the objects in a set and record the number.*

Purpose: To give additional practice in simple counting and in writing numbers.

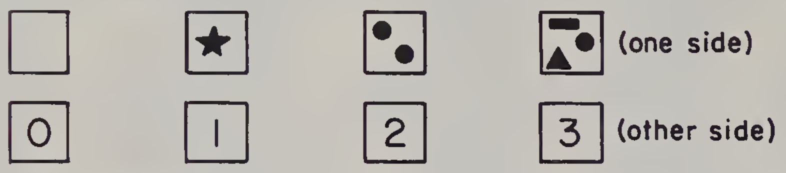
Unifying Ideas: Sets; number and counting.

Materials for Teacher: Two sets of cards, as pictured below.

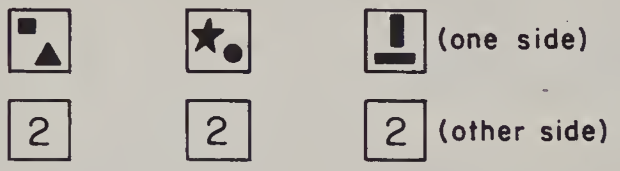
Materials for Pupils: Cards with the numbers 0-5; counters.

The Lesson:

You may find useful a set of cards that has numbers on one side and that many objects pictured on the other side.



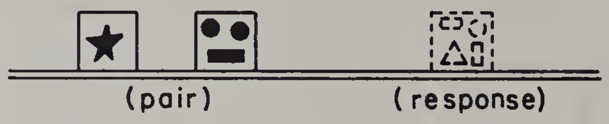
Preferably, you can make several cards for each number with different objects pictured on the other side.



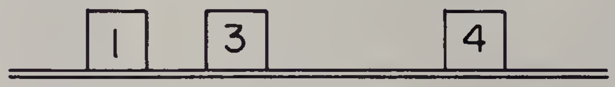
One use for these cards is to place several on the chalkrail in random order with the objects sides showing. A pupil selects a card, counts the objects shown—one, two, three, and turns it over, showing the 3. Each card is used similarly.

Then, "How many pictures on the back of this card?" . . . pointing, perhaps, to a 2-card. As soon as any response is given, turn the card over and count. This will help initiate the idea that ought to grow—"When in doubt, count."

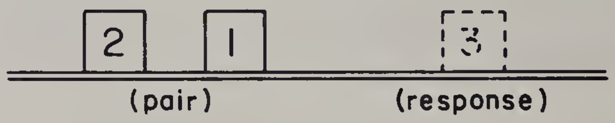
Later, these cards can be used in a way that anticipates addition. Display a pair of cards. "Can you find a single card that has the same number of pictures as the two together have?"



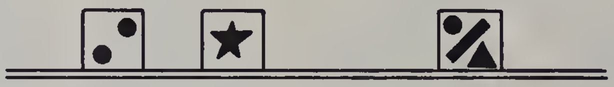
Now turn the cards over. The pupils see:



Later, begin with the numbers:



When the cards are turned over, the total number of objects for the pair taken together is to be the same as the total number of objects on the card selected.

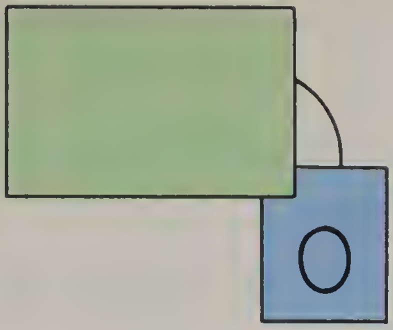


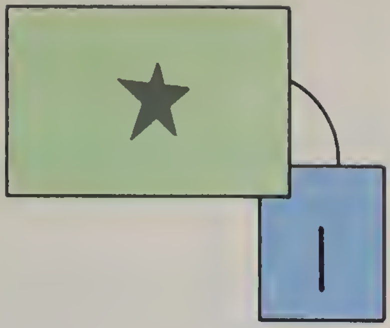
Further counting experiences involving senses of movement and sound include: (1) a version of *Simon Says*, in which pupils step forward or backward a certain number of steps, and (2) listening for how many taps of a pencil or notes struck on a piano.

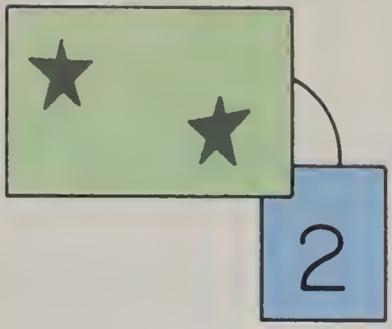
After the pupils have completed page 3, discuss it with them. Ask them whether there are any interesting things they can say about it. There are patterns in rows, columns, and diagonals on this page which some pupils may have discovered. You may wish to point out the Roman numerals which are used along the left margin, or to speak of the words written across the top of the big box.

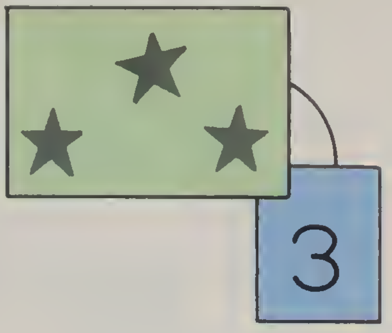
Also, you may initiate a sense of a two-dimensional coordinate system and check answers at the same time by asking questions such as, "What did you write in the box which is straight across from Roman numeral I and straight down from the letter *y*?" (You will find cells around the edges the easiest to work with.)

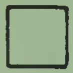
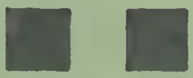
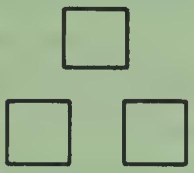
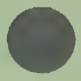
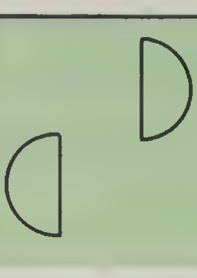
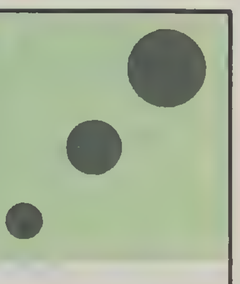
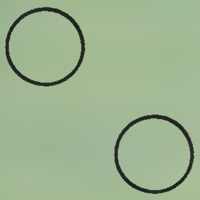
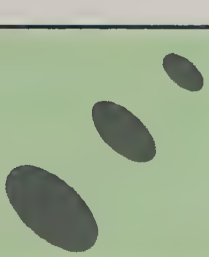
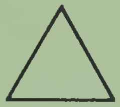
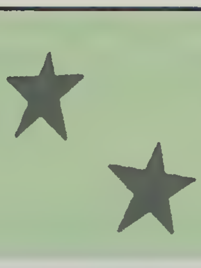
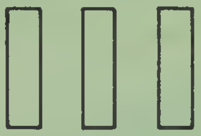
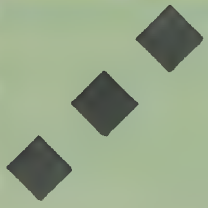
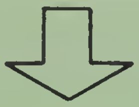
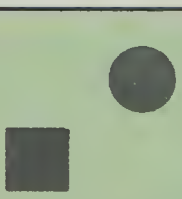
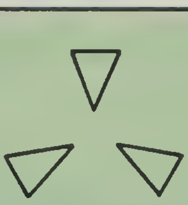
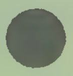
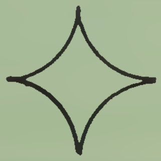
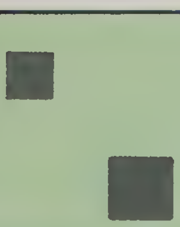
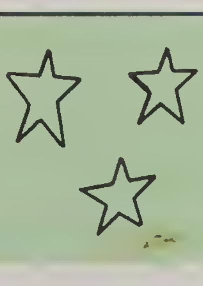
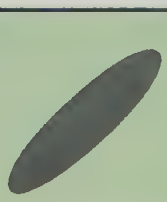
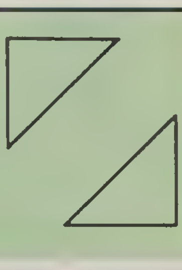
Headline Story: (a) Tell a story with 2 in it . . . (b) another with 1 in it.









	H	o	w	M	a	n	y
I	 1	 2	 3	0	 1	 2	 3
II	 2	 3	0	 1	 2	 3	0
III	 3	0	 1	 2	 3	0	 1
IV	0	 1	 2	 3	0	 1	 2

Notes on Pupil Page 4

Tasks: *Pupils count the number of objects in each set and record the counts.*

Purpose: To continue the development of the ability to do simple counting and to write numbers.

Unifying Ideas: Sets; number and counting.

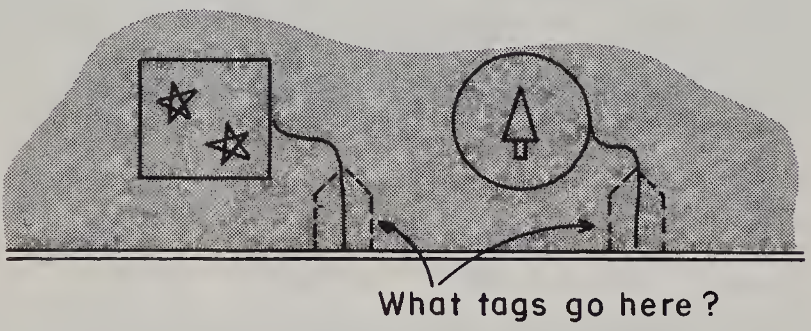
Materials for Teacher: Number tags; cards with pictures of 0-5 objects.

Materials for Pupils: Number tags.

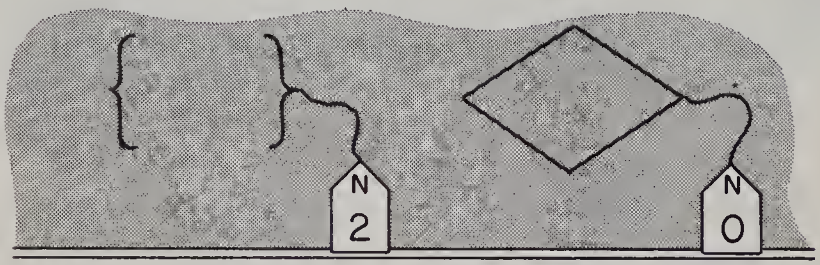
The Lesson:

The exercises on this page are somewhat more difficult than those on the preceding pages because no samples are given.

A chalkboard activity that parallels the work for pupils on page 4 will enable most to work independently. Draw pictures in various-shaped frames and draw lines down to the chalkrail.



In another version, tags are supplied—and pupils draw appropriate sketches:

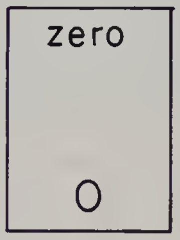


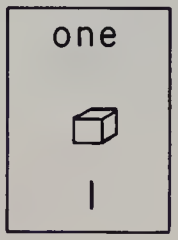
You can hope some pupil will tell you that the second picture is already finished.

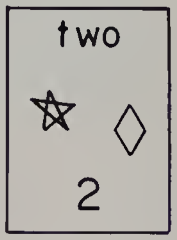
Other classroom activities include the following: With some appropriate object, a coin, perhaps, tap twice (once, three times, not at all) on the chalkboard. Have the pupils hold up their number tag which goes with the number of sounds they just heard. Have them lay the card down and clap that same number of times.

In doing page 4, a pupil may have tagged the word BOY or the three-part ring with 1. Do not say that his answer is incorrect; let him explain how he got it. Similarly, the pupil who counted the number of ears, or other parts, on the rabbit should be asked what number he would write for how many rabbits there are in the picture.

Begin now to make number charts for your classroom:

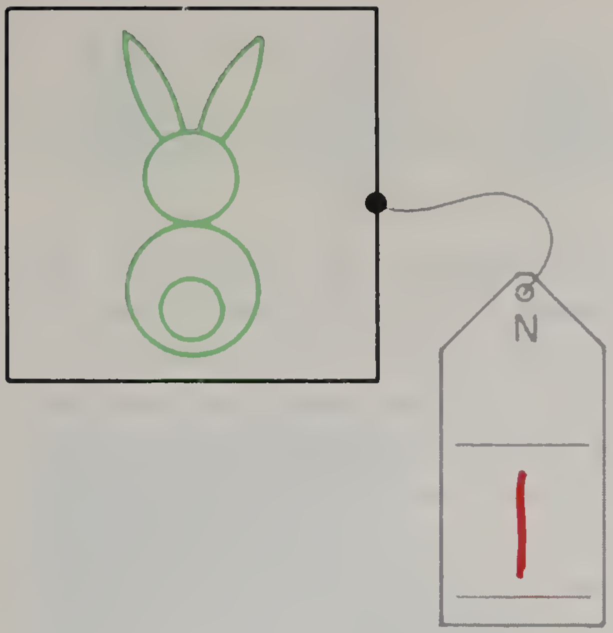


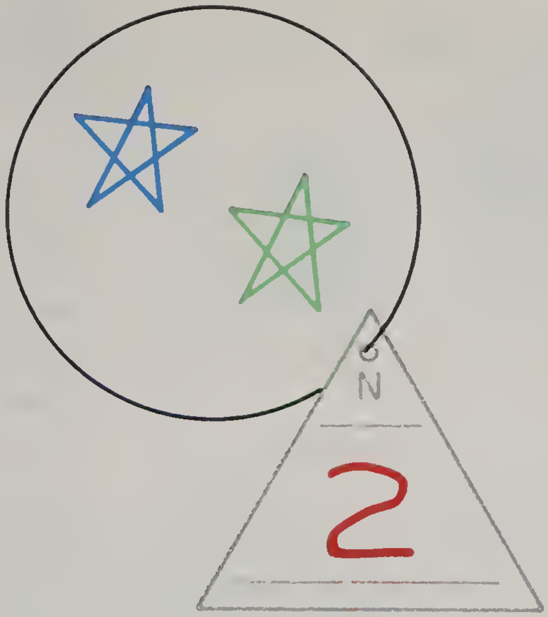


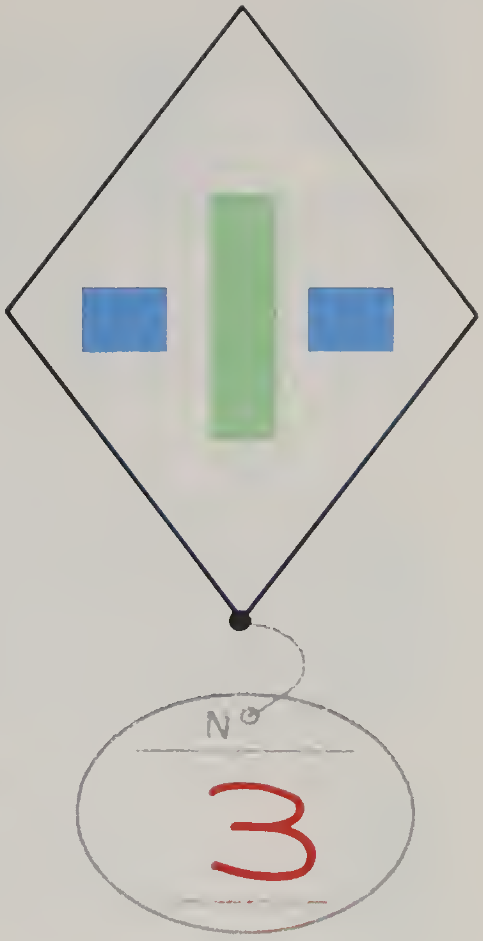


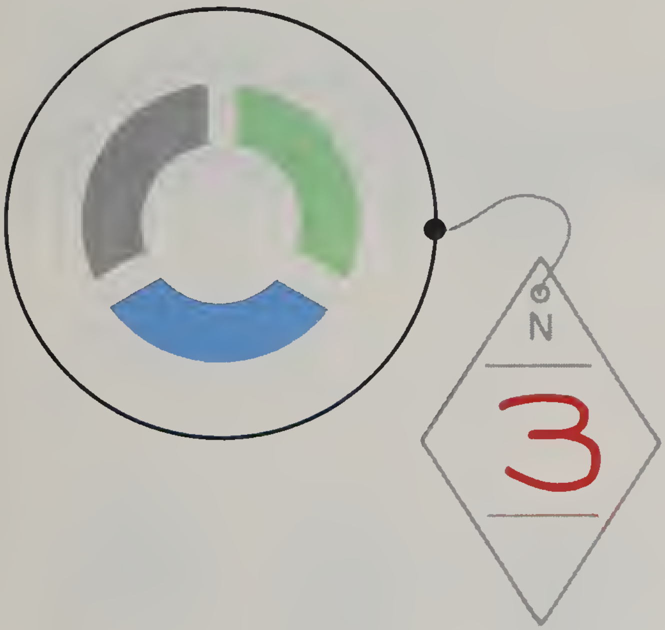
NUMBER CHART		
0	zero	
1	one	△
2	two	○ ○
3	three	□ □ □

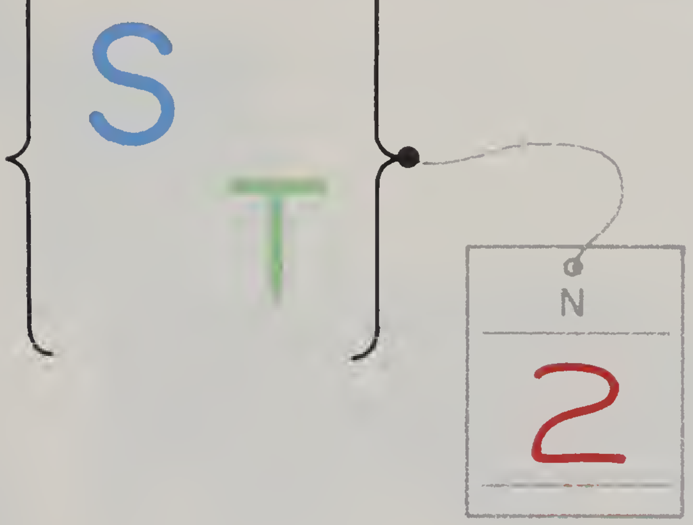
Headline Story: (a) Tell a story with 0 in it . . . (b) with 2 in it.

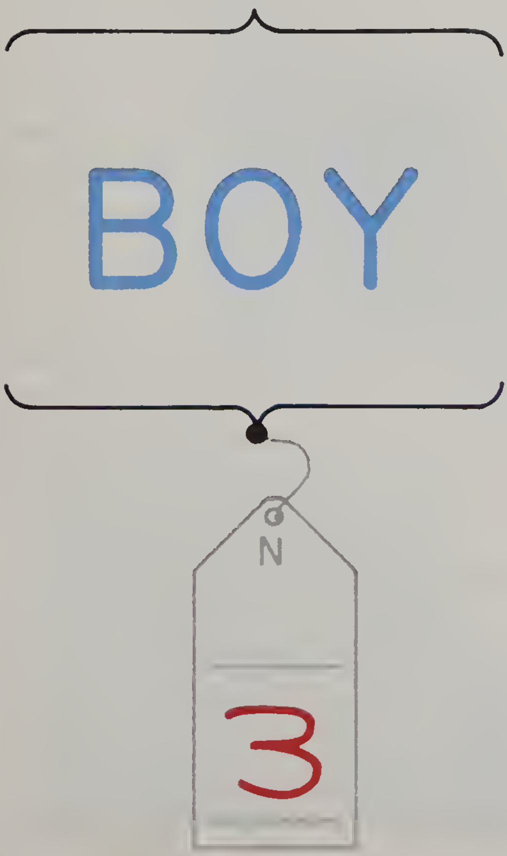

N
1

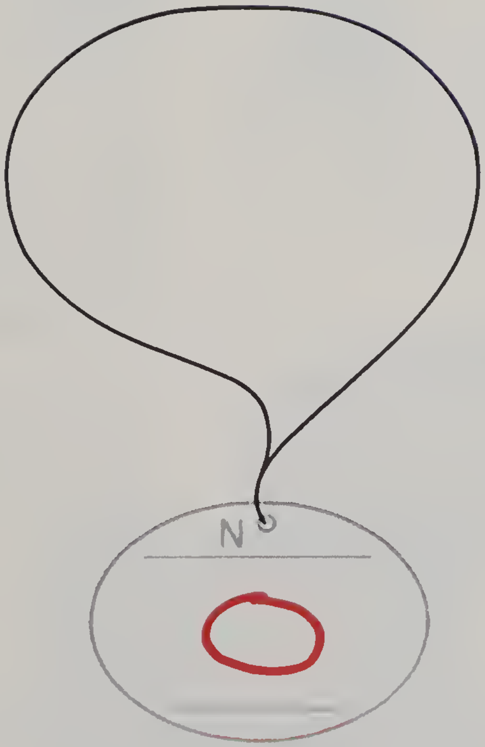

N
2

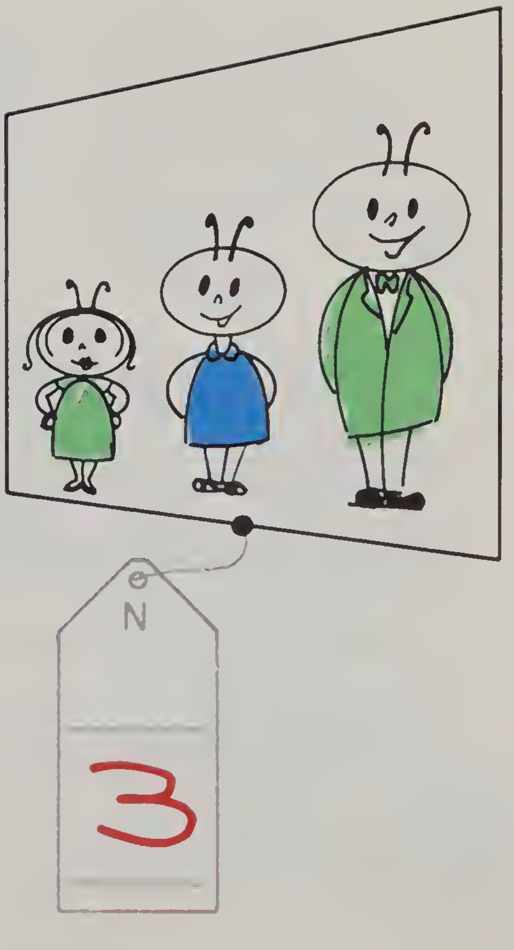

N
3


N
3


S
T
N
2


BOY
N
3


N
0


N
3

Notes on Pupil Page 5

Tasks: *Pupils complete the given number sequences, count the objects in the boxes and record the counts, and draw pictures of objects in the boxes at the bottom of the page in accordance with the given number tags.*

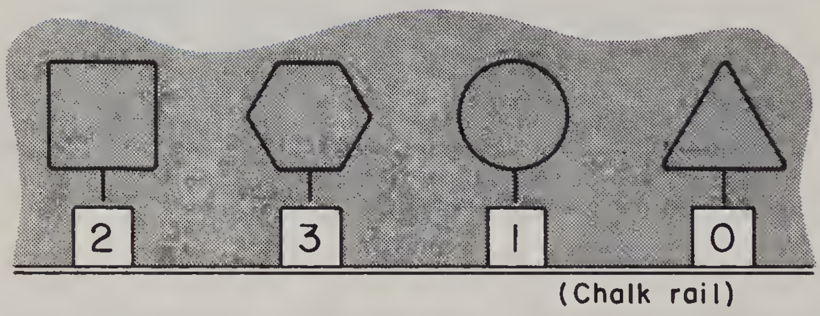
Purpose: To give practice in recognizing the natural sequence of whole numbers, in simple counting, and in writing numbers, and in associating numbers with points on a line; to present the idea of making physical representations (or sketches of them) for numbers.

Unifying Ideas: Sets; number and counting.

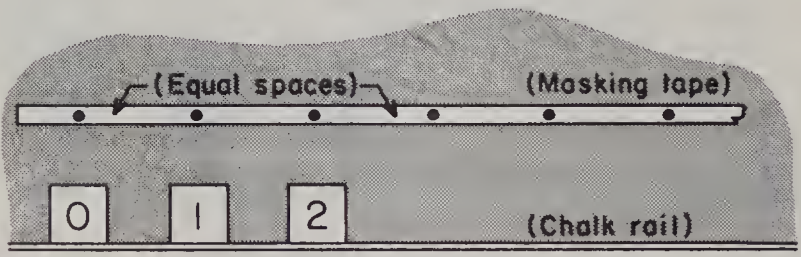
Materials for Teacher: Masking tape for use as number lines; number cards.

Materials for Pupils: Number cards; a piece of plain paper.

The Lesson:

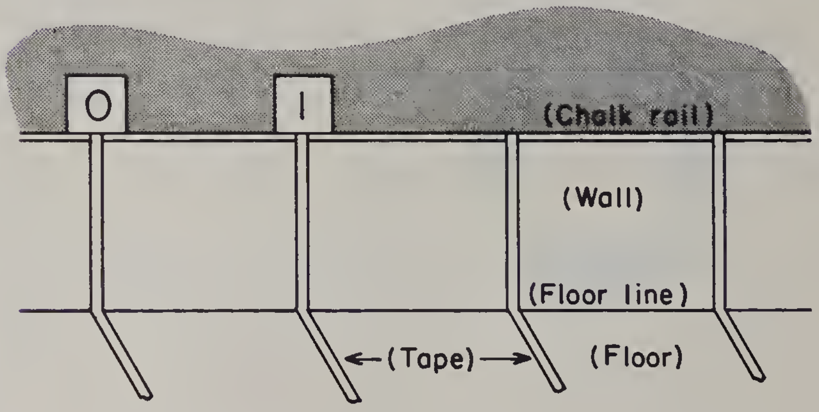


Use your number cards as shown above. Have the children draw pictures to go with each number —tally marks, circles, stars, anything easy and quick to draw. Erase the pictures, change the order of the cards, and repeat the exercise.



Install a line of masking tape with dots at equal intervals, as shown. Put the card for 0 below the first dot, the card for 1 below the second dot, and see if a child can tell what should be done next. Numbers must be used in sequential order in this context. Go up to at least 5 the first time, then pick up the cards and put just two of them back in place (2 and 3, perhaps), having the children place the rest. Repeat this, putting down two other numbers. You may have the children write the numbers to go with the dots, leaving the cards aside, after they have the idea.

Decide how much of what is on the board or the chalkrail you wish to leave there while the pupils do the work on page 5.

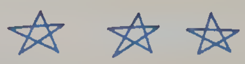


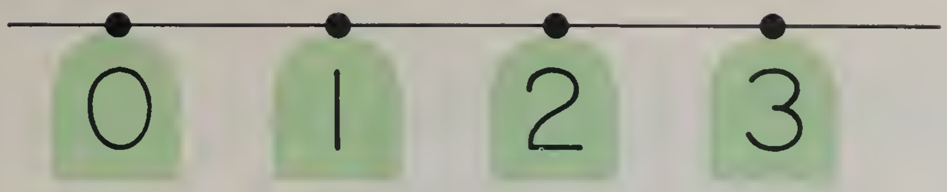
Install masking tape as shown to make what looks like a ladder lying against the wall and another lying on the floor. This gives you two "number lines," the chalkrail and the floor line, with the masking tape marking the units.

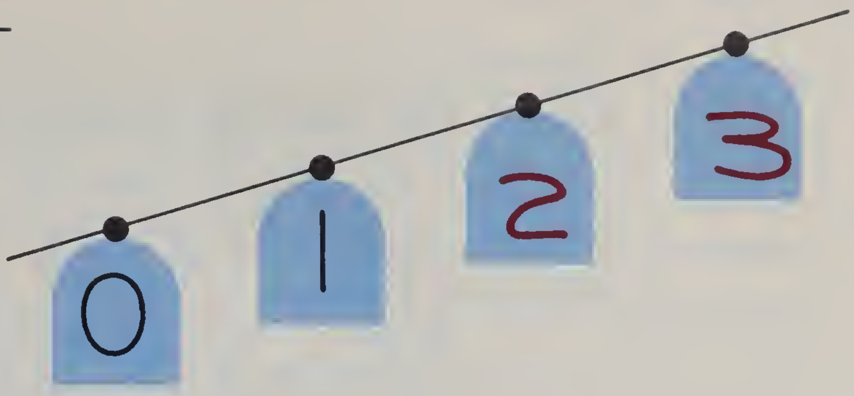
Have the pupils use number cards or chalk to number the "points" on the chalkrail. Have a pupil come and stand on a place on the floor that goes with 2; have another pupil stand at 4; ask what number a pupil would choose if he were to stand between these two, at one more than 4, at two less than 2, and so on.

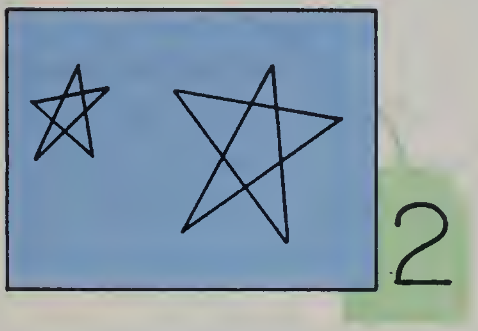
Be sure to review the accompanying booklet, *Games and Enrichment Activities,* for suggestions you may wish to incorporate in your daily lesson plan.

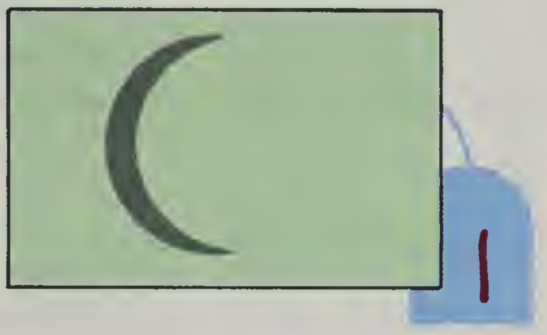
Headline Story: Tell a story suggested by the picture:

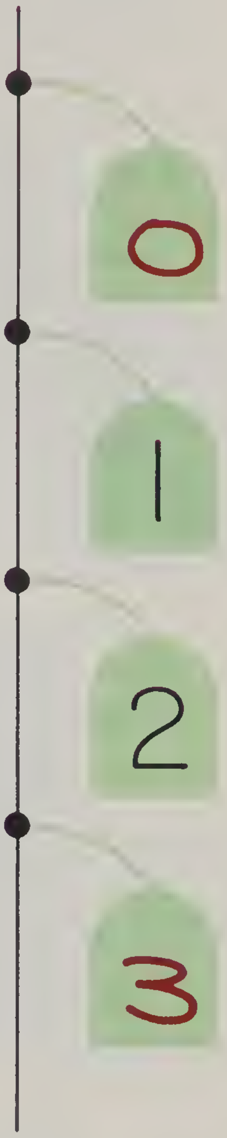


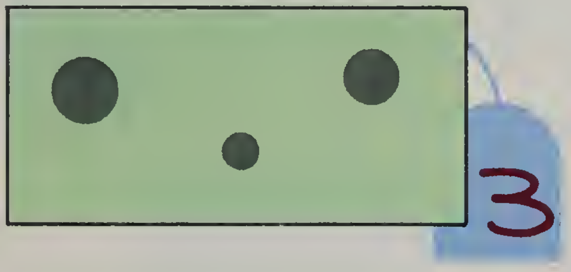

0
1
2
3

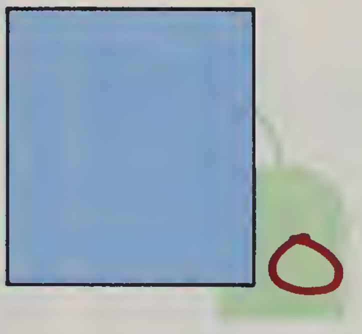

0
1
2
3

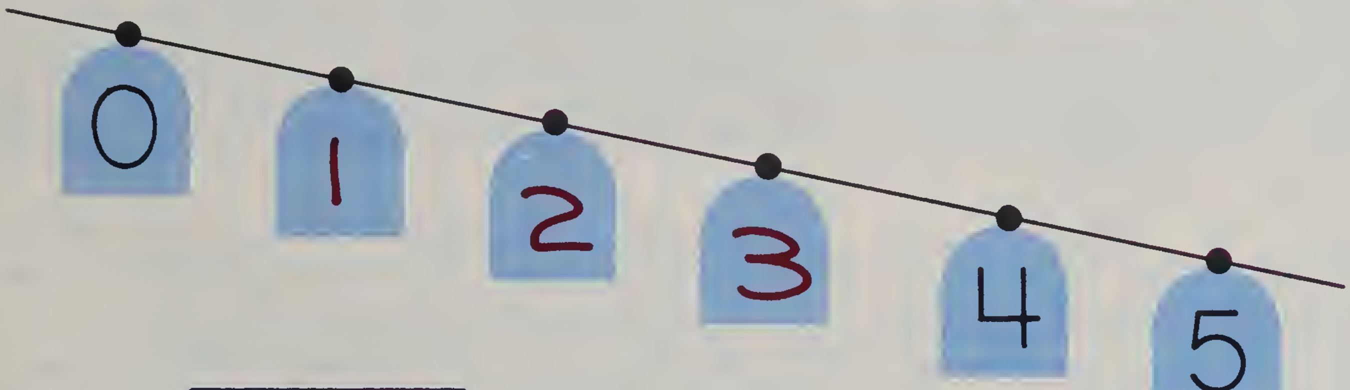

2

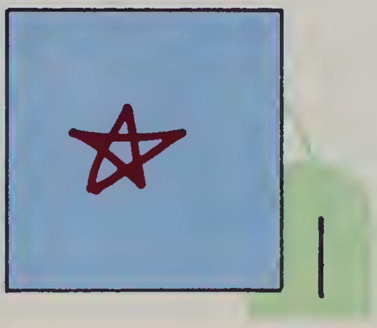

1

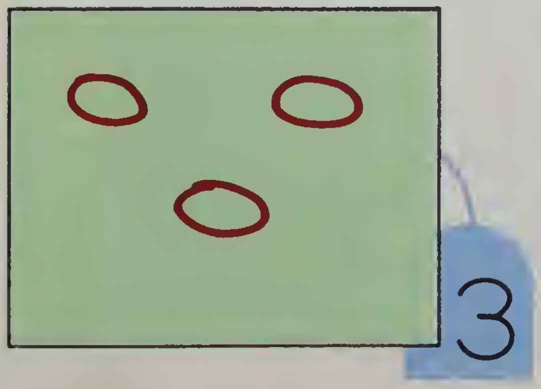

0
1
2
3

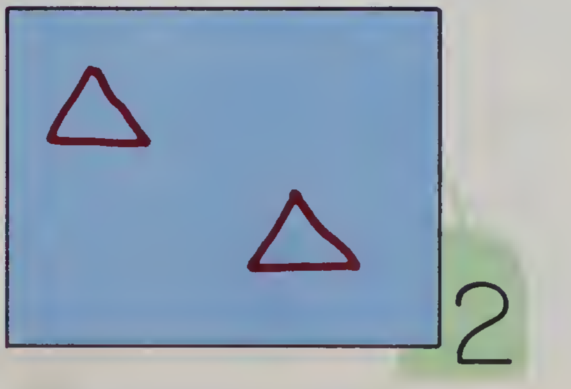

3

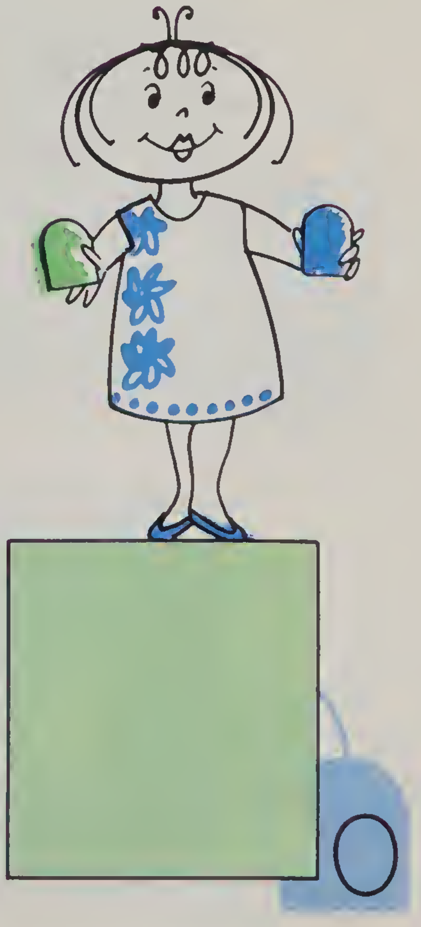

0


0
1
2
3
4
5


1


3


2


0

Notes on Pupil Page 6

Tasks: *Pupils count the objects in the boxes and record the numbers.*

Purpose: To provide practice in counting and recording, and to anticipate addition by working with pairs of numbers whose sum is 3.

Unifying Ideas: Sets; number and counting; structure.

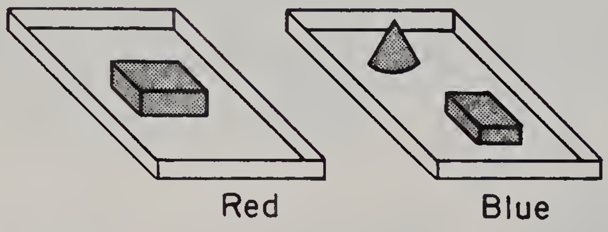
Materials for Teacher: Two boxes and three objects, as shown below.

Materials for Pupils: No special materials are needed.

The Lesson:

Here are two boxes—one red and one blue. We have 3 objects. We can put them all in the boxes in several ways.

How many different ways are there?
(using all of the objects each time)



Anne volunteers one arrangement—1 object in the red box and 2 in the blue. A recordkeeper (one of the pupils or the teacher) fills in the top line of a chart.

	Red	Blue	
Anne	1	2	1 + 2 = 3

$\left(\text{or } \begin{array}{r}1\\+2\\\hline 3\end{array}\right)$

Other pupils are urged to find another arrangement. Each proposal is checked against the record to see whether or not it is different (different numbers of objects in the red and blue boxes) from arrangements already noted. If it is, its originator is given credit.

Eventually the record may look like this:

	Red	Blue		
Anne	1	2	1 + 2 = 3	4 ways
Fred	2	1	2 + 1 = 3	
Ruth	0	3	0 + 3 = 3	
Bill	3	0	3 + 0 = 3	

Are there any more different ways?

When we place 3 objects in the boxes, can we have the same number of objects in each box?

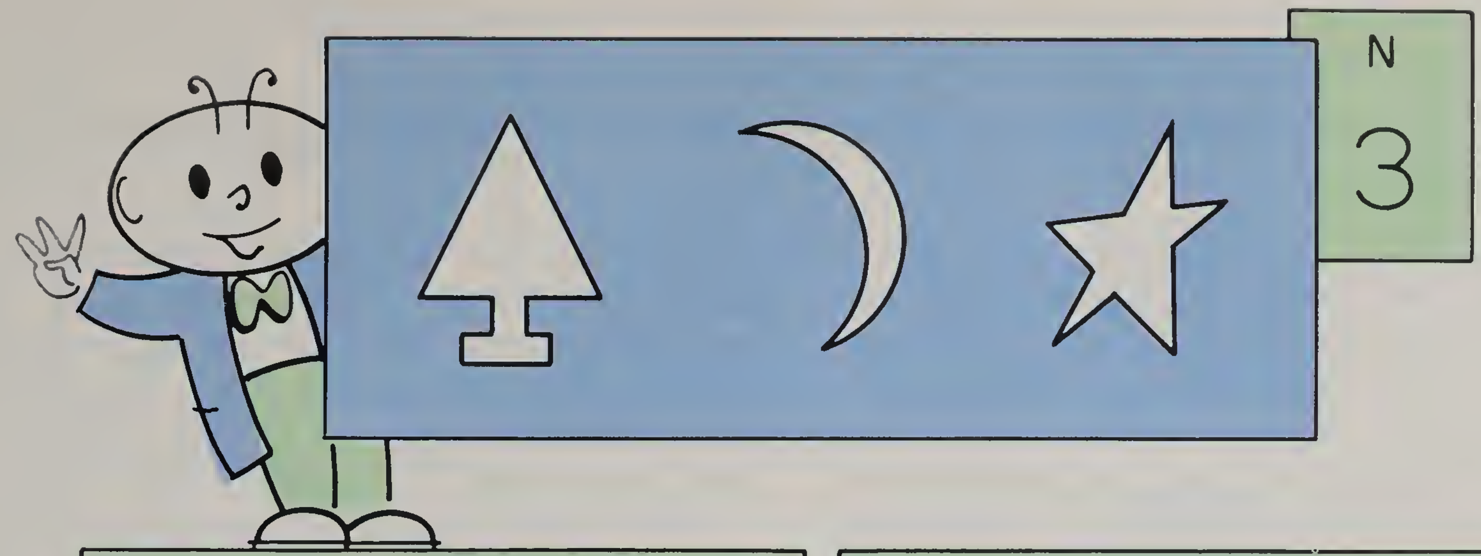
Can we have two more in one box than in the other?

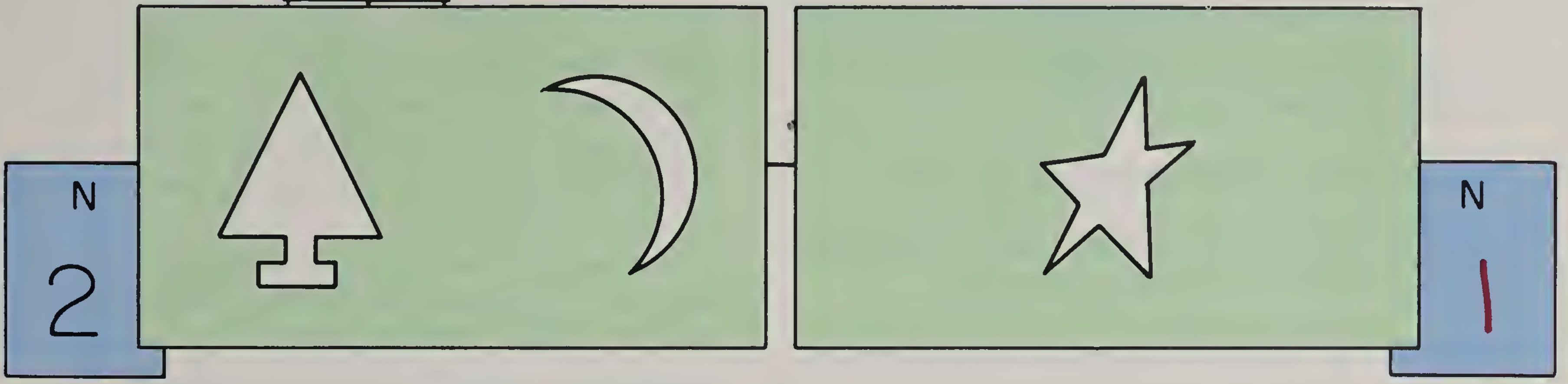
Can we have twice as many in one box as in the other?

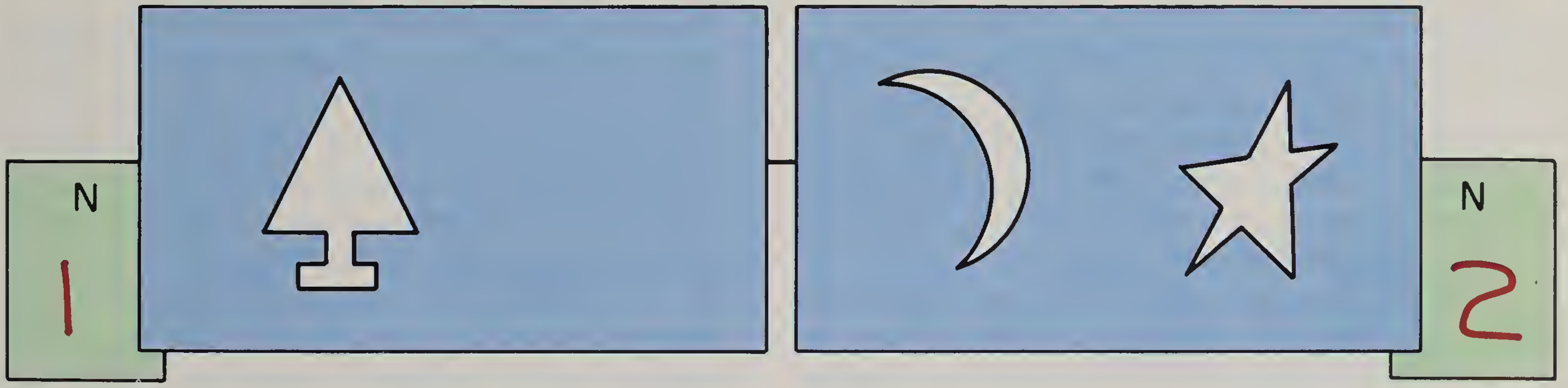
Let's try again with another set of objects. (Leave the original record on the board so it can be compared with the next one.)

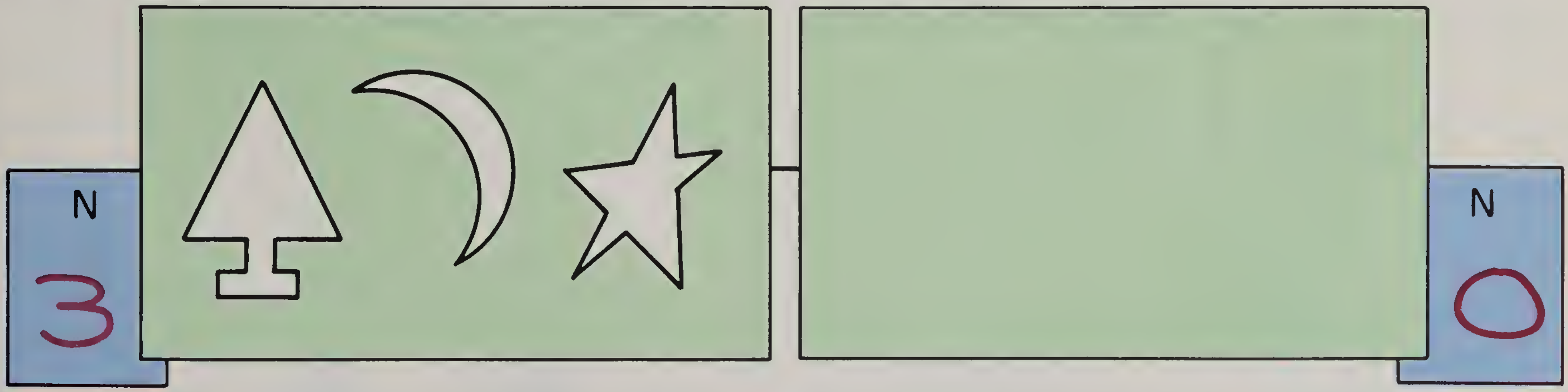
Assign page 6 for individual study. The pupils should note that the same three objects are used in each of the four situations.

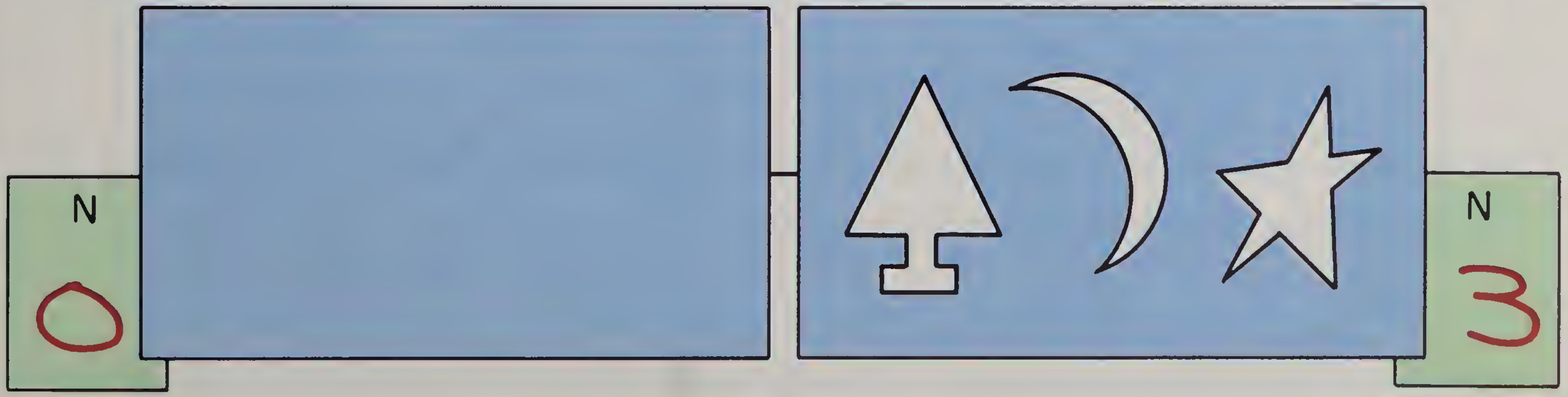
Headline Story: Three children were playing. If ___ were girls, then ___ were boys.


N
3


N
2
N
1


N
1
N
2


N
3
N
0


N
0
N
3

Tasks: *Pupils draw lines connecting members of a set of objects with an ordered set of points on a line and circle the number which tells the count of the objects in the set.*

Purpose: To show pupils that the number of objects in a set can be determined by establishing a one-to-one correspondence between the objects in the set and a "standard" set, the standard set being, in this case, points on a line.

Unifying Ideas: Sets; number and counting.

Materials for Teacher: Masking tape, clothesline, chalk, or tempera for use as number lines.

Materials for Pupils: No special materials needed.

The Lesson:

A number line can be used to keep a record of counting. Matching the elements of a set to the number line is a graphic way to suggest the idea of number. When a number line is used to establish a one-to-one correspondence, the first number on the line should be 1 rather than 0.

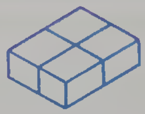
Before using this page, make a large number line on the floor with chalk, a clothesline, tape, or tempera. Number points 1 to 5. Have two, three, or four pupils find a place on the number line starting at 1. Ask, "How many more pupils do we need to make five? How do you know?" After five pupils have found places on the number line, have another join them. Ask how many are there now. Have the pupils number more points on the line. Then they can find out how many more pupils they need to make 7, or 9, or 10.

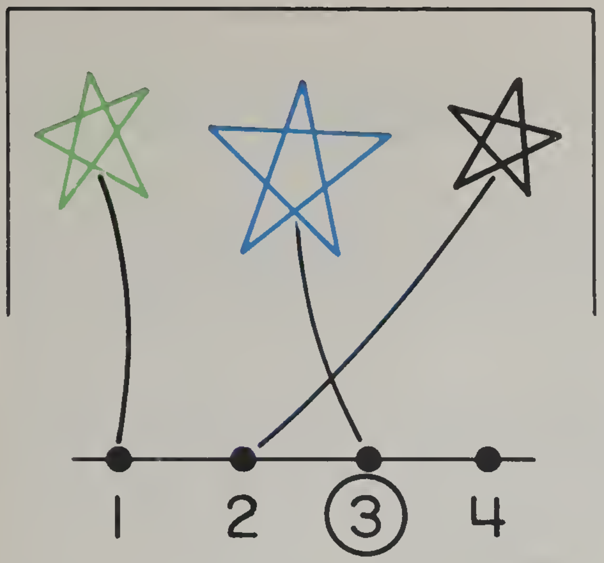
Place number lines on the bulletin board. These number lines can be made quickly with strips of paper. Place groups of objects on a table close to the bulletin board and attach to each object a string long enough to reach the number lines. Pupils should count as they attach the strings to numbers on the line.

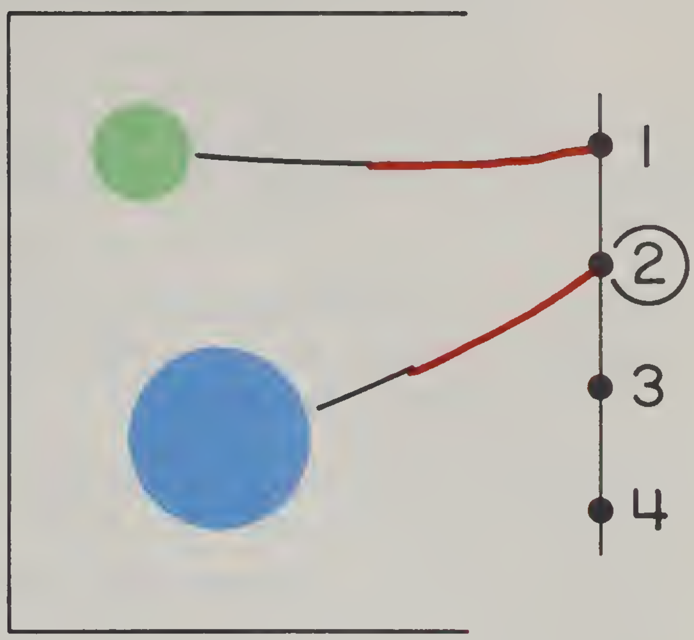
You may also want to copy on the board the pictures and number lines from page 7 so that, before doing the page, children will be able to practice connecting pictures to the number lines.

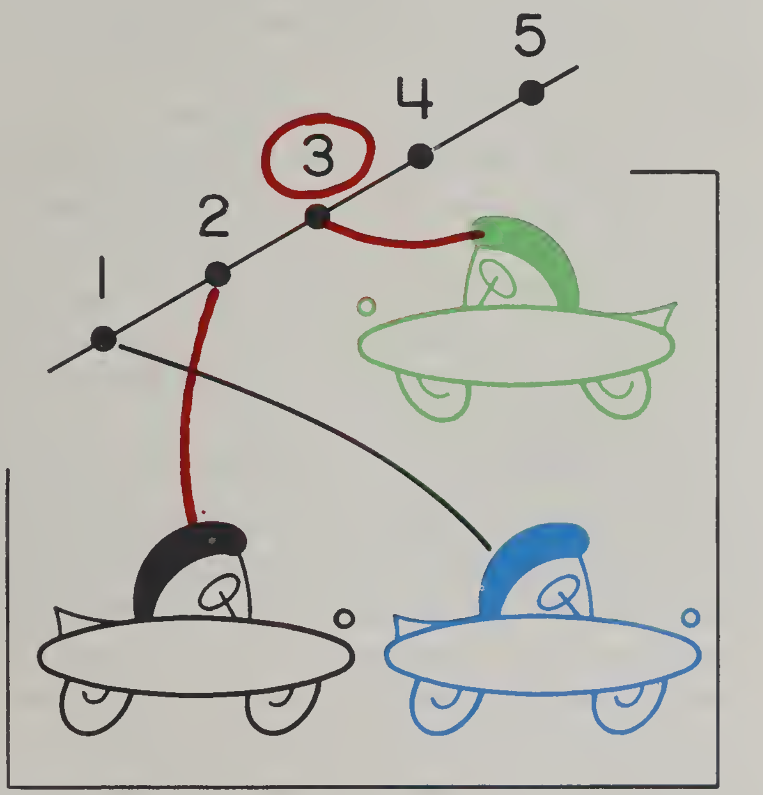
It is extremely important that pupils recognize that there are many ways of establishing a one-to-one correspondence (a "comes out even" matching) between two sets. One way to foster this realization is to have some of the strings used in the preceding activity crossing each other. (They may cross, or they may not; the important criterion is that each object is matched with one number and each number is matched with one object.) Another way is to ask the pupils to count a given set of objects, with the teacher indicating which object he should count first. The teacher should vary the starting object and the arrangement of objects.

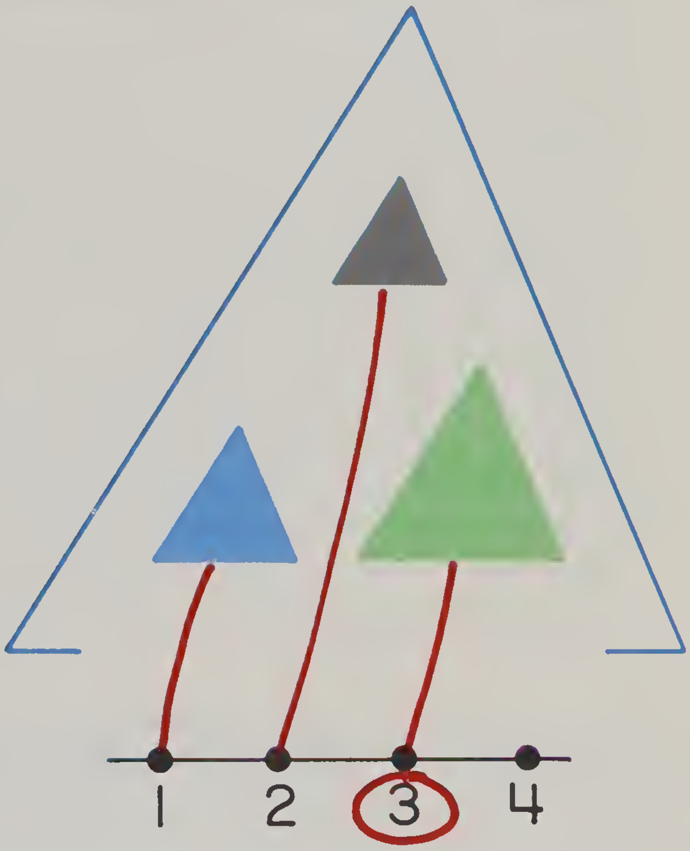
Headline Story: Tell a story suggested by the sketch:

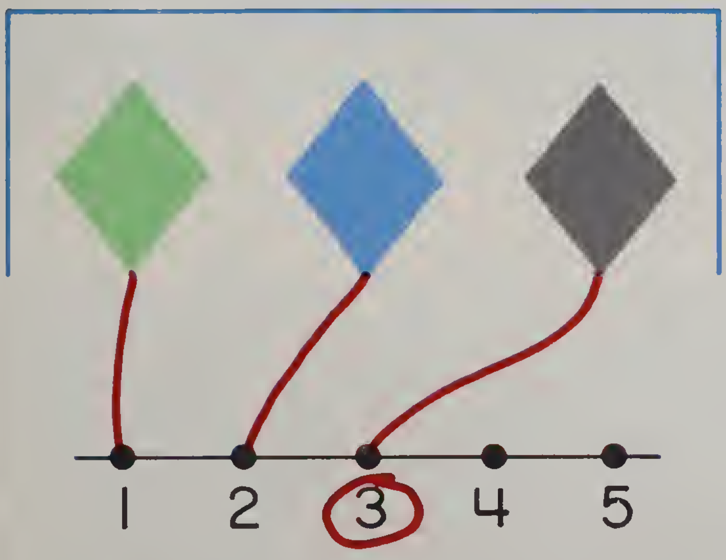


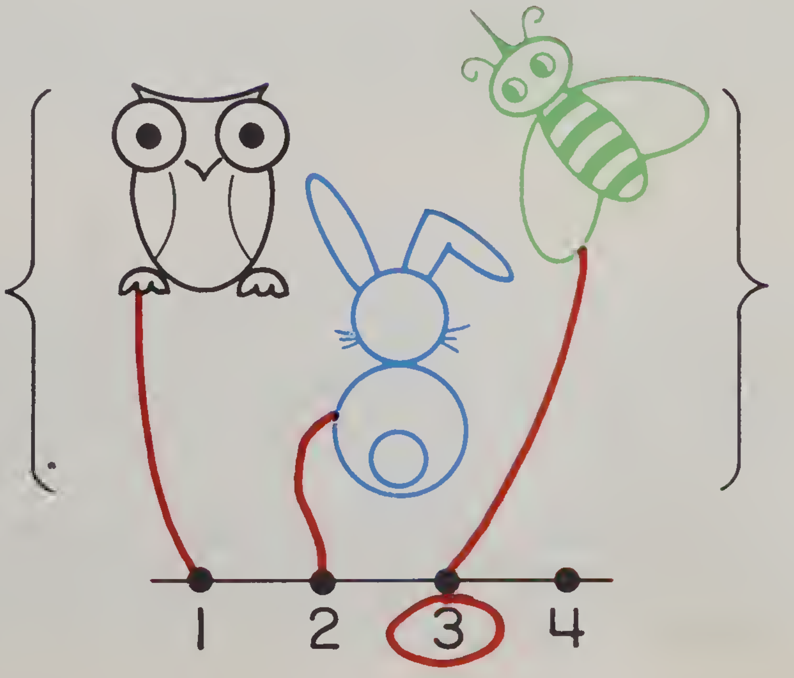

1
2
③
4


1
②
3
4


1
2
③
4
5


1
2
③
4


1
2
③
4
5


1
2
③
4

Notes on Pupil Page 8

Tasks: *Pupils complete the pattern of jumps on each number line and write the appropriate numbers.*

Purpose: To provide experience in recognizing and completing patterns, and to prepare for addition by making jumps on a number line.

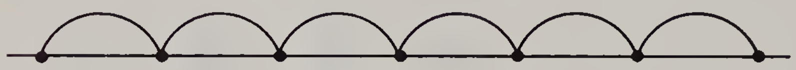
Unifying Ideas: Functions and relations.

Materials for Teacher: Number lines as described below; beads of two or three different colors; and wire or string.

Materials for Pupils: No special materials needed.

The Lesson:

Use the tape strips which extend out on the floor from your chalkrail number line (see the manual for page 5) or another number line you have installed on the floor. Have points equally spaced along this line, but no number symbols. Tell the pupils you are going to play "Rabbit." Have them watch you hop along the line. The first time, move one unit each hop. Ask who can do the same thing you did. Have two or three pupils do this. How did they know what to do? On the chalkboard, make a pictorial representation of their answers:



Repeat the exercise, using other patterns for hopping—2, 2, 2 . . .; 1, 2, 1, 2 . . .; 2, 2, 1, 2, 2, 1 . . .; and so on.

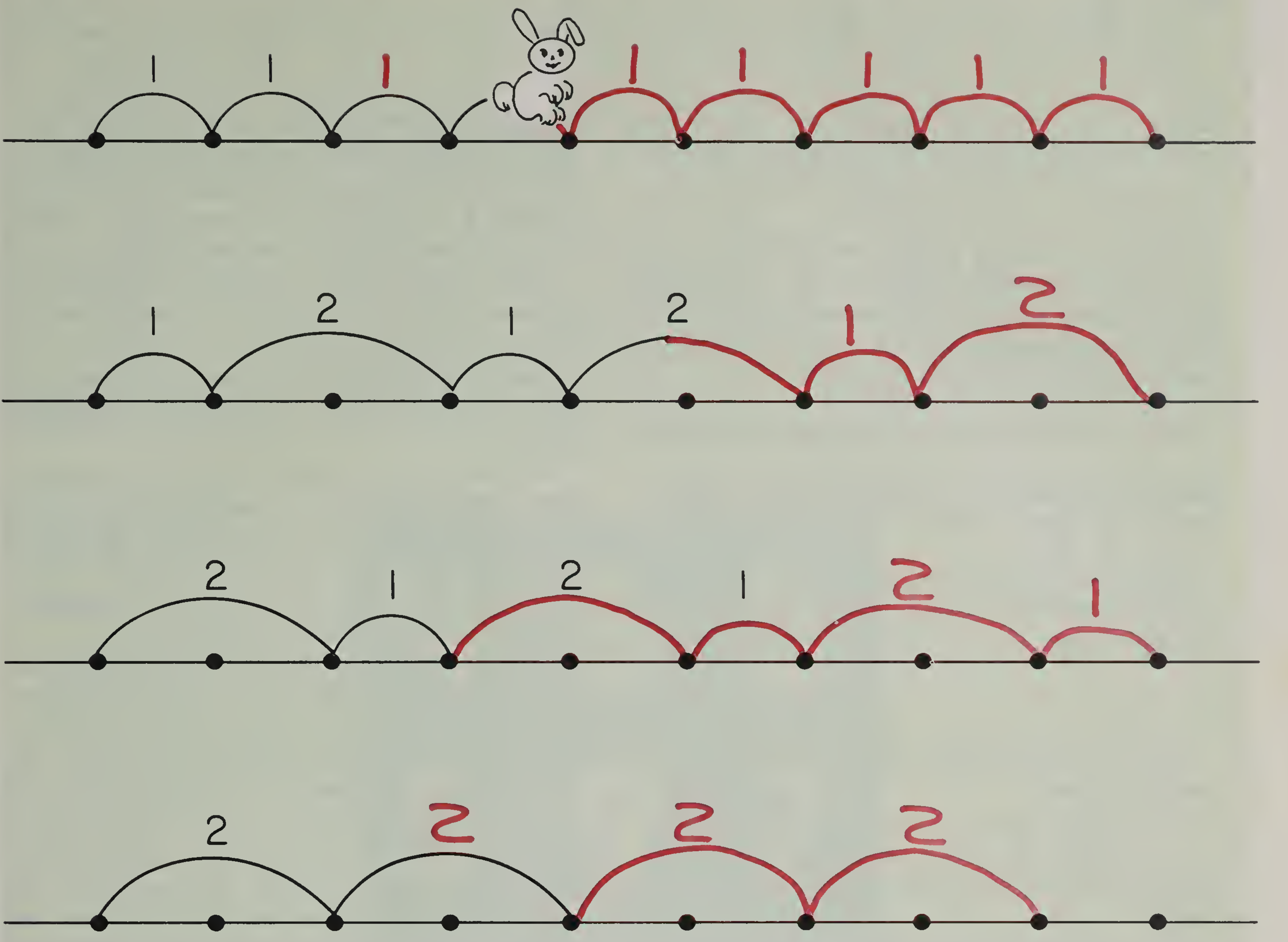
Beads of various shapes, sizes, or colors may be strung in specified patterns for the pupils to recognize and continue. The result of this activity can be pictured as jumps along the number line.

The "Giant Step, Baby Step" game can be modified for use in the context of this lesson. Again, the results may be pictured ultimately as jumps along the number line.

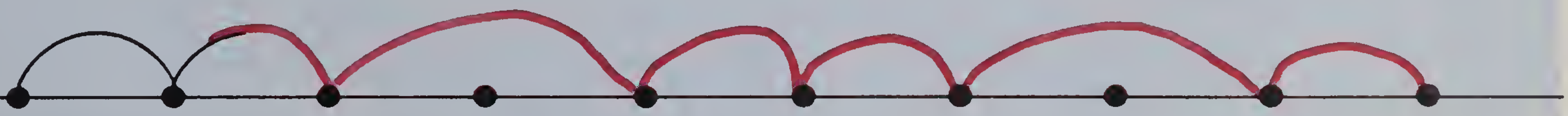
After your pupils have completed the work on page 8, you may provide more experiences of the same kind by preparing worksheets similar to this page, but using other patterns, such as: 3, 3, 3 . . .; 1, 3, 1, 3 . . .; 1, 2, 3, 1 . . .; and so on.

You may wish to use the following extension of this lesson with a selected group of pupils. Assign numbers to points on the line. Ask questions such as: If I start at 4 and jump forward 1 space, where do I land? If I start at 1 and jump forward 2 spaces, where do I land? If I start at 0 and jump forward 2 spaces and then 1 more space, where do I land? This exercise can become as complicated as you choose to make it by starting at larger numbers, taking larger jumps, or having the pattern of jumps repeated a stated number of times. It usually turns out to be interesting and productive if you ask the pupils to suggest starting points and patterns of jumps. You might also consider the idea of jumping backward, or perhaps the pupils will suggest it.

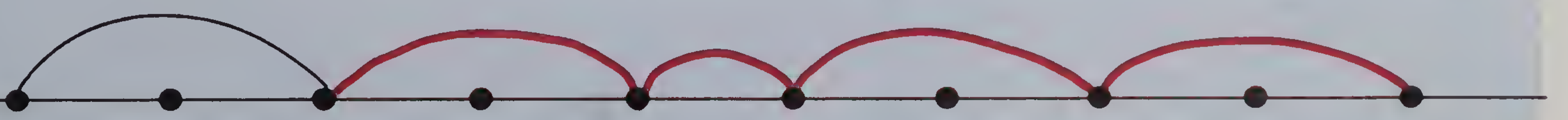
Headline Story: Mary had 2 cookies. (Pupils extend the story.)


1 1 1 1 1 1 1 1
1 2 1 2 1 2
2 1 2 1 2 1
2 2 2 2

1, 1, 2, 1, 1, 2, 1, 1, 2, 1, 1, 2, 1



2, 2, 1, 2, 2, 1, 2, 2, 1, 2, 2, 1, 2



Notes on Pupil Page 9

Tasks: *Pupils color like numbers the same color, follow guide lines, and write the number of objects in each set pictured at the bottom of the page.*

Purpose: (a) To help pupils learn to read, match, and write the numbers 4 through 10; (b) to help pupils perfect their skill in counting while they learn to recognize the cardinal numbers of various sets (4 through 10 objects in a set).

Unifying Ideas: Number and counting; numeration.

Materials for Teacher: Demonstration number cards 4-10; colored chalk.

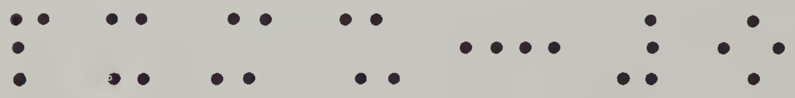
Materials for Pupils: Ten counters of the same shape and color; one counter of the same shape but different color.

The Lesson:

Display a set of number cards at random on the chalkboard tray.

Have pupils pick a card from a pile of similar cards located face down on your desk. Each matches his card with one like it on the chalk tray.

Ask pupils to construct their favorite arrangement of four counters of the same shape and color. Show several on the board. Some possibilities might be:



Sketch pictures of their patterns on the board above the appropriate number tag placed on the chalk tray. Proceed in like manner for each of the numbers 5 through 10. Now remove the number cards from the chalk tray and shuffle them.

Select pupils to pick a number card from the pile and place it on the chalk tray under the appropriate sketches.

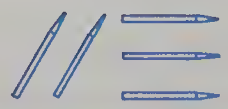
Have pupils arrange four counters of the same shape and color into a square pattern on their desks.

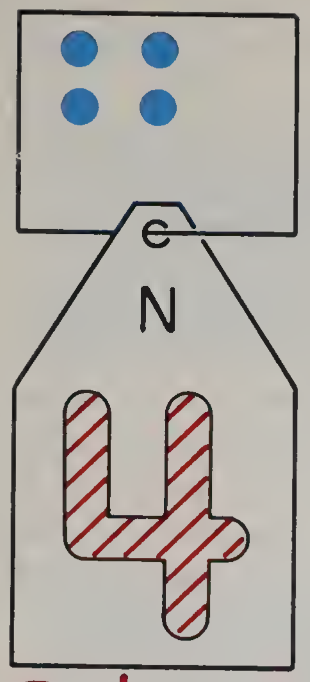
Ask them to place another counter of the same shape but different color beside the top row of counters, as shown on the pupil page. Sketch this construction on the chalkboard (with colored chalk). Ask the class which number tag goes on the chalk tray beneath your sketch.

Have them replace the new counter with one of the same shape and color as the remaining four. They now have five counters of the same shape and color arranged as before. They can take the counter of the same shape but different color and place it beside the bottom row, as shown on the pupil page. Sketch this pattern on the board. Ask the class which number tag goes with their new set of counters. Pupils should replace the newest counter with one of the same shape and color as the remaining five.

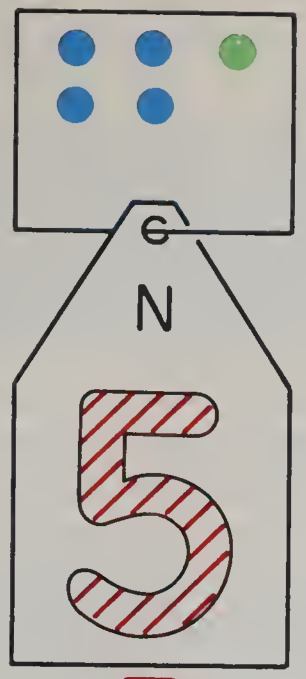
Proceed with the activity until the "and one more" patterns for the numbers 7 through 10 have been constructed, sketched, and tagged. Each pupil now works independently on his own MATH WORKSHOP page 9.

Headline Story: Tell a story suggested by the picture of pencils:

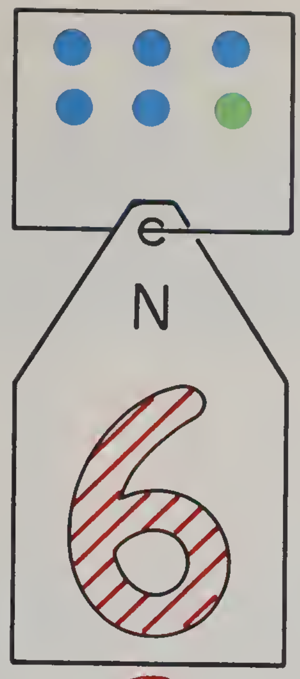




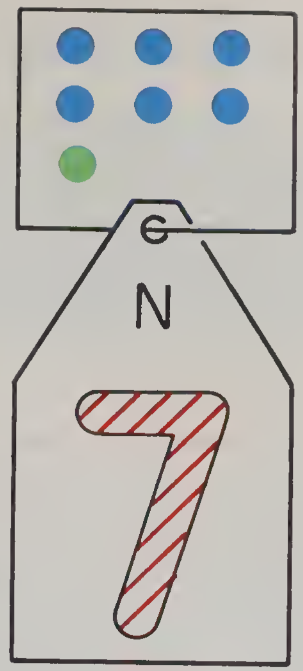
Color A



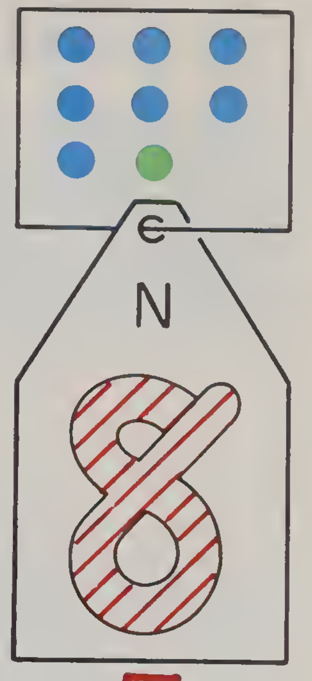
B



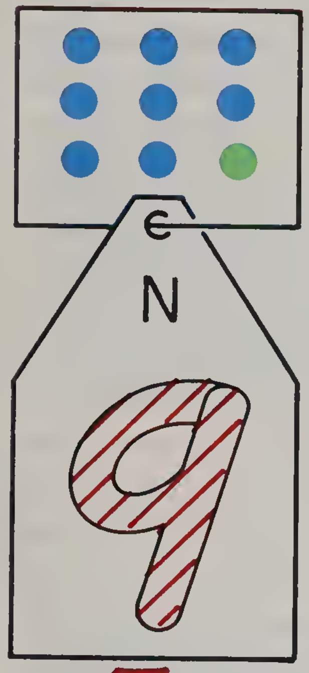
C



D

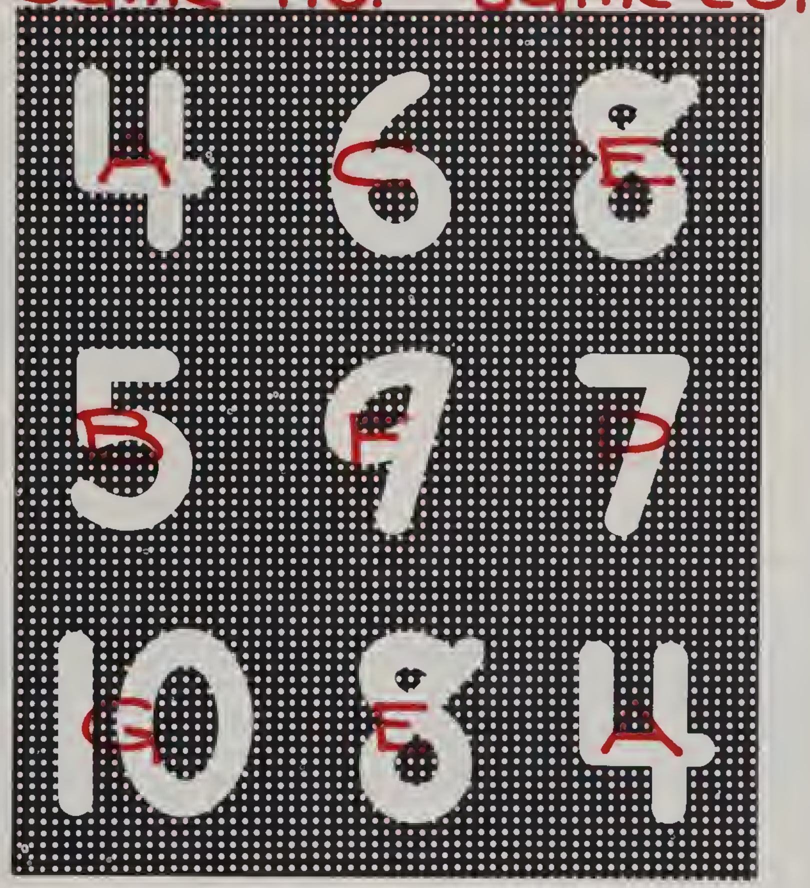


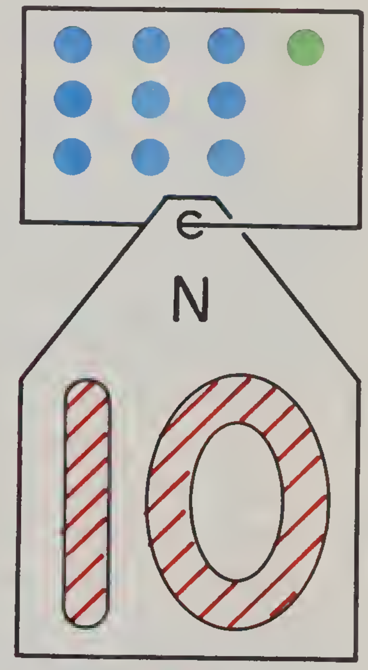
E



F

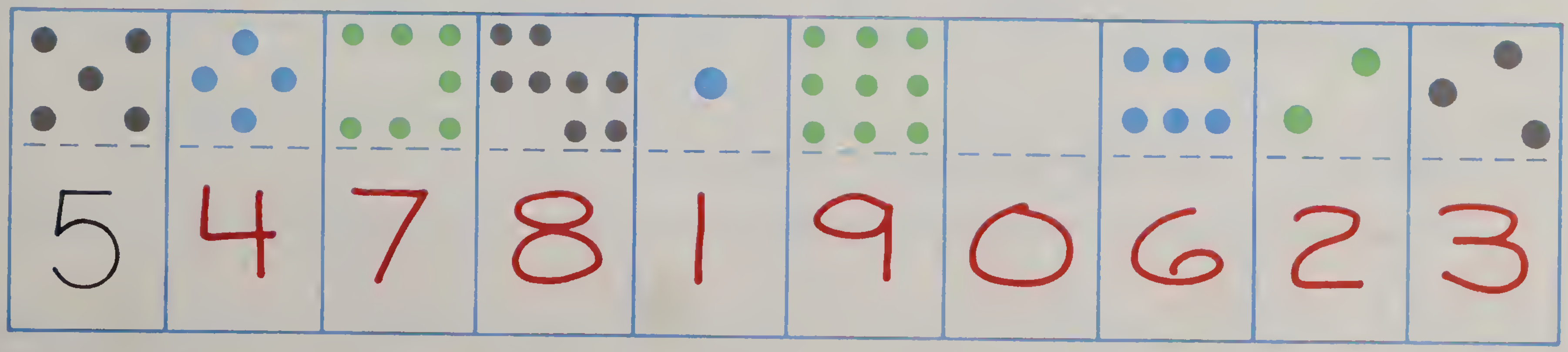
Same no.- same color


4 A
6 C
8 E
5 B
9 F
7 D
10 G
8 E
4 A



G

0 1 2 3 4 5 6 7 8 9 10


5
4
7
8
1
9
0
6
2
3

Tasks: *Pupils write the number that tells how many objects are pictured for each of the boxes in the first three rows, draw pictures to go with the numbers in the fourth row, and trace the numbers which are sketched in at the bottom of the page.*

Purpose: To provide practice in recognizing the numbers, in counting, in writing numbers, and in seeing that a given number can have different physical representations; to introduce the number words *zero* through *thirteen*.

Unifying Ideas: Sets; number and counting.

Materials for Teacher: Demonstration counters (flannel cutouts, wooden blocks, etc.).

Materials for Pupils: At least 10 small objects to count.

The Lesson:

This is just the beginning of work with numbers larger than 3, so there is little need to hope for complete understanding of these numbers at this time. Provide numerous experiences of a great many kinds with physical representations of these numbers, and understanding will come in due course.

Write a number, say 5, on the chalkboard. Ask the pupils to show this number with their counters. Check to see that all pupils have used the correct number of counters, and use your demonstration counters to show those who may have made mistakes. Note the different arrangements of counters the pupils may have made. Can each pupil rearrange his 5 counters in a different pattern? In another different pattern? Repeat this activity with other numbers 4 through 10.

Draw a rectangle on the board, attach a tag with a number on it, say 6, and draw perhaps three pictures inside the rectangle. Have a pupil draw more pictures so that the tag will tell the right story. The other pupils act out this activity with their counters. You may then erase the pictures and the number, put in a part of a new picture and write another number, and repeat the activity.

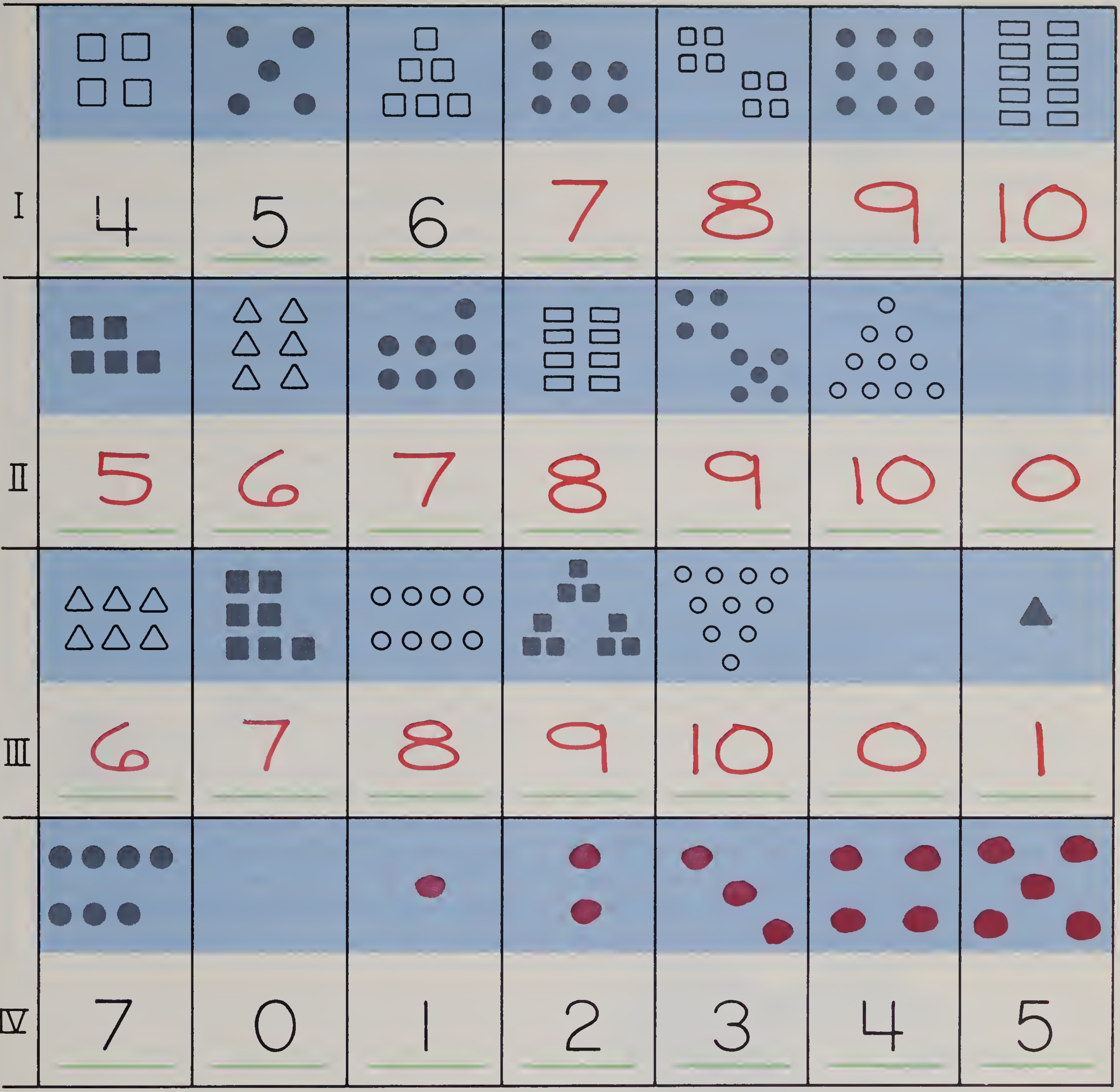
You will notice that this page is similar to page 3. Do any of your pupils recognize the words *How Many* and the Roman numerals?

The number words *zero* through *thirteen* appear on this page for the first time. It is not expected that the pupils will learn all these words at this time. It is intended that this page will provide one more experience in the essential category of building visual imageries of the ways we can speak and write about numbers. Three ways of referring to numbers are included here—a picture of the members of an appropriate set of objects, the number symbol, and the number word. The pupils should have at least a nonverbal awareness that the same three ways would apply to numbers of any size. One way to treat the material at the bottom of this page is merely to point out that this is a way the numbers and the number words go in order. Decide in the light of your own reading program which of the words, if any, you wish the pupils to learn to read at this time.

Be sure to review the accompanying booklet, *Games and Enrichment Activities*, for suggestions you may wish to incorporate in your daily lesson plan.

Headline Story: Tell a story with 10 in it . . . with 8 in it . . . etc.

How Many



I	4	5	6	7	8	9	10
II	5	6	7	8	9	10	0
III	6	7	8	9	10	0	1
IV	7	0	1	2	3	4	5

zero	one	two	three	four	five	six
0	1	2	3	4	5	6
seven	eight	nine	ten	eleven	twelve	thirteen
7	8	9	10	11	12	13

Notes on Pupil Page 11

Tasks: *Pupils draw lines connecting objects to numbered points on a line, circle the number that tells the total, and record the counts as indicated by the tags.*

Purpose: To provide further experiences in counting and establishing one-to-one correspondences, and to suggest the useful concept of "seeing smaller numbers inside big numbers."

Unifying Ideas: Sets; number and counting.

Materials for Teacher: Ten demonstration counters, alike in some respect and different in another; a number line on the chalkboard.

Materials for Pupils: No special materials needed.

The Lesson:

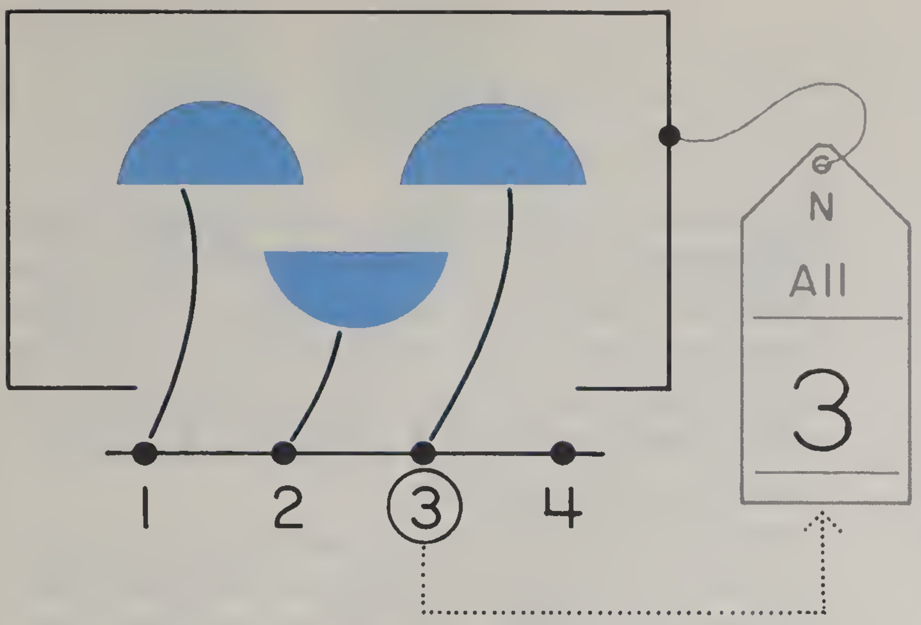
The mathematical signs + and = are to be introduced informally on this page. By way of leading into the use of these signs, you might have a discussion of signs. The pupils are familiar with traffic signs, and it is sometimes essential that they observe these signs as they walk to school. Details vary in the different states, but there is always some scheme for classifying the traffic signs by shape, by color, and by the kinds of characters on them. It is universally true in this country that a stop sign is octagonal (it looks like a square with the corners cut off, eight sides in all). There may be signs around your school showing the way to the cafeteria or to the nurse's office. If applicable, you might ask how these signs could be changed so that a person would not have to know how to read words to tell what the sign meant.

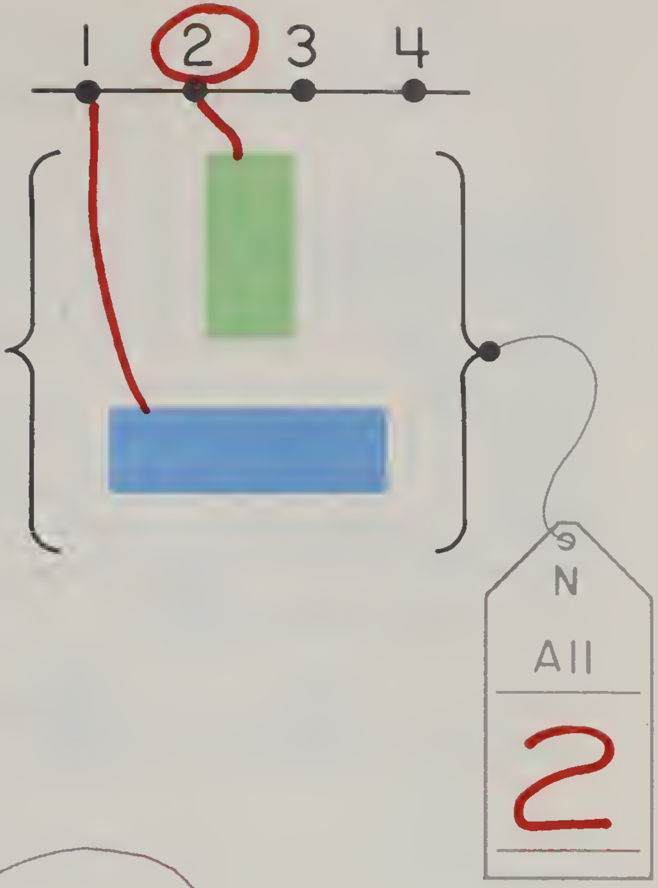
Take a few minutes to teach your pupils to read the word *All.* It appears three times on page 11, and frequently throughout the book. The words *Blue* and *Green* occur frequently, also, but there are color clues which make it unnecessary for the pupils to read them.

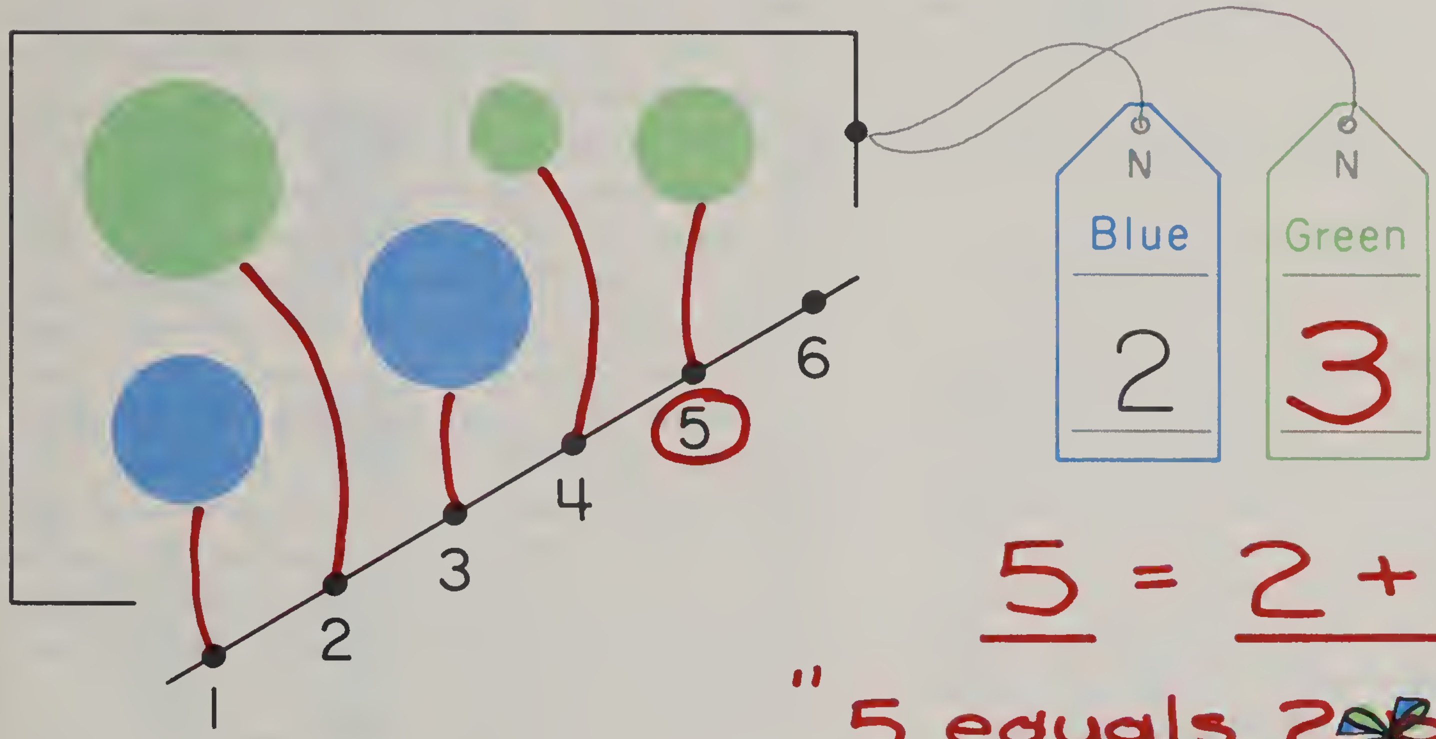
You might set the stage for this lesson by picking up 2 or 3 counters and asking the pupils for suggestions of ways to talk about the counters so that everyone can be sure which one (or which ones) you mean. At the very least, they should suggest size, shape, and color. Then choose specific questions to conform to the things you are using. You want to hold counters of one kind in one hand and those of the other kind in the other hand, recording the results each time. If a pupil should say, "You have 2 round counters in one hand and 1 square counter in the other, 3 in all," you would write casually: $2 + 1 = 3$. As you write this, say, "Two plus one equals three." If the pupil mentions the number *in all* first, write: $3 = 2 + 1$.

Draw pictures of your counters near the chalkboard number line and have the pupils draw lines to show the matching of objects and numbers. They should circle the number which tells "how many in all."

Headline Story: Tell a story with 1 and 2 in it.

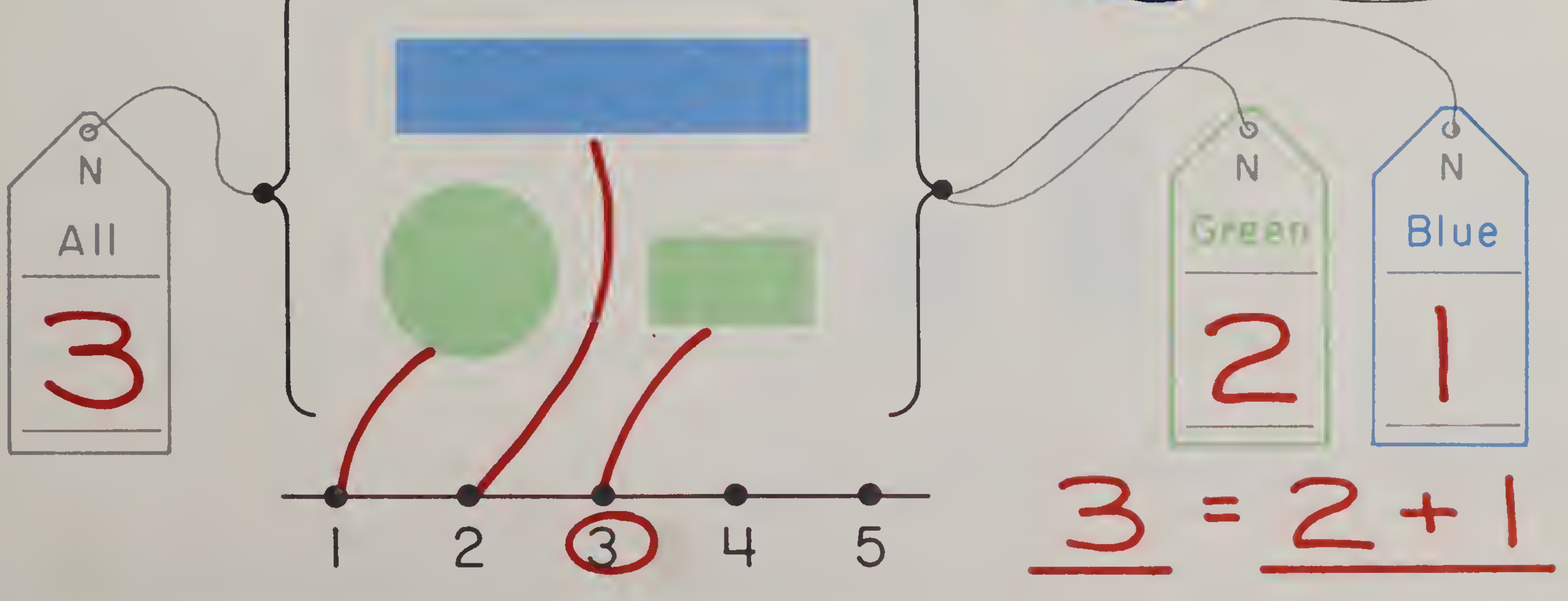





N
Blue
2
N
Green
3
1
2
3
4
5
6

5 = 2 + 3

"5 equals 2 plus 3"


N
All
3
N
Green
2
N
Blue
1
1
2
3
4
5

3 = 2 + 1

Notes on Pupil Page 12

Tasks: *Pupils count how many of one kind, how many of another kind, how many in all, and record the counts as indicated by the tags. The tasks for the first activity are the same as those for page 11.*

Purpose: To provide practice in counting and establishing one-to-one correspondences, in "seeing smaller numbers inside big numbers" (foreshadowing addition and subtraction), and to introduce the writing of equations.

Unifying Ideas: Sets; number and counting; structure; addition and subtraction.

Materials for Teacher: No special materials are needed.

Materials for Pupils: Two kinds of counters, if needed for help.

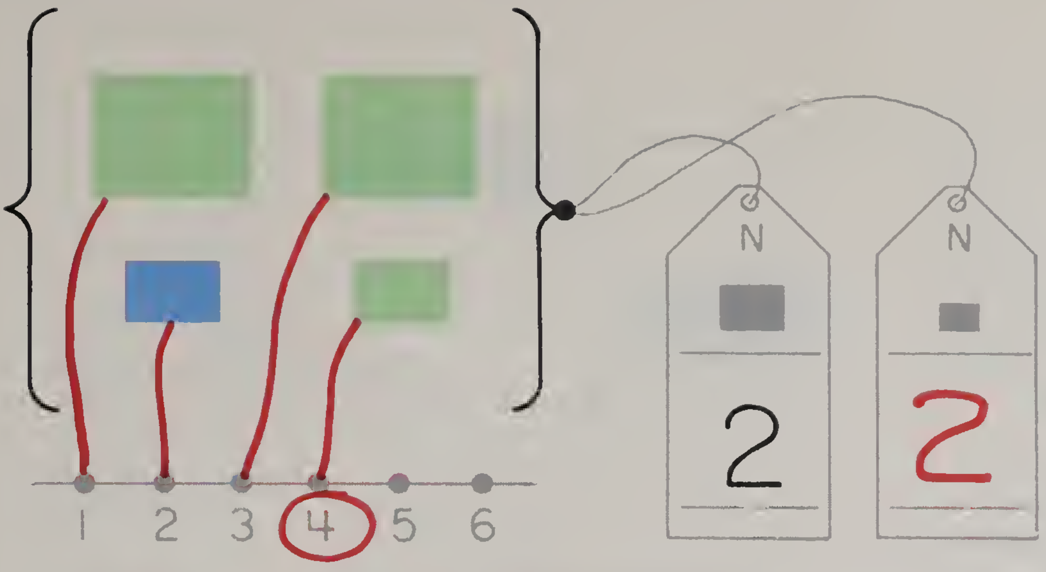
The Lesson:

One way to introduce this lesson is to have two pupils come to the front of the room and stand facing the class. Then, you call out a number, say 5. One pupil holds up a certain number of fingers and calls out the number of fingers he is holding up, say 4. The other pupil then holds up the number of fingers necessary to make the teacher's total and calls out the number he is holding up. Other pupils should check and you should write the appropriate equation on the board: $5 = 4 + 1$. Have the two pupils exchange places, still holding up the same fingers, and write that equation: $5 = 1 + 4$. Repeat the activity with different pupils and different numbers. If you are lucky, some pupil will hold up a number of fingers larger than the number you called out. Pupils who regard this as a big joke probably know what they need to know about the number you are using. Those who see nothing funny, unless the reason is a matter of deportment, need some individual help.

Assign page 12 for individual study to those pupils who are ready for it. You may help pupils who have trouble counting correctly up to 10 by using a variety of representations of the number line. Begin by using some sort of number line the pupils can stand on. Have the pupils help you number the points, beginning with 1, using your number cards. Have a group of pupils come to the front of the room. Who can count to find out how many pupils are there? Check by having the pupils, one at a time, find places on the number line as the pupil who is counting says the number he is matching with them. As each pupil takes his place, ask, "How many are there now?" When the pupils appear to be quite secure with this activity, begin to consider subsets: "How many girls? How many boys? How many with red hair? (If there are none, the number is zero.) How many with green shirts?", and so on. The equivalent of this activity can be done with a number line on a pupil's desk or on a demonstration table, and counters which the pupils can pick up and lay on the appropriate points on the line.

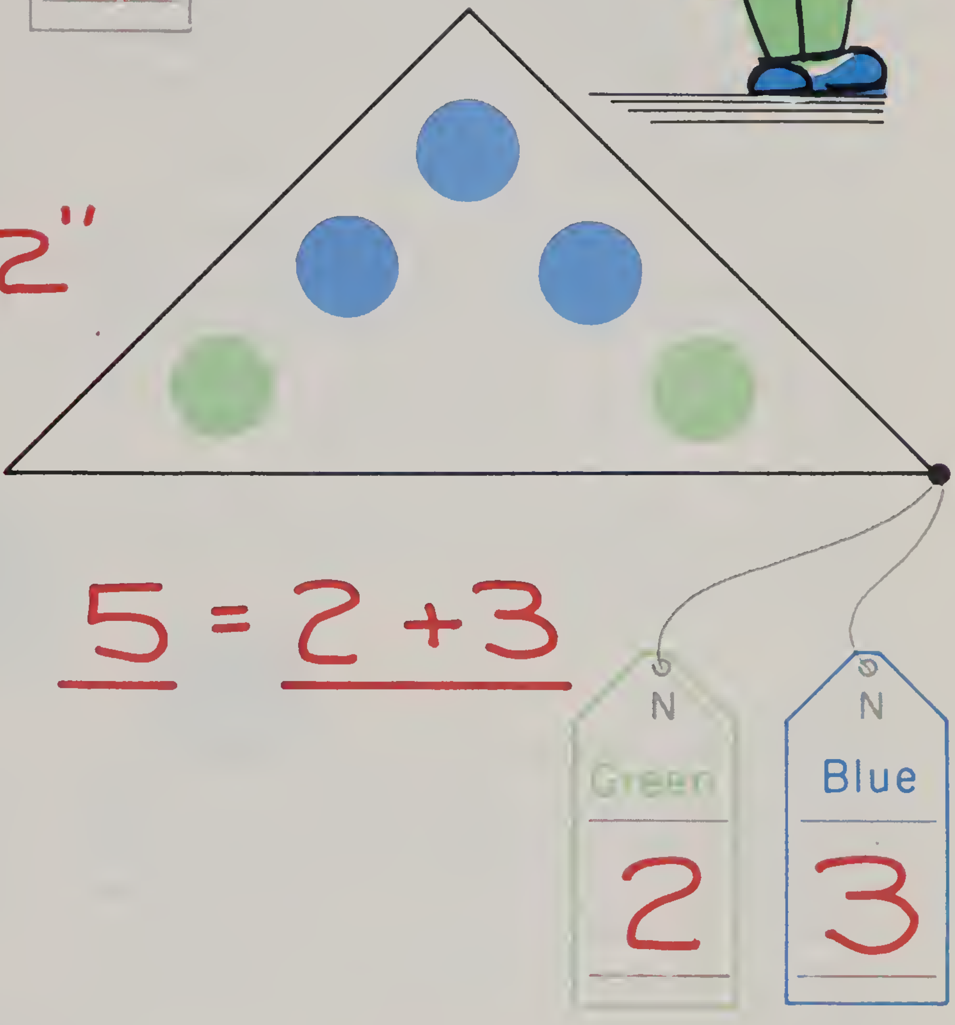
When the pupils have finished page 12, discuss it with them. Show them how to write the appropriate equations and teach them how to read what they have written. Do not expect that they will master either of these skills at this time.

Headline Story: Tell a story with 3 and 2 in it.

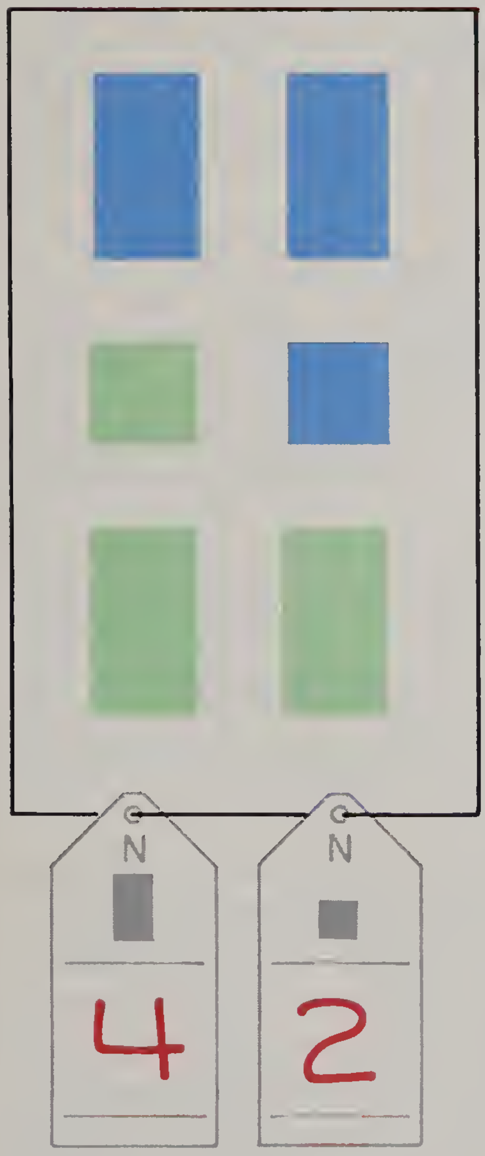

N
N
2
2
1 2 3 4 5 6

$\underline{4} = \underline{2 + 2}$

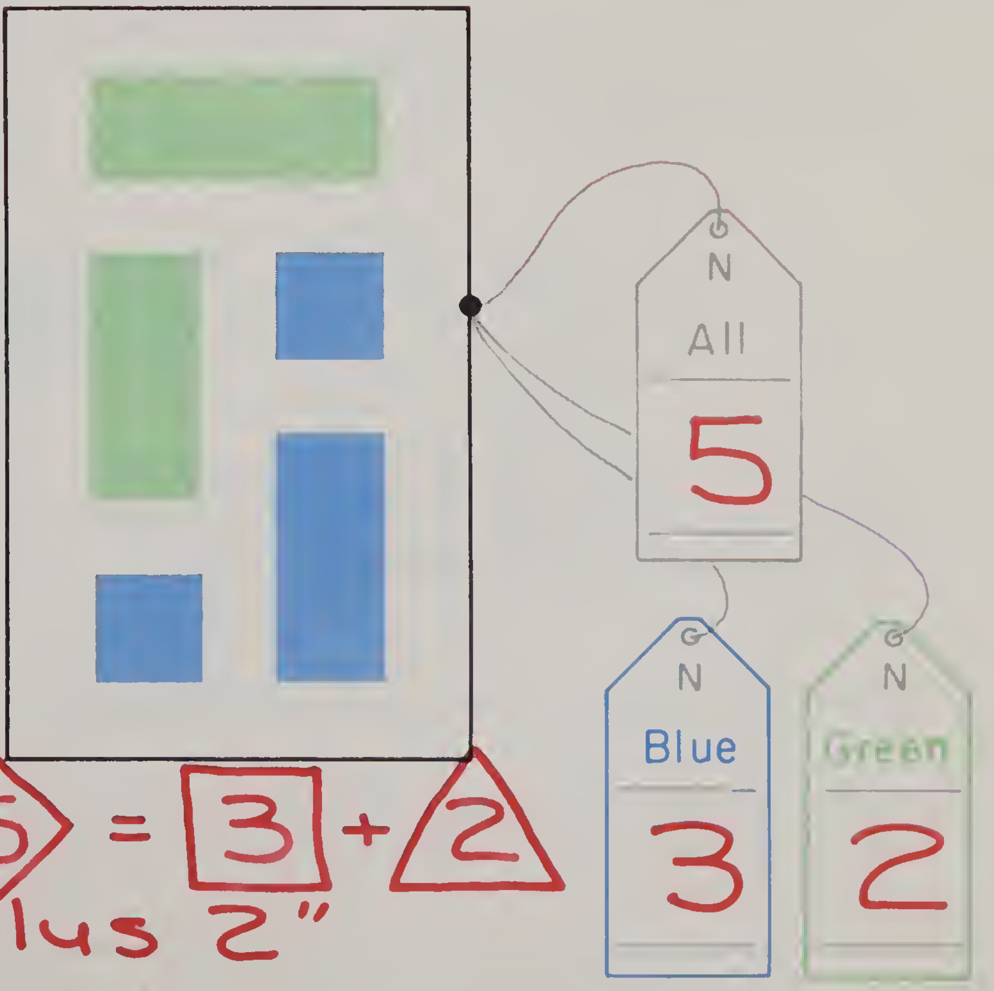
"4 equals 2 plus 2"


N
Green
2
N
Blue
3

$\underline{5} = \underline{2 + 3}$



$\underline{6} = \underline{4 + 2}$


N
All
5
N
Blue
3
N
Green
2

$\Diamond 5 = \boxed{3} + \triangle 2$

"5 equals 3 plus 2"

Notes on Pupil Page 13

Tasks: *Pupils write the correct number in the blank on each tag according to the directions on the tag, e.g., the number of blue or square shapes.*

Purpose: To introduce the idea of decomposing sets into subsets determined by criteria such as color, shape, size, and position. To introduce the writing and reading of mathematical sentences.

Unifying Ideas: Sets; number and counting.

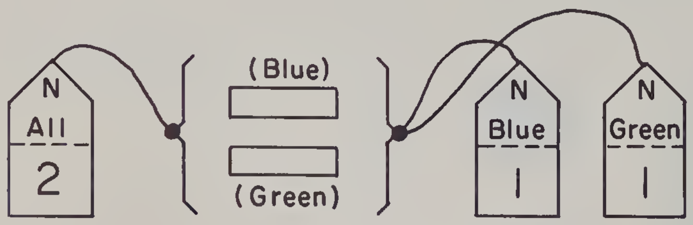
Materials for Teacher: Chalk (white, green, blue), number tags labeled 0 through 2. Two demonstration rectangular-shaped strips of the same size, one blue and one green. A blue square shape, a blue triangular shape, and a green circular shape.

Materials for Pupils: All the above materials except chalk appropriate in size for each pupil to work with at his own desk

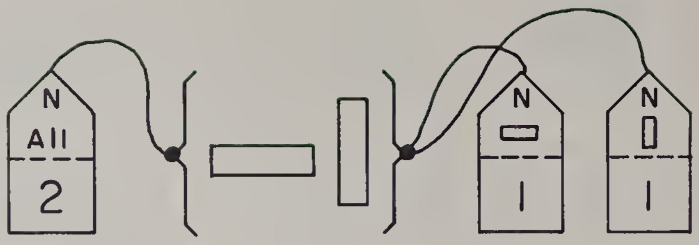
The Lesson:

Hold up your demonstration strips. Have pupils arrange similar strips on their desks. Invite pupils to compare their arrangements with their neighbors' and with yours. Discuss the similarities and differences in size, shape, color, and position of various arrangements. Sketch some of their patterns on the board.

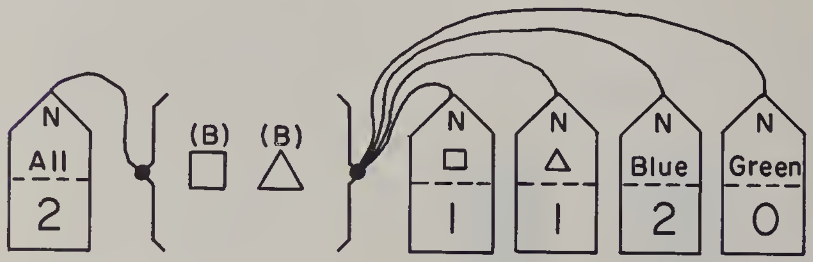
Sketch the following diagrams on the board and discuss the labeling. In each example have pupils arrange their strips in the same way.

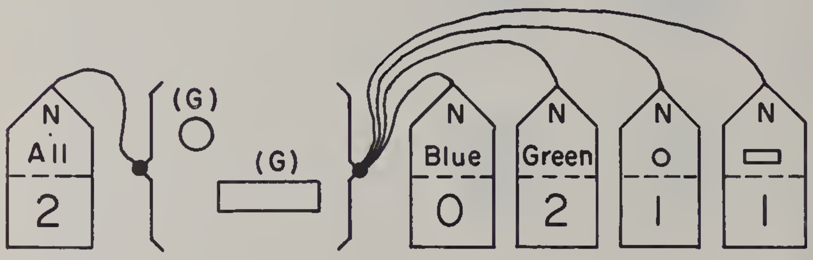


Hold your strips so that one is vertical and the other horizontal. How many in all? How many are lying down? How many are standing up?



Pupils who need no more guided activities may now proceed to complete page 13 on their own. You may need to conduct a sequence of activities similar to those diagrammed below involving the use of the square, triangular, and circular shapes.

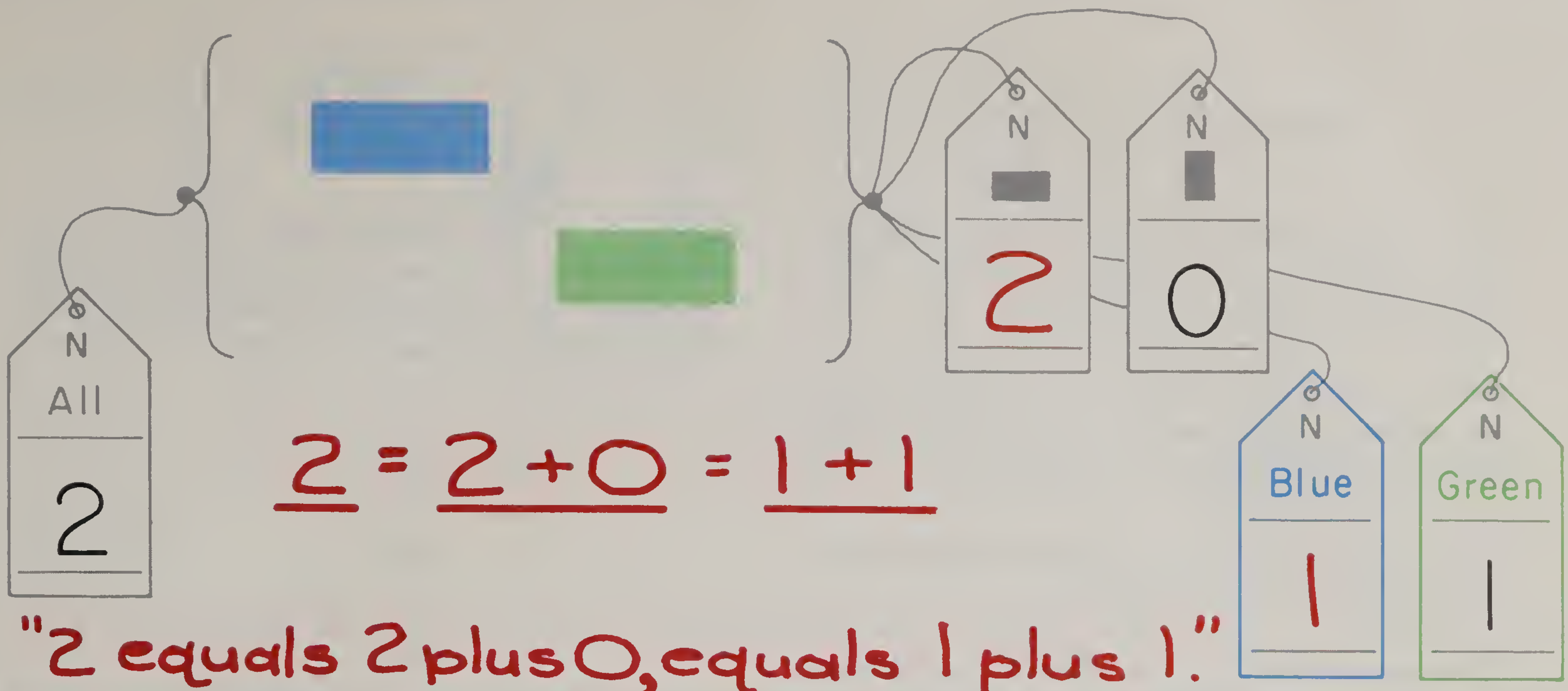




After all pupils have filled in the number tags on page 13 correctly, help them write the equations that summarize what they found out in each row. There are several correct ways to do this. For the first row, one might write: $2 = 2 + 0$ and $2 = 1 + 1$, or: $2 + 0 = 2$ and $1 + 1 = 2$, or $2 + 0 = 2 = 1 + 1$.

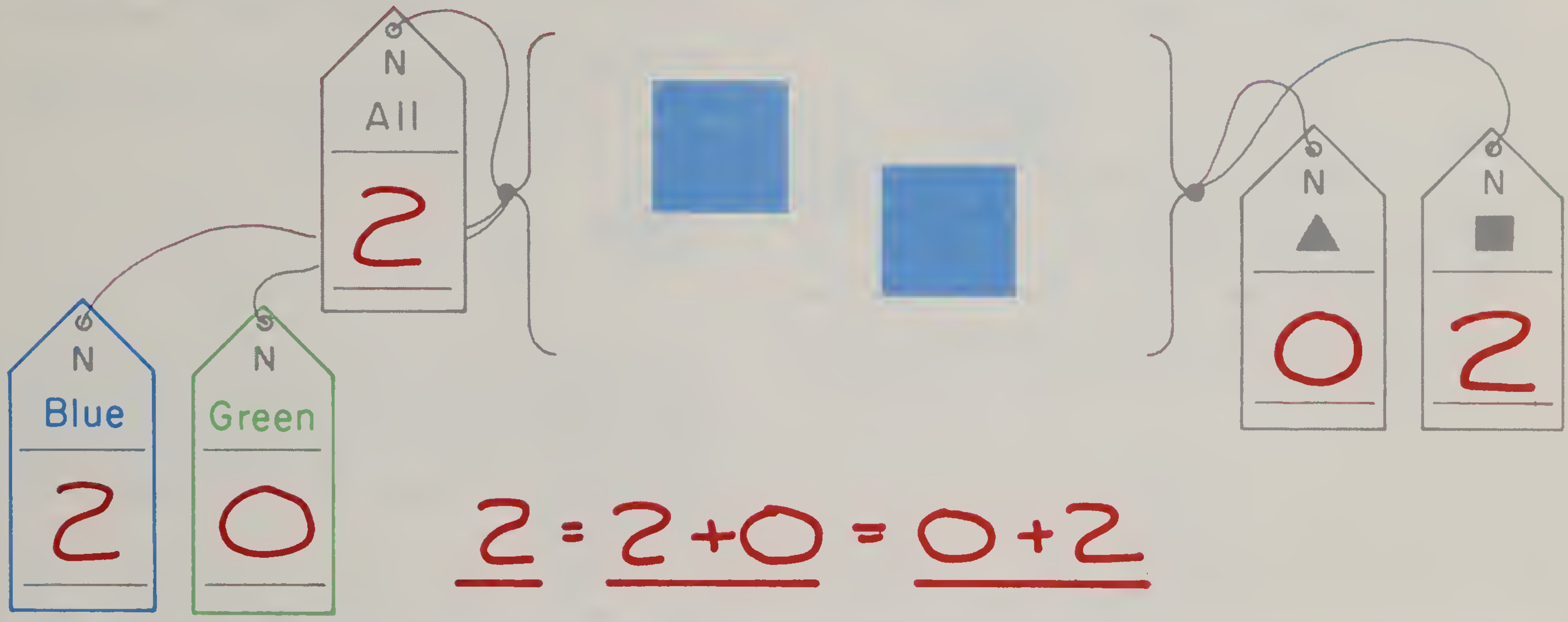
When the pupils have written correct equations of some kind, point out to them that the sign $=$ is the equals sign and is read as "is" or as "equals." The sign $+$ is the plus sign and is read as "plus." Putting these signs in appropriate conjunction with number symbols gives us a *mathematical sentence*, or *number sentence*. Let pupils practice reading their number sentences.

Headline Story: Tell a story that fits: $2 = 1 + 1$ $2 = 0 + 2$

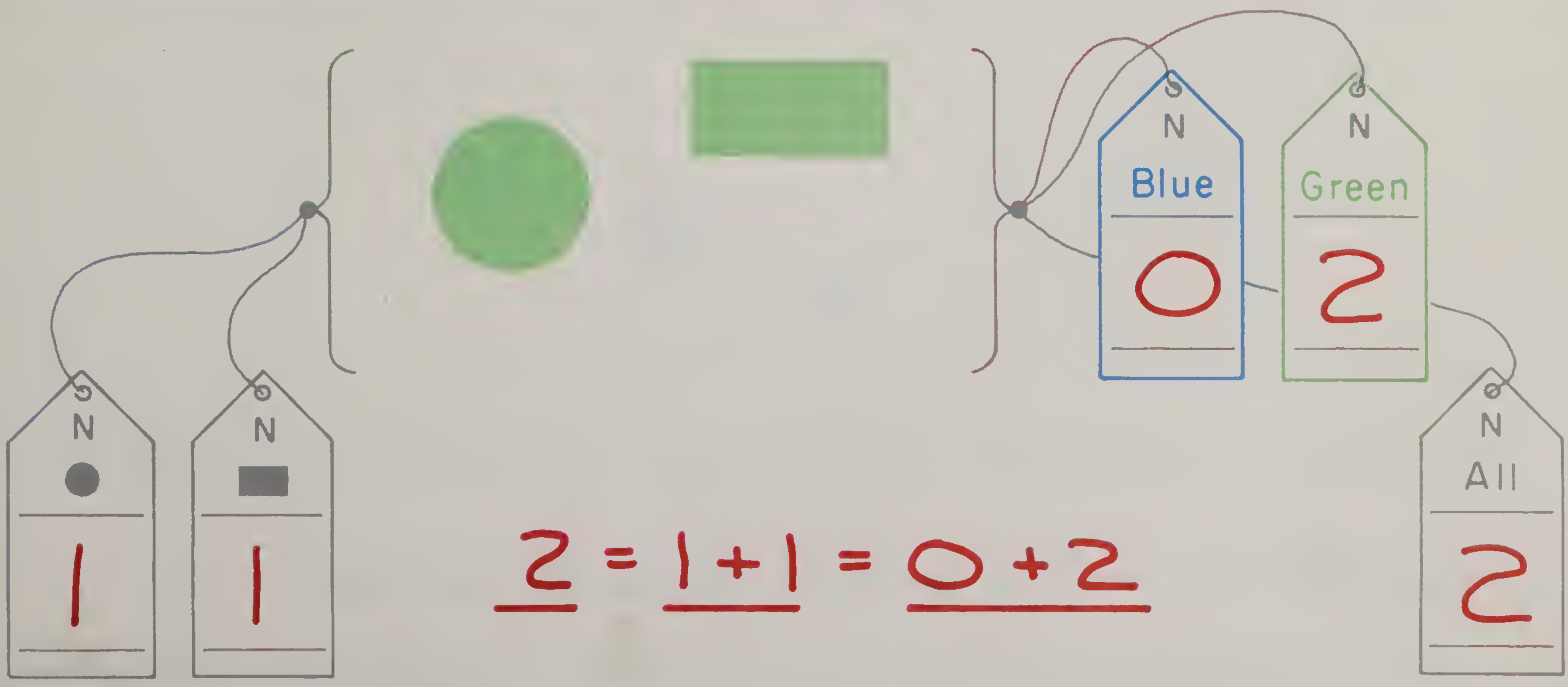

N
All
2
N
2
N
0
N
Blue
1
N
Green
1

$\underline{2} = \underline{2+0} = \underline{1+1}$

"2 equals 2 plus 0, equals 1 plus 1."


N
All
2
N
Blue
2
N
Green
0
N
0
N
2

$\underline{2} = \underline{2+0} = \underline{0+2}$


N
Blue
0
N
Green
2
N
1
N
1
N
All
2

$\underline{2} = \underline{1+1} = \underline{0+2}$

Notes on Pupil Page 14

Tasks: *Pupils count the number of like objects in a set, count the total number, record the counts, and write the appropriate equations.*

Purpose: To teach addition facts with sums of 3 and to show that a set of three objects can be partitioned into subsets in several ways.

Unifying Ideas: Sets; number and counting; addition and subtraction; structure.

Materials for Teacher: No special materials are needed.

Materials for Pupils: A piece of paper folded down the middle; 3 counters.

The Lesson:

Write headings for two tables on the chalkboard: 3 is and: 3 = . Say nothing about the second one at this time. Ask different pupils to tell you the ways they could put their 3 counters on the two parts of their paper, and record what they say as follows, in whatever order the items are mentioned:

3 is
2 and 1
0 and 3
1 and 2
3 and 0

Then go to the second heading and say, "Let's write the same ideas using mathematical sign language." Do the first line or two, and have the pupils help you complete the table.

3 =
2 + 1
0 + 3
1 + 2
3 + 0

Now have the pupils do page 14 in their books. Suggest that they write equations as indicated on the answer page. Practice reading these mathematical sentences.

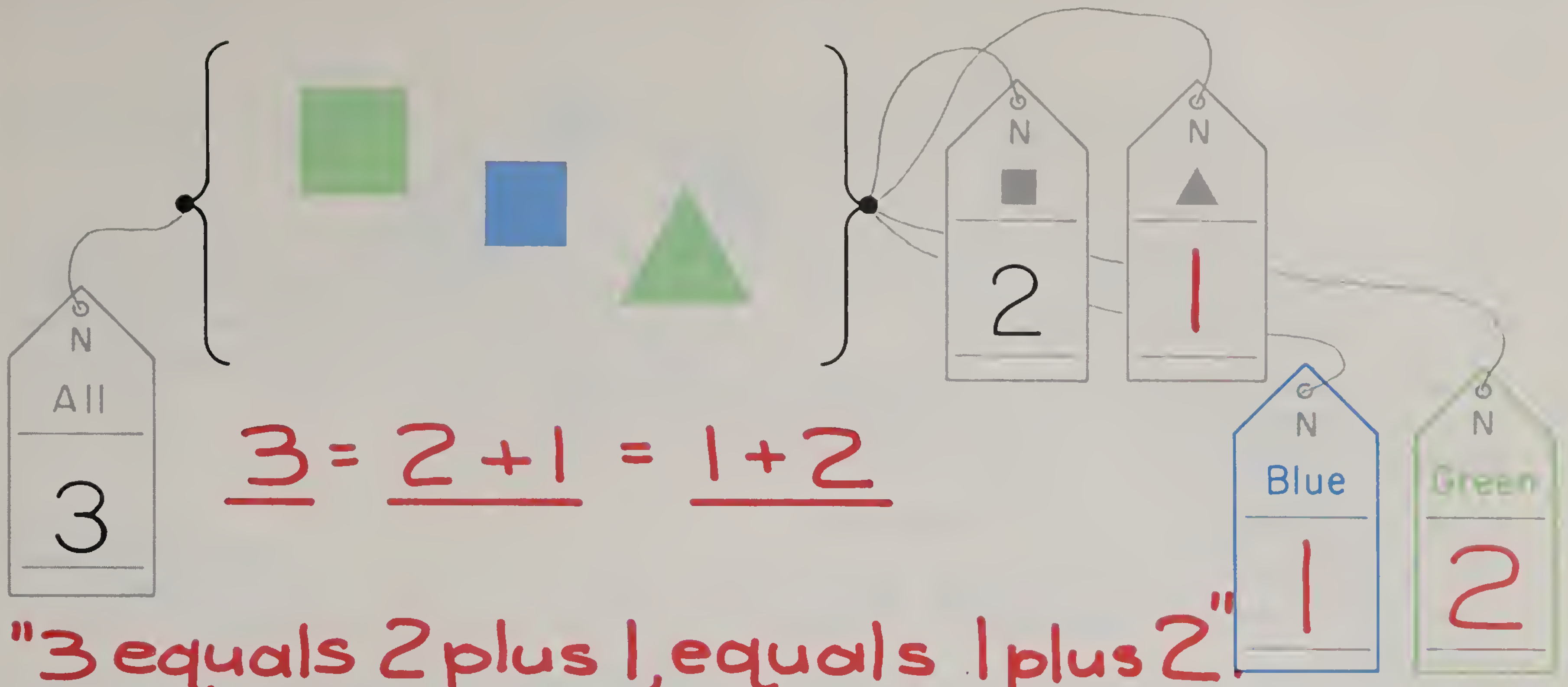
Show the pupils another way to write simple sentences with the same content.

$$2 + 1 = 3 \qquad \begin{array}{r} 2 \\ +\ 1 \\ \hline 3 \end{array}$$

They might deduce that, in the vertical form, the bar takes the place of the = sign, and that the sentence makes good sense whether read from top to bottom or bottom to top. Then assign some practice exercises:

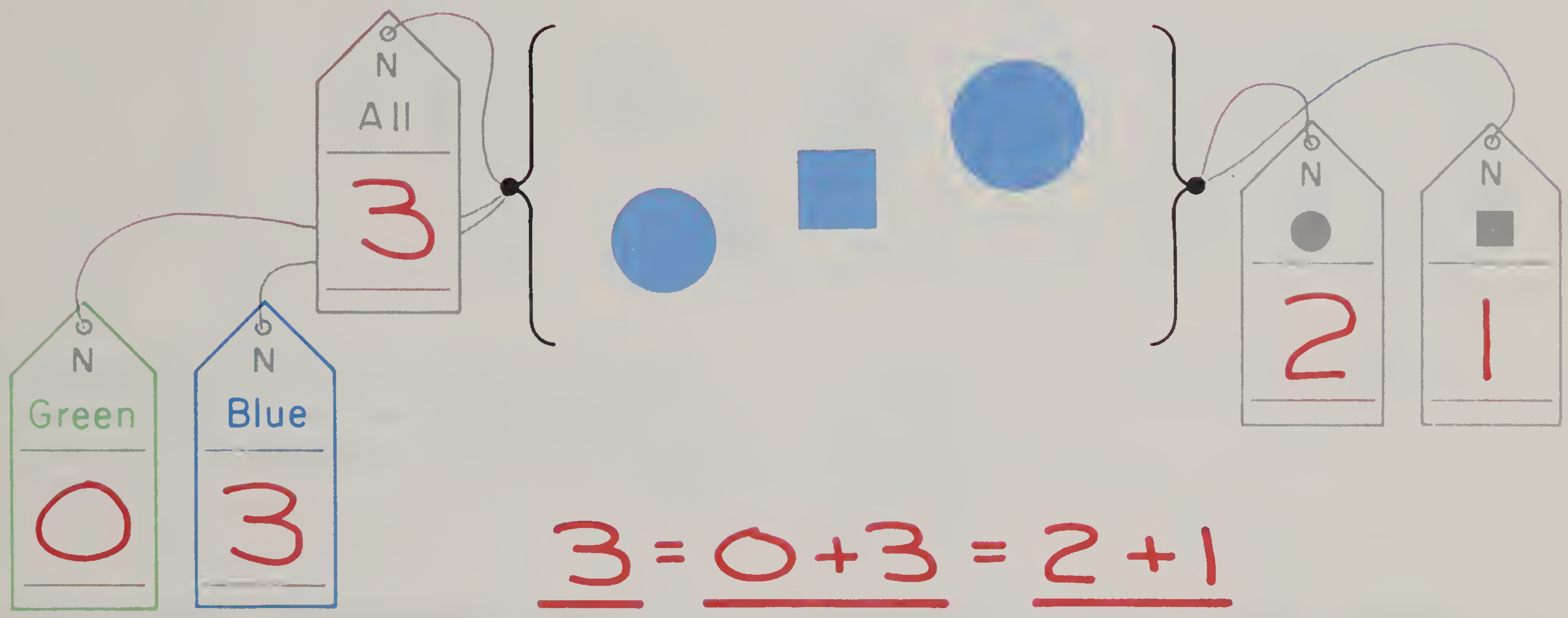
$$\begin{array}{r} 1 \\ +\ 2 \\ \hline \end{array} \quad \begin{array}{r} 2 \\ +\ 1 \\ \hline \end{array} \quad \begin{array}{r} 0 \\ +\ 3 \\ \hline \end{array} \quad \begin{array}{r} 3 \\ +\ 0 \\ \hline \end{array} \quad \begin{array}{r} 1 \\ +\ 1 \\ \hline \end{array} \quad \begin{array}{r} 2 \\ +\ 0 \\ \hline \end{array} \quad \begin{array}{r} 0 \\ +\ 2 \\ \hline \end{array}$$

Headline Story: There were 2 big apples and 1 small apple. How many apples?

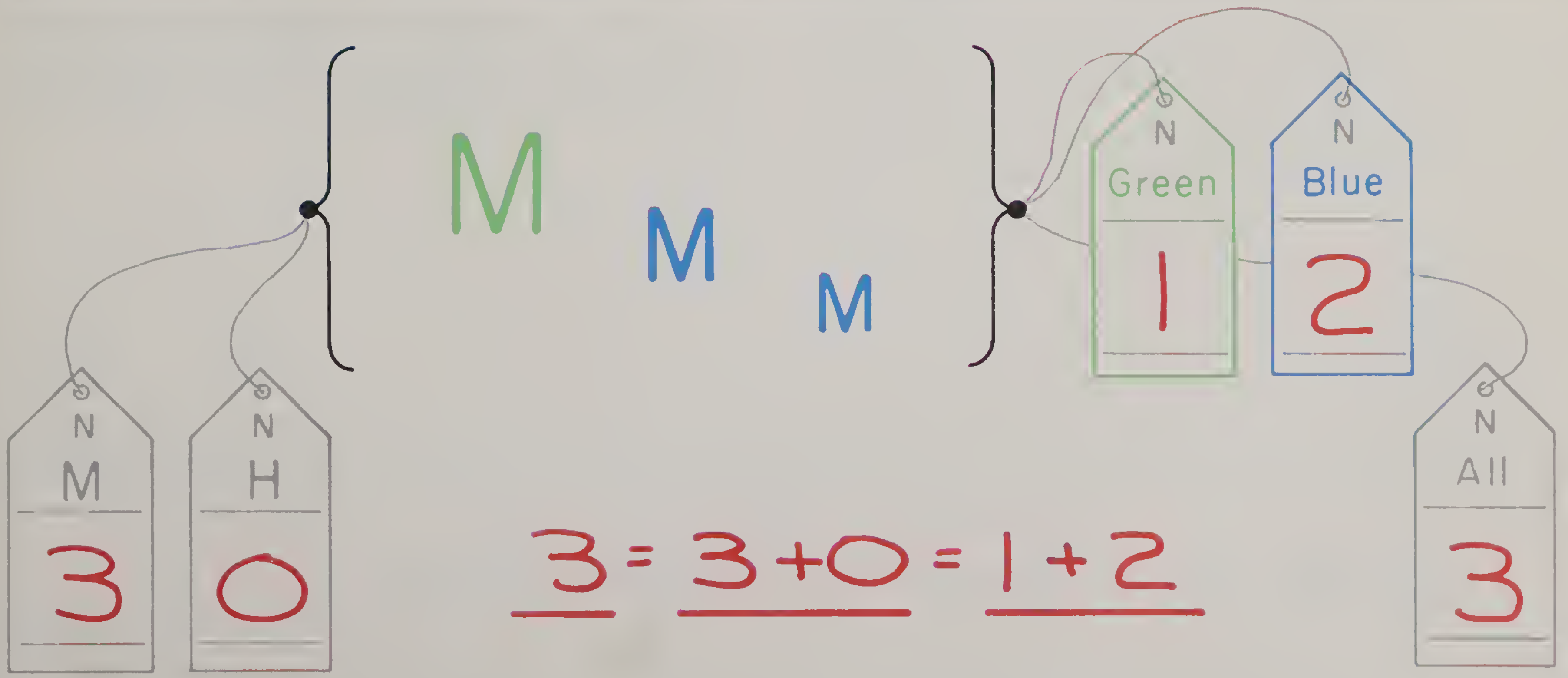

N
All
3
N
2
N
1
N
Blue
1
N
Green
2

3 = 2+1 = 1+2

"3 equals 2 plus 1, equals 1 plus 2"


N
All
3
N
Green
0
N
Blue
3
N
2
N
1

3 = 0+3 = 2+1


M
M
M
N
M
3
N
H
0
N
Green
1
N
Blue
2
N
All
3

3 = 3+0 = 1+2

Notes on Pupil Page 15

Tasks: *Pupils count the number of like objects in a set, count the total number, record the counts, and write the appropriate equations.*

Purpose: To teach the addition facts with sums of 4 and to show that a set of four objects can be partitioned into subsets in several ways.

Unifying Ideas: Sets; number and counting; addition and subtraction; structure.

Materials for Teacher: A representation of the number line.

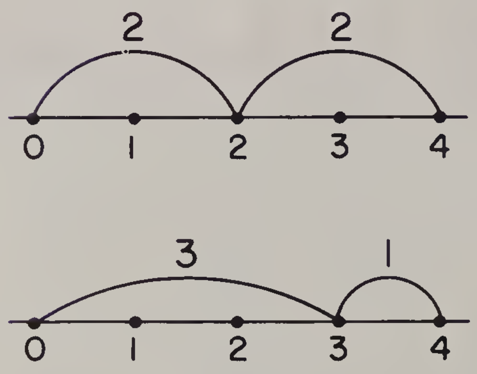
Materials for Pupils: Four each of two kinds of counters.

The Lesson:

Ask the pupils to take out 8 counters, 4 each of two kinds. Tell them they are to discover all the ways they can put 4 counters in a row on their desks using these two kinds. Help each pupil keep a record of what he finds out, as shown below. (Of course, the order in which entries are made may be different.)

Arrangements of counters:	Record:		
	N ☐	N ○	N All
☐ ☐ ☐ ☐	4	0	4
☐ ☐ ☐ ○	3	1	4
☐ ☐ ○ ○	2	2	4
☐ ○ ○ ○	1	3	4
○ ○ ○ ○	0	4	4

Do the equivalent activity in terms of jumps on a number line. You may have the pupils actually do the jumping on a number line on the floor, or you may use pictures on the chalkboard or a piece of paper.



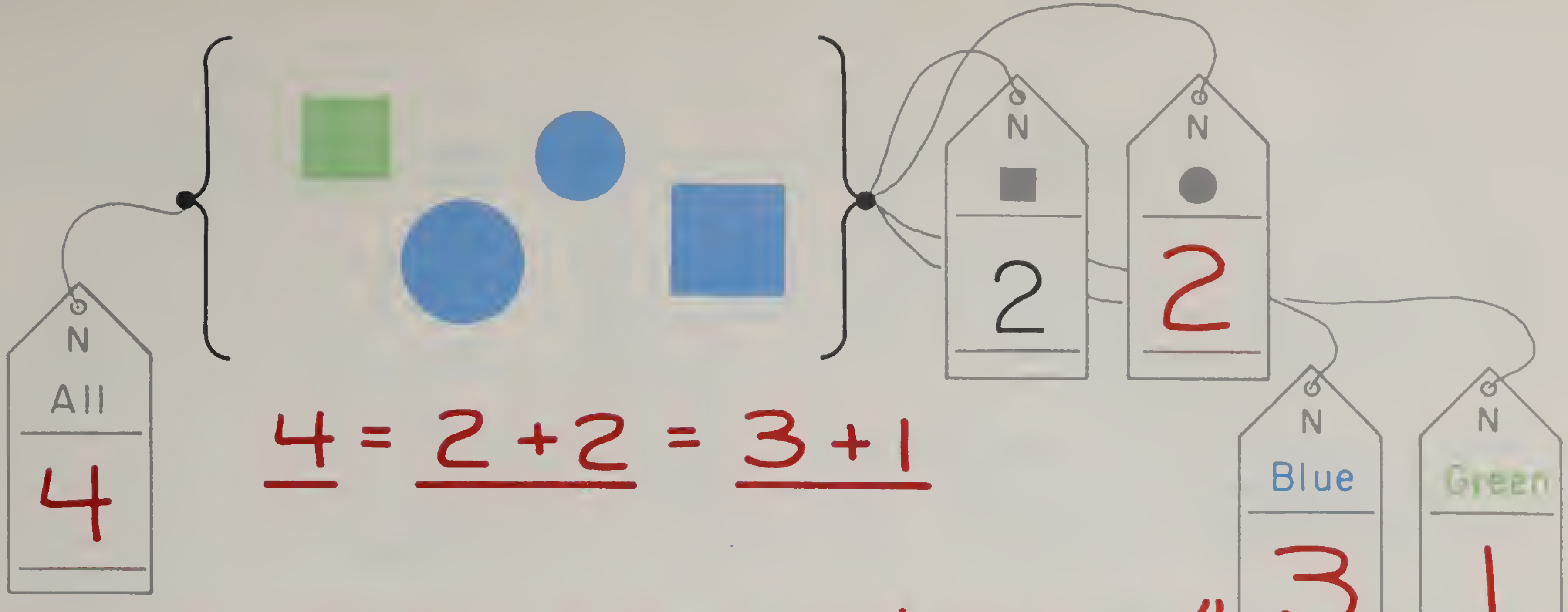
First jump	Second jump	Land
2	2	4
3	1	4

etc.

Compare the results of these two activities. Be sure that each pupil has been involved actively in each of them. Then assign page 15 for individual study.

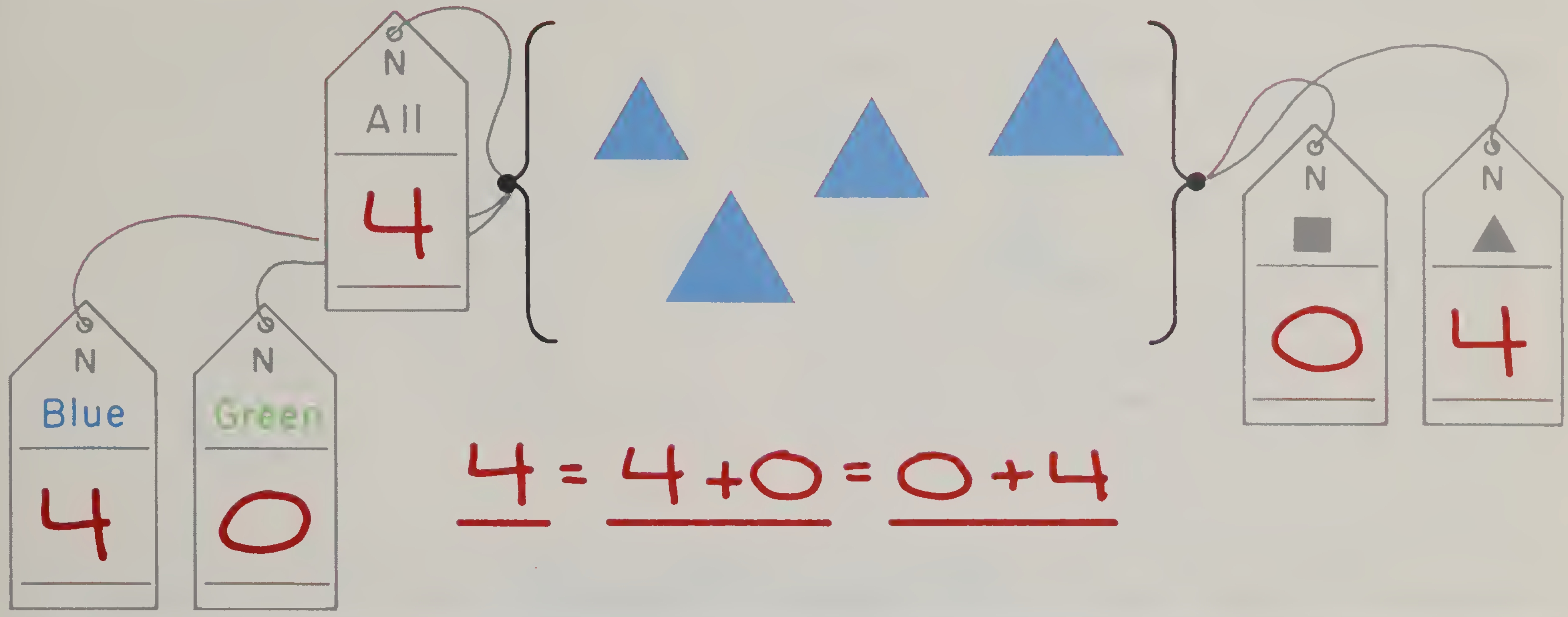
Be sure to review the accompanying booklet, *Games and Enrichment Activities*, for suggestions you may wish to incorporate in your daily lesson plan.

Headline Story: Bill had 4 wheels on his wagon. (Pupils extend the story.)

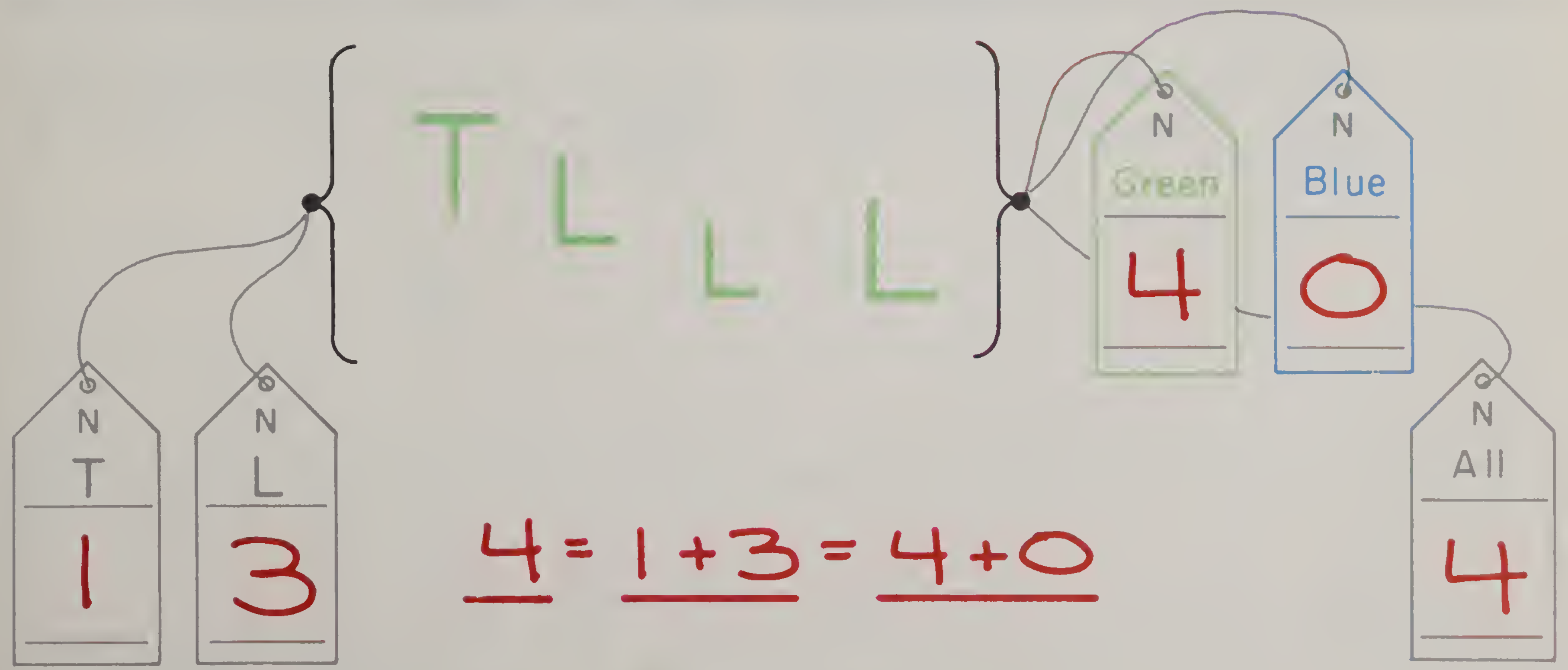

N
All
4
N
■
2
N
●
2
N
Blue
3
N
Green
1

$\underline{4} = \underline{2+2} = \underline{3+1}$

"4 equals 2 plus 2, equal 3 plus 1"


N
All
4
N
Blue
4
N
Green
0
N
■
0
N
▲
4

$\underline{4} = \underline{4+0} = \underline{0+4}$


T L L L
N
T
1
N
L
3
N
Green
4
N
Blue
0
N
All
4

$\underline{4} = \underline{1+3} = \underline{4+0}$

Notes on Pupil Page 16

Tasks: *Pupils count the number of like objects in a set, count the total number, record the counts, and write the appropriate equations.*

Purpose: To teach the addition facts with sums of 5 and to show that a set of five objects can be partitioned into subsets in several ways.

Unifying Ideas: Sets; number and counting; addition and subtraction; structure.

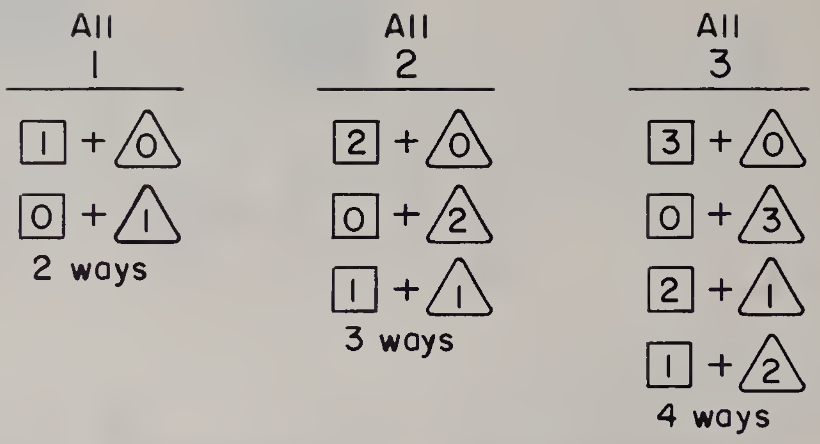
Materials for Teacher: Two boxes; 5 small objects; number lines.

Materials for Pupils: Duplicates of the teacher's materials, as needed.

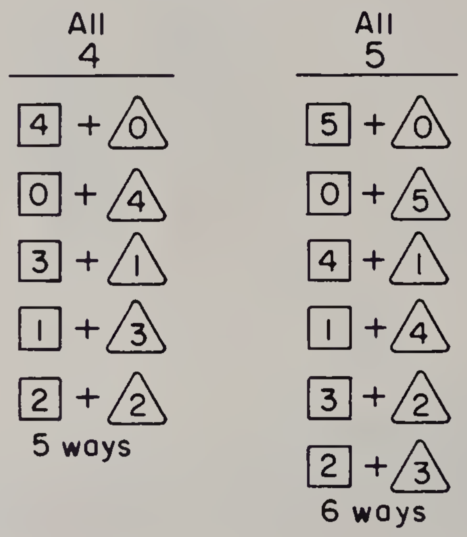
The Lesson:

Refer to the suggestions for pages 13, 14, and 15, and adapt the activities described in the teacher's guide for those pages to working with the number 5. (As the numbers you work with get larger, the number of possible partitions increases; so, you should expect the time required in preparing pupils for the work on pupil's page 16 to be more than the time used in preparing pupils for the corresponding work on pupil's page 13—15.)

When the pupils have finished page 16, take time to summarize the addition facts they have been taught to date. One way to do this is to use two boxes, one identified with a square shield and the other by a triangular shield. (This will pave the way for the use of frames.) Record all results, somewhat as follows:

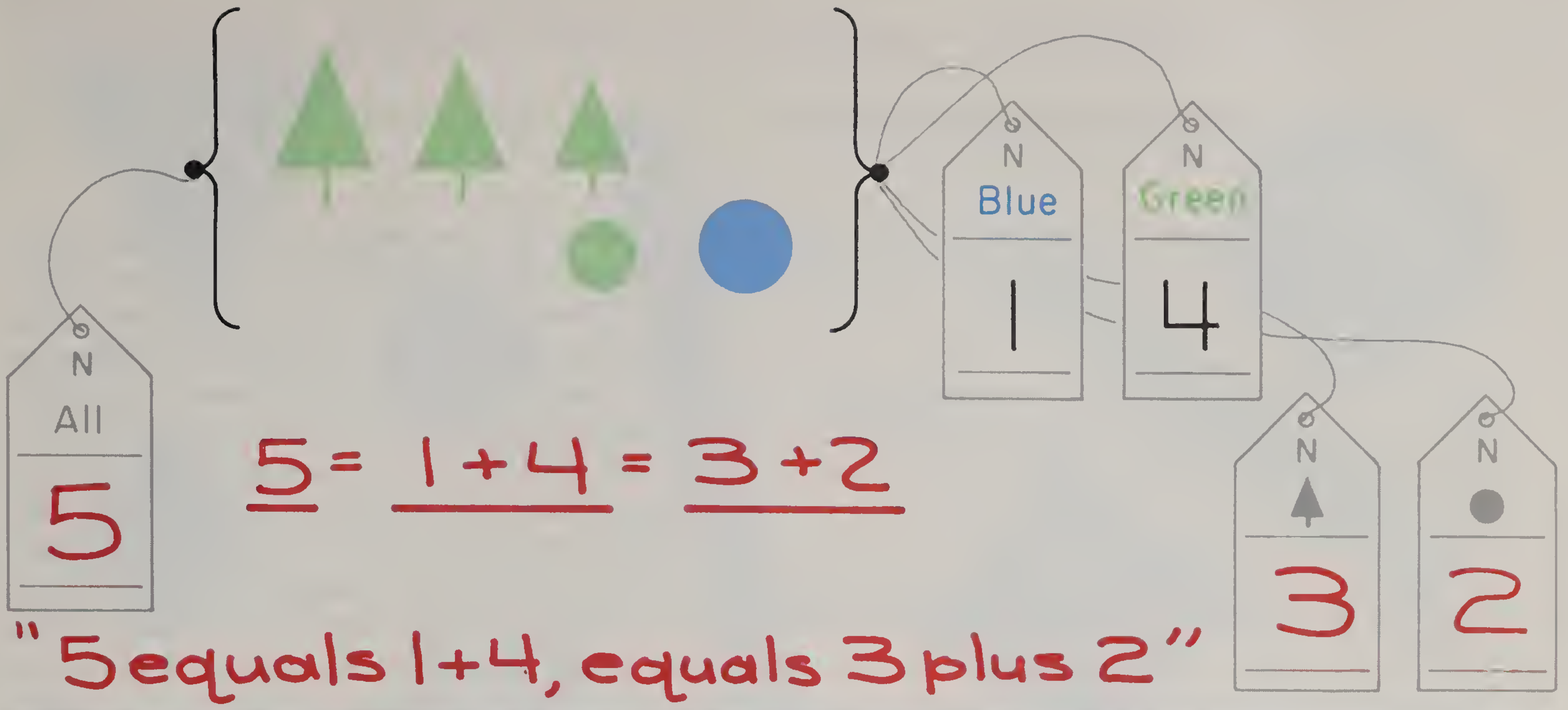


At this point, can anyone predict how many ways there are to make 4 in this manner? Test the prediction. Then do the same with 5.



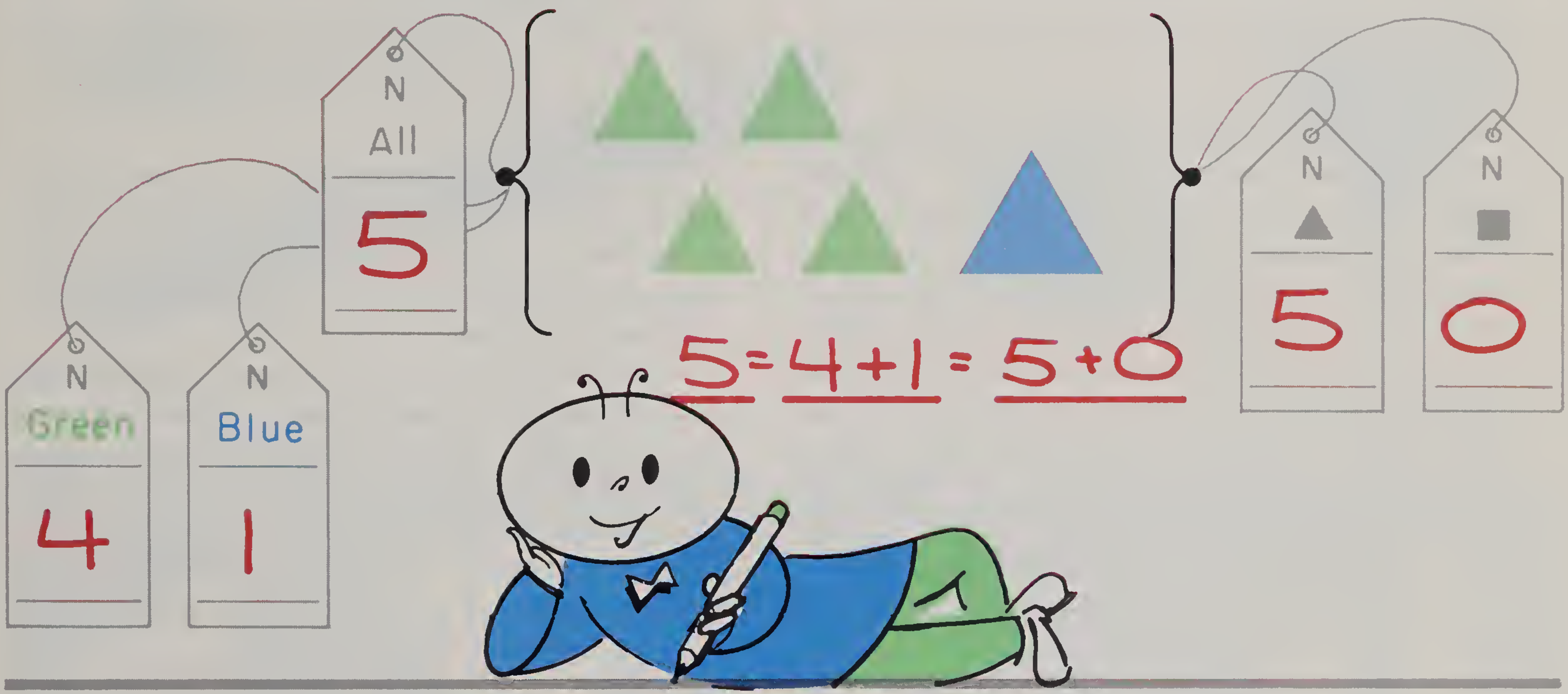
Now, go back and look at all the tables. Be sure that every pupil notices that when the same two numbers are used in the opposite order, the total remains the same. For instance, under 3, he will find 2 in the triangle and 1 in the square; also under 3, he will find 1 in the triangle and 2 in the square. Have him find other such situations. These observations can lead to economy in learning the addition facts.

Headline Story: Tell a story with 1 and 4 in it . . . another with 5 in it.

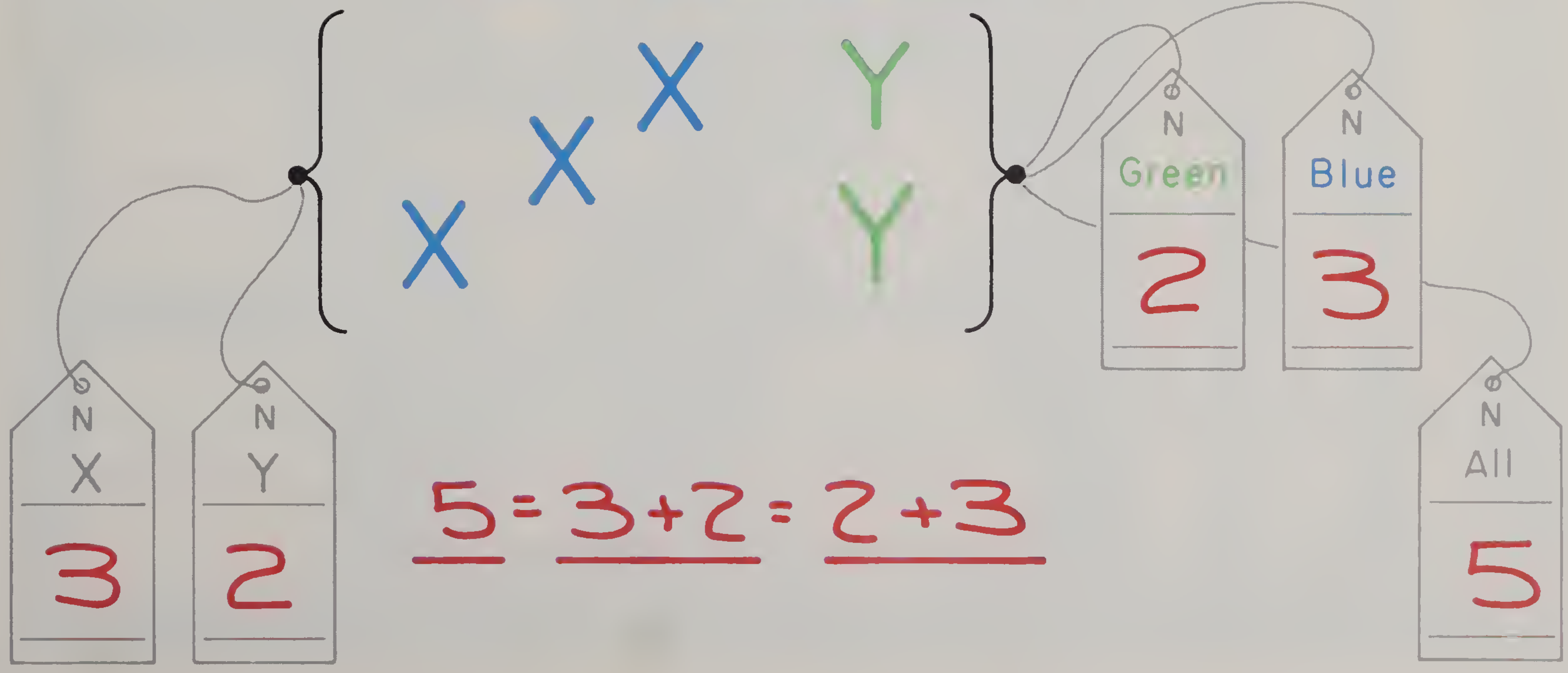

N
All
5
N
Blue
1
N
Green
4
N
▲
3
N
●
2

$\underline{5} = \underline{1+4} = \underline{3+2}$

"5 equals 1+4, equals 3 plus 2"


N
All
5
N
Green
4
N
Blue
1
N
▲
5
N
■
0

$\underline{5} = \underline{4+1} = \underline{5+0}$


X
X
X
Y
Y
N
Green
2
N
Blue
3
N
X
3
N
Y
2
N
All
5

$\underline{5} = \underline{3+2} = \underline{2+3}$

Notes on Pupil Page 17

Tasks: *Pupils color the shapes in the gray section to match colors of shapes of the same size at the top of the page.*

Purpose: To provide experience in recognition of shapes and recognition of sequences of similarly-shaped objects which differ in size.

Unifying Ideas: Geometry.

Materials for Teacher: No special materials are needed.

Materials for Pupils: A duplicate of the top half of page 17, prepared as described below; a box of crayons; scissors.

The Lesson:

Trace the shapes on the top part of this page on a Ditto master, make duplicates, and distribute them to the pupils. Have each pupil select three colors. Ask him to find three pictures of the same shape on his paper and color each of these a different color. Talk about the shapes. The pupils should learn to use the words *square*, *circle*, and *triangle*. Decide whether you want your class to call the other shape *rectangle* or *bar* for the time being. Let the pupils discuss the shapes in their own way. How can they tell when something is square? Ask them to draw a square in the air either by using one hand, as in sky writing, or by using both hands to outline the shape. How many sides (or edges) has a square? What can we say about these edges? (They are all the same length.) How many corners (or angles) does a square have? What do these corners look like? Are the corners of a square like the corners of a rectangle (or bar)? How is a square different from a rectangle? Do the pupils know any big words which have the little word *tri* in them. How many wheels has a tricycle? How many sides does a triangle have? What does a circle look like? Ask each pupil the name of the shape of the pictures he has colored. Tell him to color the three examples of each of the other shapes in the same way. When the coloring is completed, examine the results. Which triangle is the largest? The smallest? The "in-between" size? Have the pupils cut out the triangles and test their answers to these questions. Do the same with the other shapes.

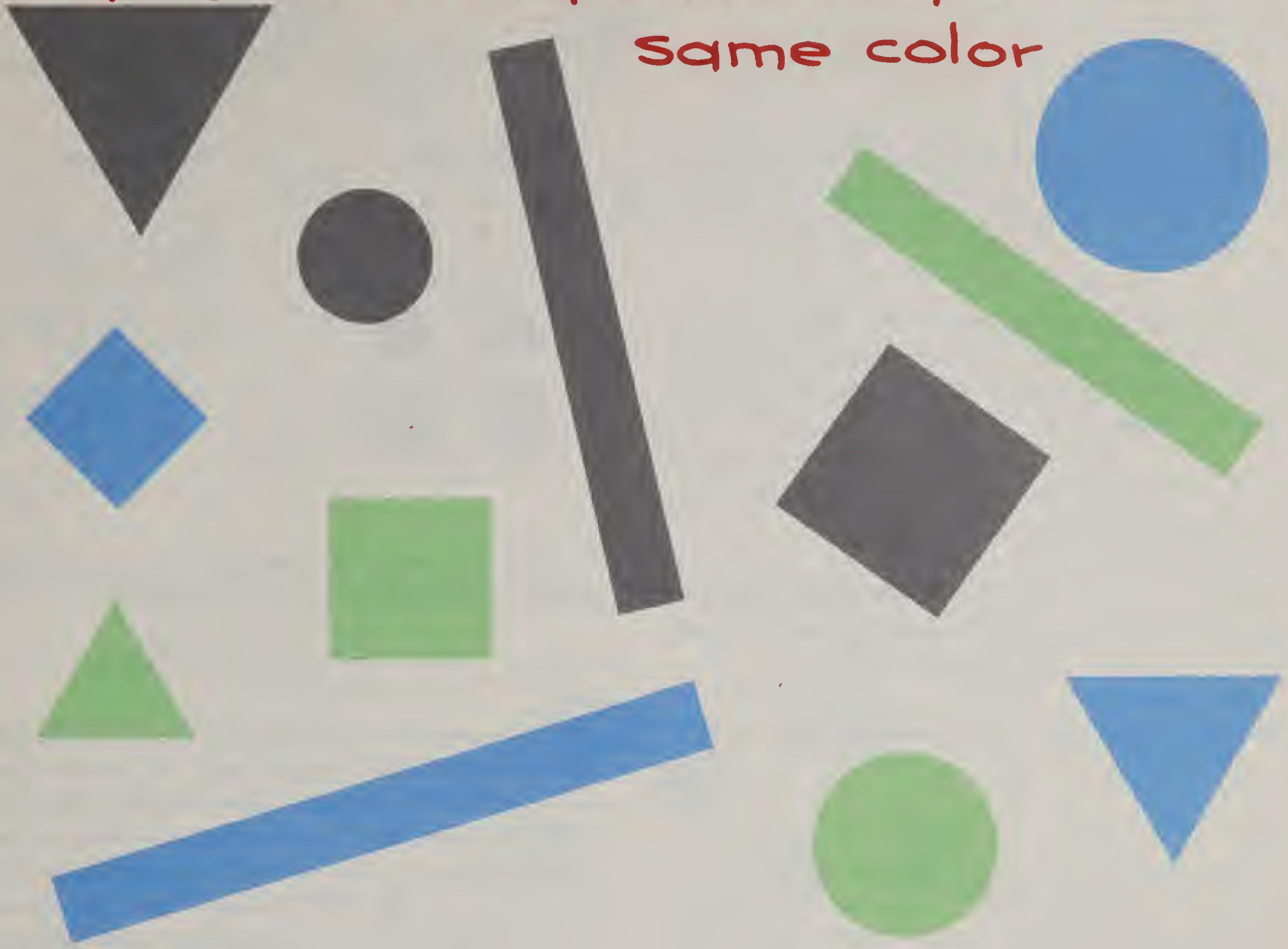
Ask the pupils to look at page 17 in their books. See if they can tell what to do with the page. Check to see that each pupil has a blue crayon, a green crayon, and a gray or black crayon; then have him complete the page.

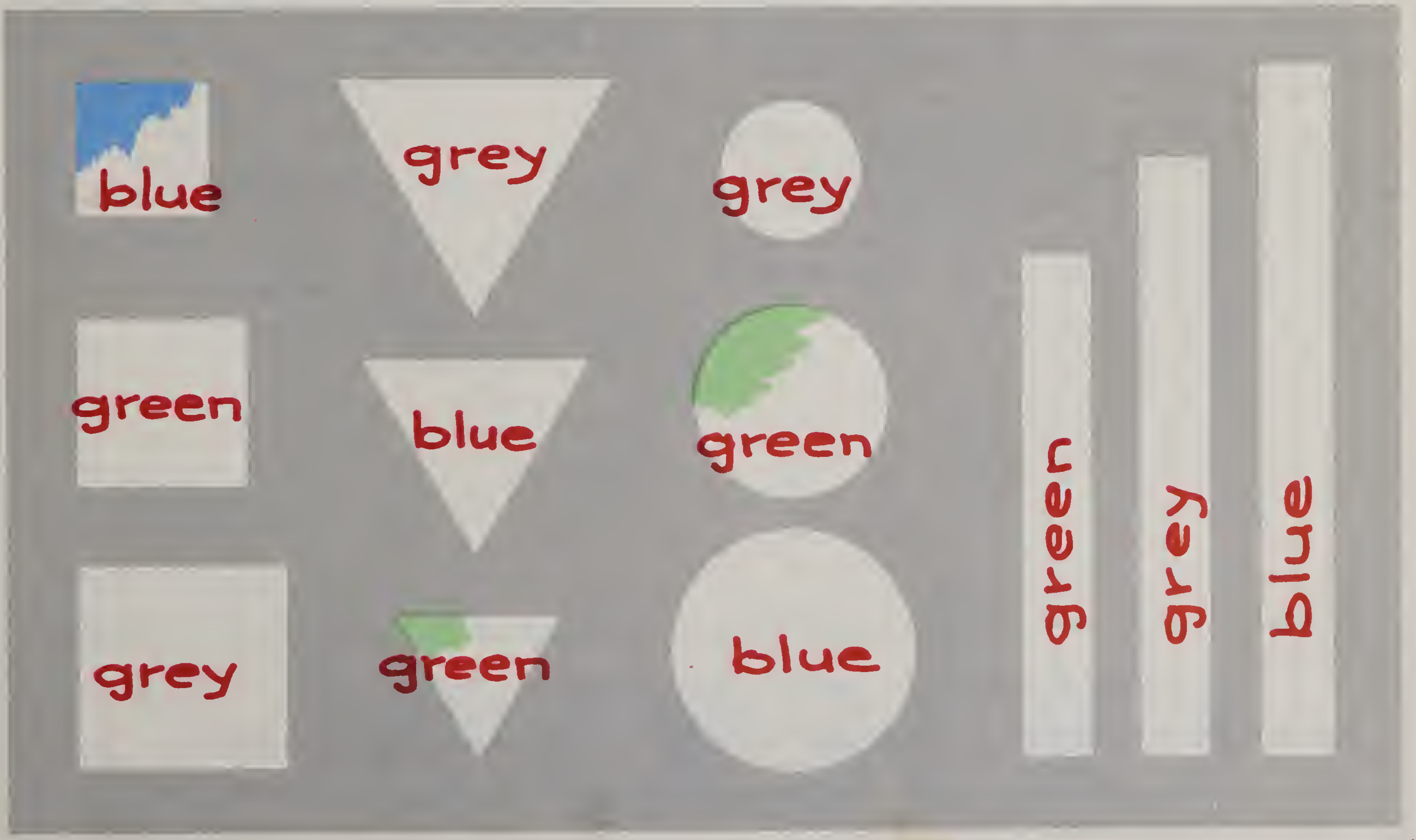
Have the pupils look around their classroom and find things shaped like squares, circles, triangles, and rectangles. Where do they see these shapes in their homes, on their way to school, in the dime store, or at a garage?

Make a poster with the caption SHAPES. Have the pupils bring in pictures to paste on this poster.

Headline Story: Tell a story including the idea that 5 is more than 3.

Same size and same shape - same color




blue
grey
grey
green
blue
green
green
grey
blue
grey
green
blue

Notes on Pupil Page 18

Tasks: *Pupils count the number of like objects in a set, count the total number of objects in the set, record the counts, and write the appropriate equations.*

Purpose: To teach addition facts with sums of 6, to show that a set of 6 objects can be partitioned into subsets in a variety of ways, and to introduce the use of frames.

Unifying Ideas: Sets; number and counting; addition and subtraction; structure.

Materials for Teacher: Two boxes and a variety of shapes of shields (triangle, circle, square, hexagon, etc.); 6 objects to put into the boxes.

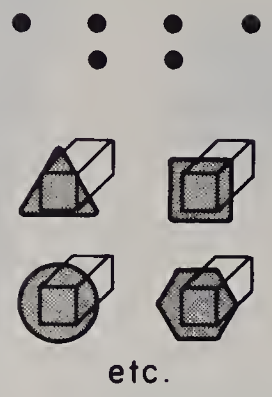
Materials for Pupils: No special materials needed.

The Lesson:

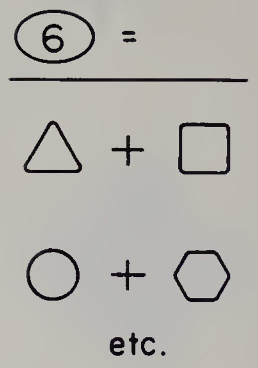
The simple performance of filling in the tags on this page should be easy for your pupils now, but there is an unfamiliar item at the bottom of the page. It is not intended that the pupils become thoroughly familiar with the use of frames and = signs and + signs all in one lesson. It is intended that they become acquainted with these things in one of the contexts in which they may be used.

Use the 2 boxes, 6 objects, and 2 shields at a time in determining the different ways 6 objects can be put in two boxes. Build the following table, row by row, on the board. Make the frames the shape of the shields you are using at the time. Ask the pupils to help you fill in the frames, a way of recording the activity.

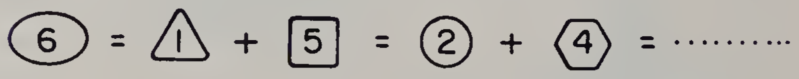
On the table:



On the board:



When the table is completed, summarize the activity by writing a series of equations.



Assign page 18 for individual study. When the pupils have finished, call attention to the equation at the bottom of the page. Remind them that the sign = is called the "equals sign" and that we can read it either "is" or "equals." Remind them that the sign + is the plus sign and that we always read it "plus." Have several pupils read this sentence. Have them write similar sentences (without frames) for the other two rows on the page.

You may wish to call attention to the chart on your bulletin board:

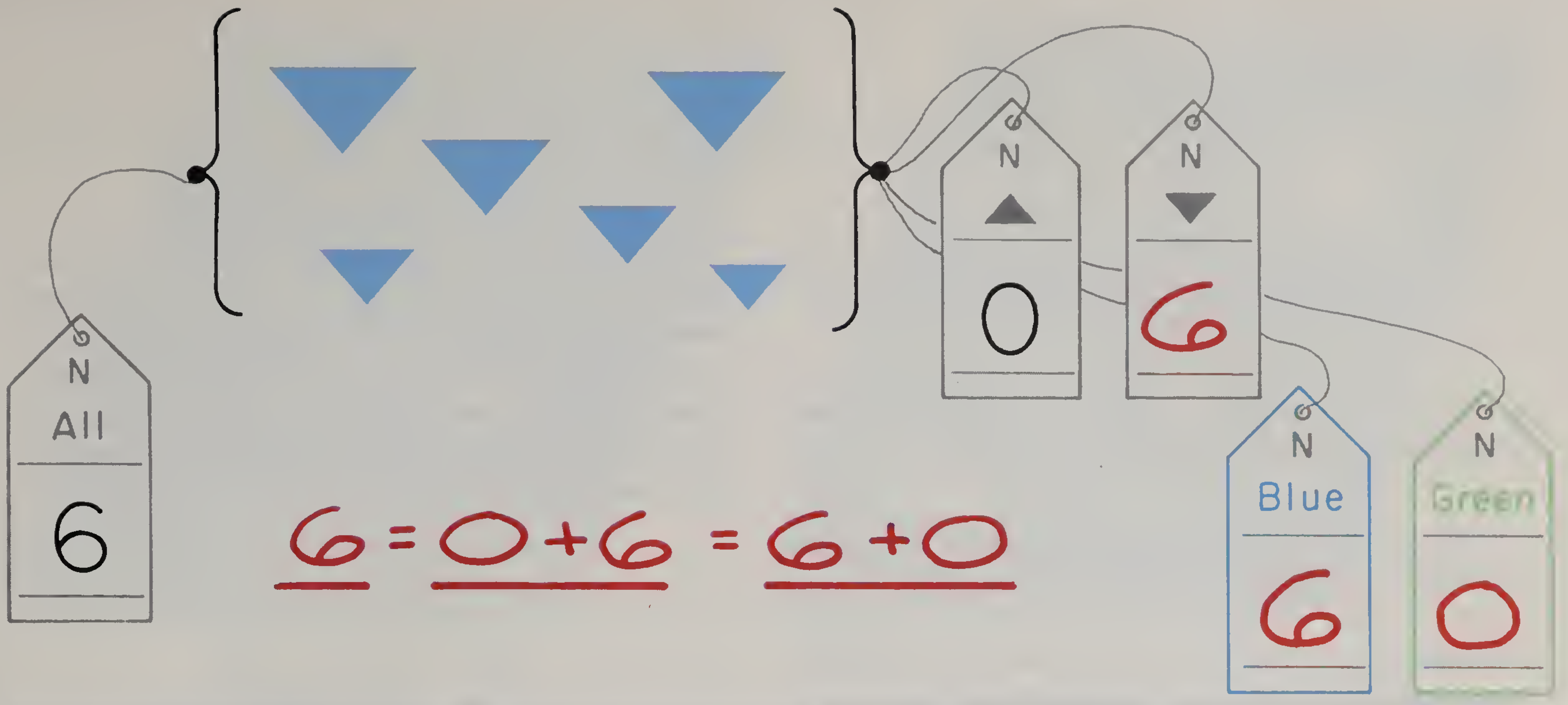
+ plus

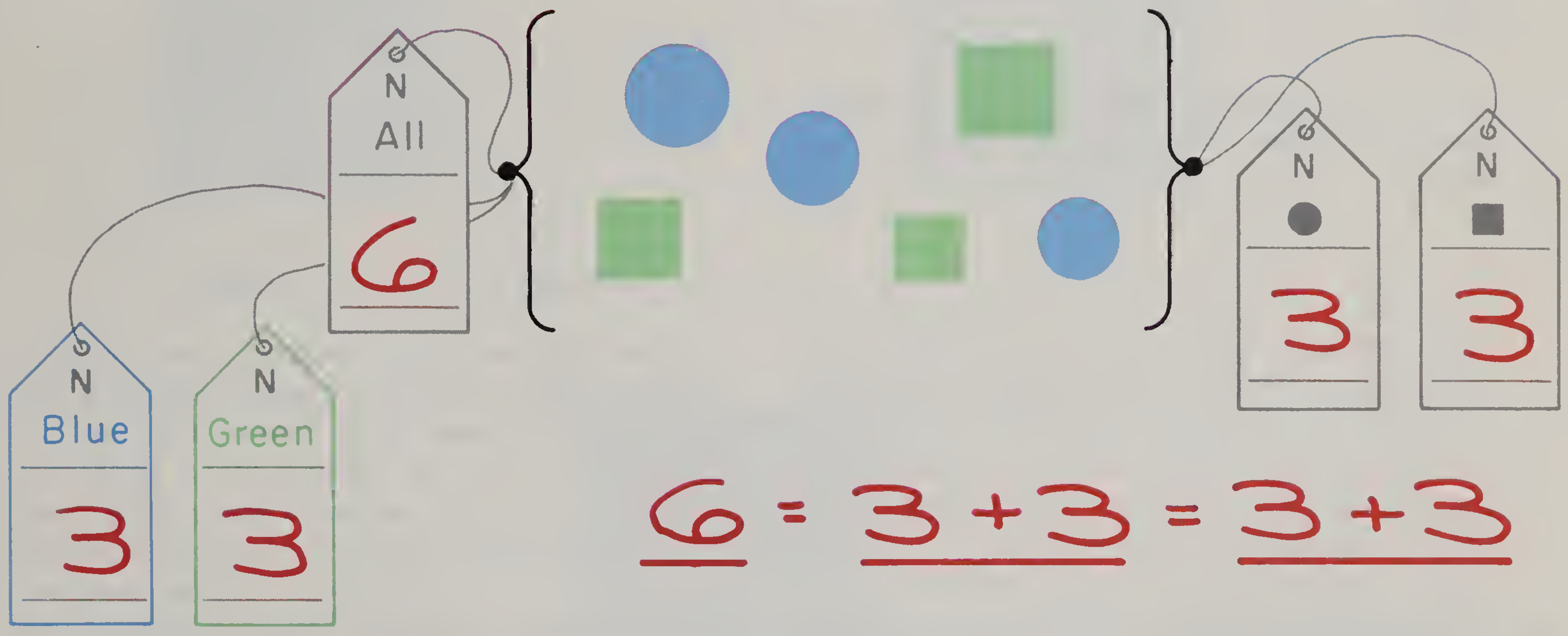
= equals

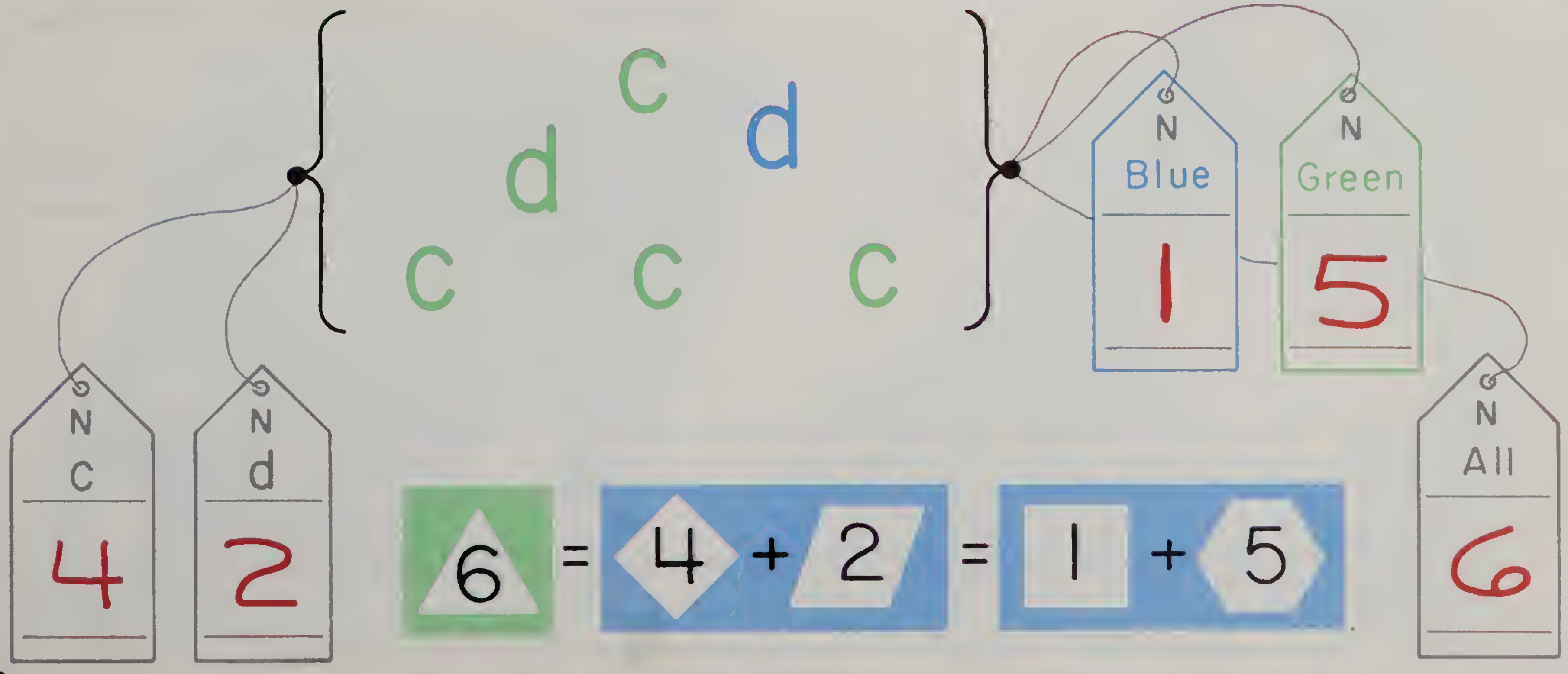
1 + 1 = 2

1 plus 1 equals 2

Headline Story: Dan had 6¢. If he spent __¢, then he has __¢ left. (Make a chart.)


N
All
6
N
▲
0
N
▼
6
N
Blue
6
N
Green
0
6 = 0 + 6 = 6 + 0


N
All
6
N
●
3
N
■
3
N
Blue
3
N
Green
3
6 = 3 + 3 = 3 + 3


c
d
d
c
c
c
N
Blue
1
N
Green
5
N
c
4
N
d
2
N
All
6
6 = 4 + 2 = 1 + 5

Notes on Pupil Page 19

Tasks: *Pupils count the number of objects in each compartment and record the number in a special two-by-two table.*

Purpose: To provide practice in simple counting and to lead into a significant new activity.

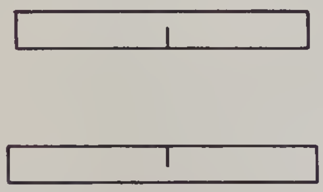
Unifying Ideas: Sets; number and counting; measurement.

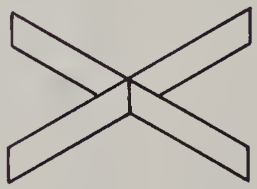
Materials for Teacher: A flat box with dividers partitioning it into four compartments; five or six objects which can be put into these compartments, or the equivalent materials for use on a flannel board.

Materials for Pupils: Two or more small pieces of paper, each folded once each way to make four sections.

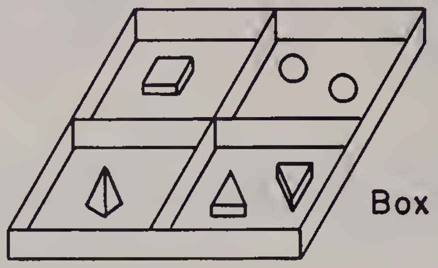
The Lesson:

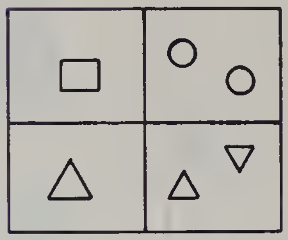
You will have frequent use for materials to be used as shown in the pictures on page 19, and you may find it worthwhile to line a box with flannel and back some cardboard cutouts with flannel so that you can pick up the box for the pupils to see with all the objects in place. (This is a way of defying gravity.) You can make dividers for the box by cutting two strips of stiff paper in appropriate lengths, cutting each of them halfway through at the middle, and fitting them together.



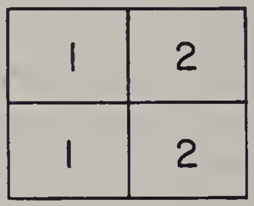


Place 1 or 2 objects in each compartment of your box and show it to the pupils. Ask them how they can use one of their pieces of paper to draw a picture of what you have in the box. When they have done this, they can use another piece of paper to record the number of pictures they have drawn in each section.





Paper 1



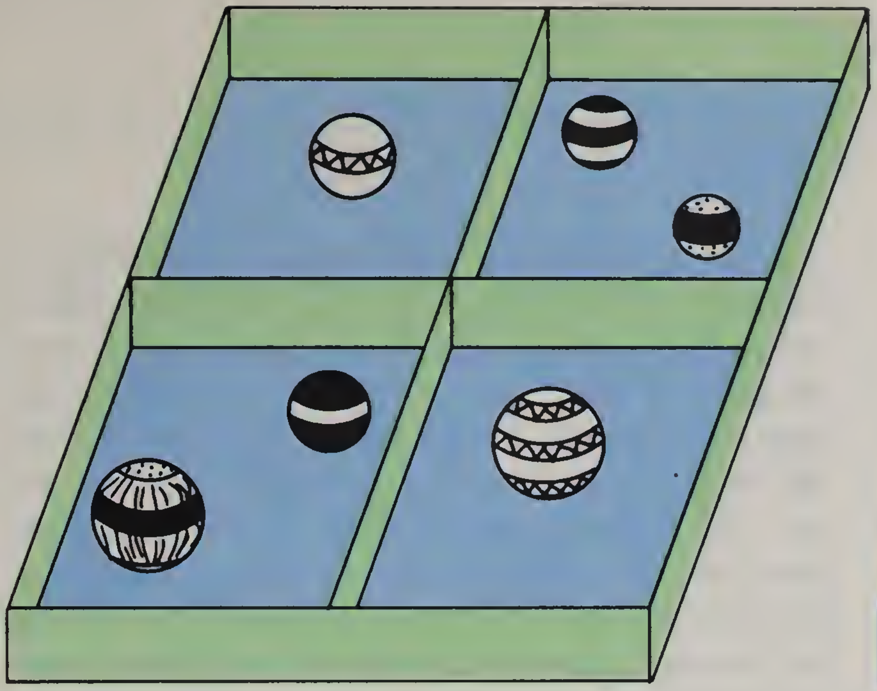
Paper 2

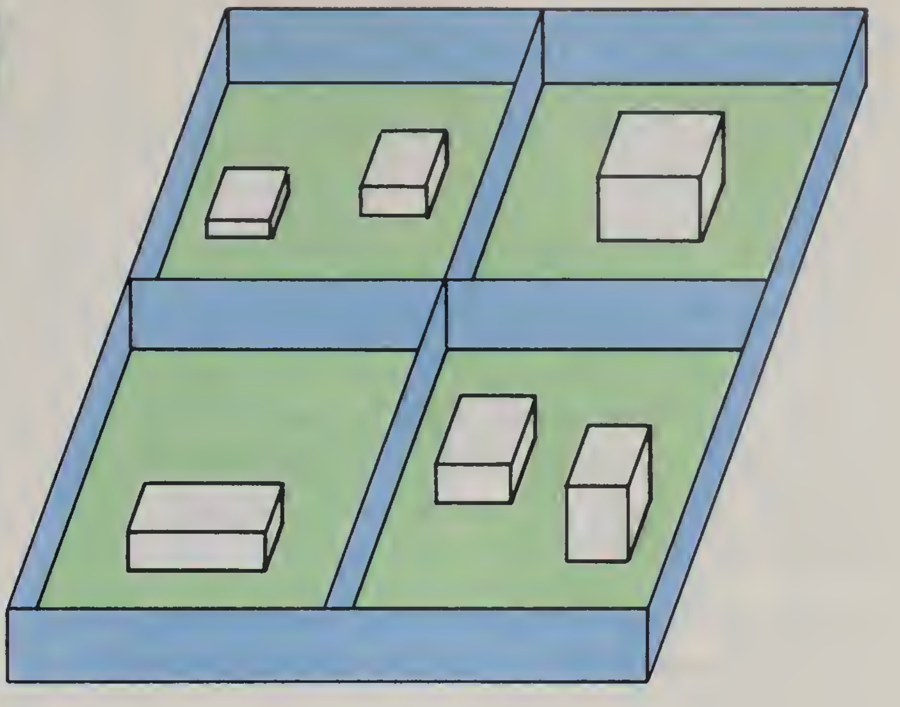
Repeat this activity, if necessary, and then do the same thing a couple of times, eliminating the picture-drawing stage, going directly from the objects in your box to the numbers in the record.

Assign page 19 for individual study. If it turns out that you have a pupil who does not see which cells in the record correspond to a given compartment in a box, work with him as follows. As he counts the objects in each compartment of the demonstration box, let him put the appropriate number card in the compartment. The completed arrangements should look very much like a completed record.

Use your demonstration clock face to guide the class through various time statements involving whole hours only. Keep a written board record as you proceed.

Headline Story: It's 3 o'clock. What time will it be in 1 hour? . . . 2 hours? . . . etc.

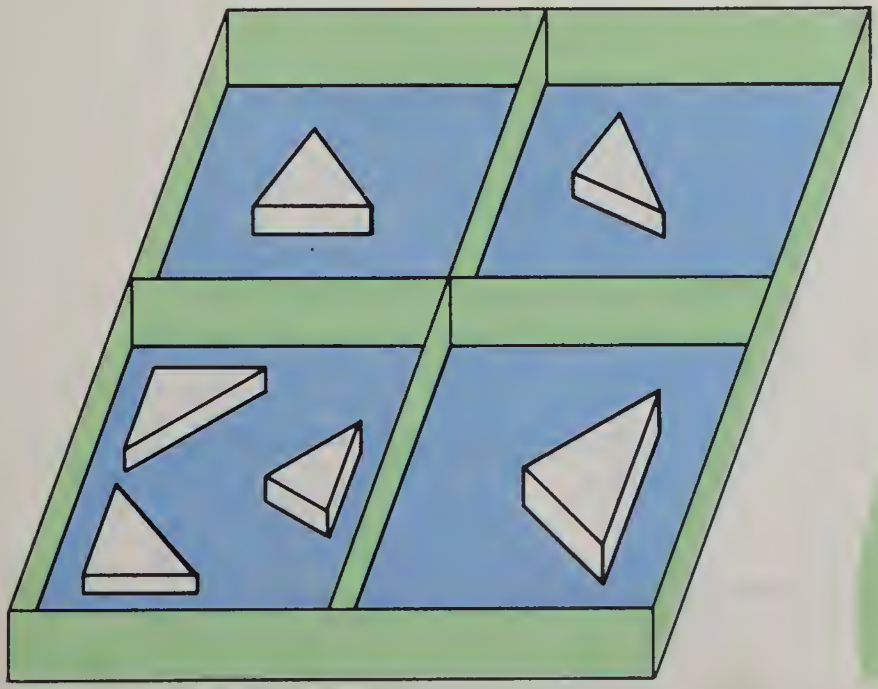


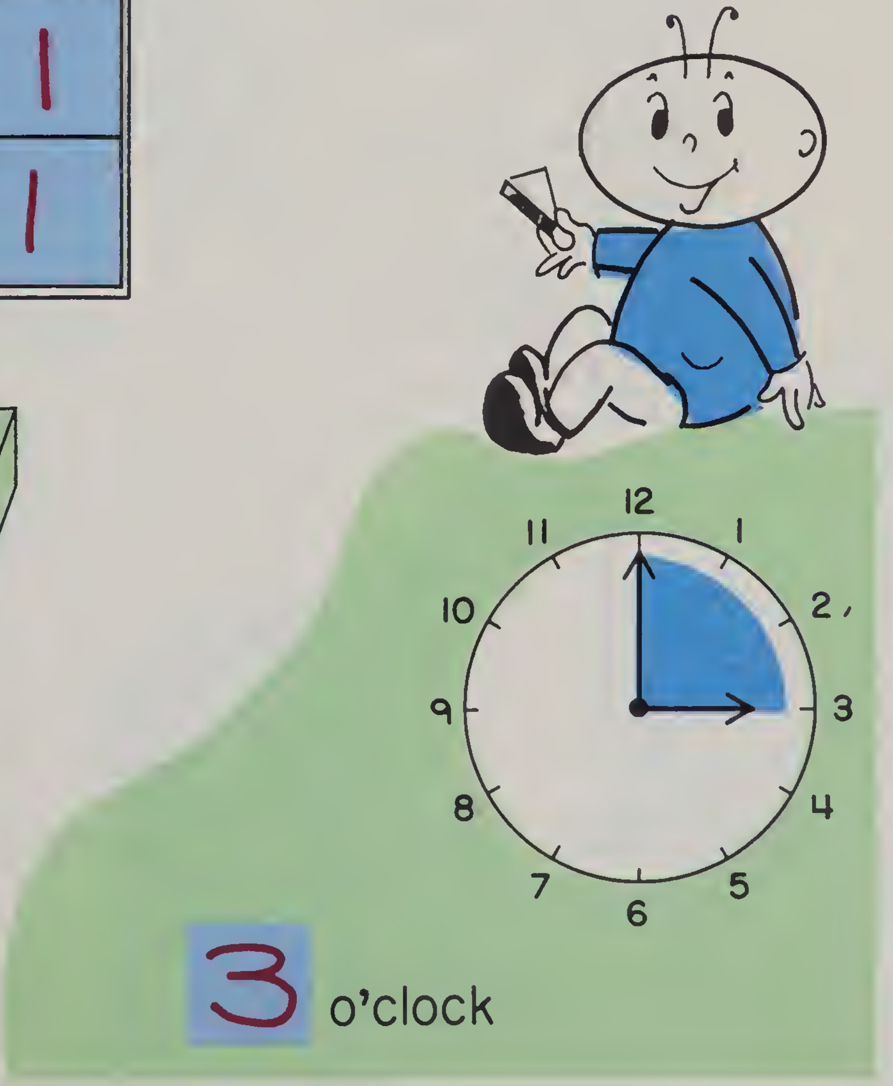


1	2
2	1

2	1
1	2

1	1
3	1




12
1
2
3
4
5
6
7
8
9
10
11

3 o'clock

Notes on Pupil Page 20

Tasks: Pupils count objects in the compartments, record the counts, and find and record the total number of objects in rows and in columns.

Purpose: An extension of the lesson on the preceding page, anticipating the fact that changing the order or grouping of numbers to be added does not affect the sum.

Unifying Ideas: Sets; number and counting; addition and subtraction; structure; measurement.

Materials for Teacher: Five to ten objects to be counted.

Materials for Pupils: No special materials needed.

The Lesson:

You may use the same materials you used in the preceding lesson for this one, altering the record to provide the indicated extension, if you wish. An alternative activity is described below.

Put four pupils' desks together so they can play the same role as the compartments in the box. Draw a picture on the board, and see that all pupils understand how it goes with the desks. (The numbers appear later, one by one.)

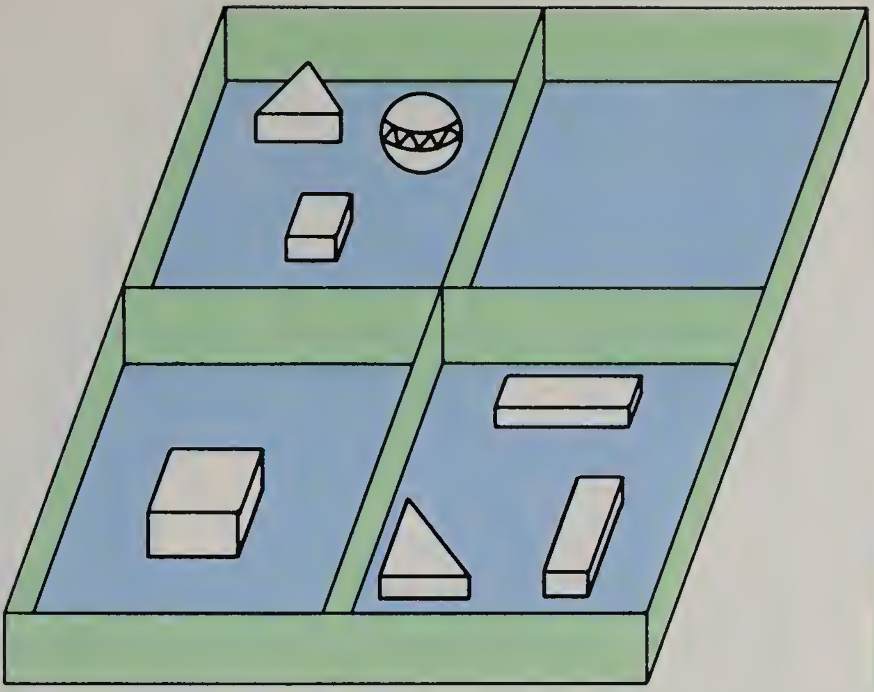
Amy's desk 2	Mark's desk 1	3
Roy's desk 1	Pat's desk 3	4

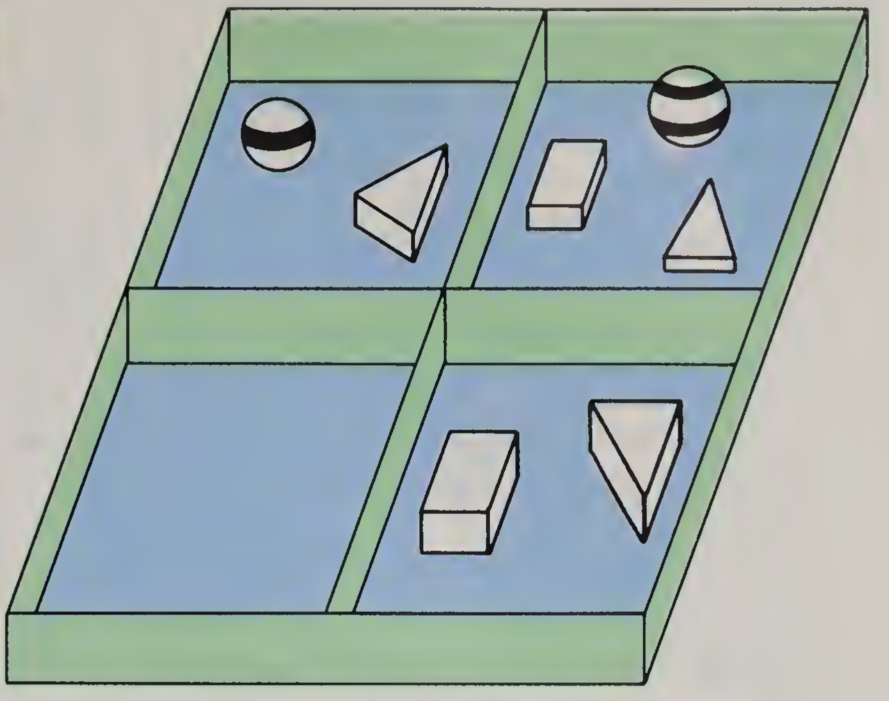
Lay a few objects on each desk and have different pupils write numbers in the corresponding cells of the picture on the board to tell how many objects are on each desk. Then, stand beside a desk in the first row and ask the pupils in that row to pass all the things on their desks to you. Who can tell from the record on the board how many objects you must be holding? Can someone suggest a place to record this number? Do the same with the objects in the second row. You have all the objects now. You want to put them back where they were. How will you know how to do this?

When the objects are all back in place, stand behind the second desk in one of the columns and ask the pupils in that column to pass all their things to you. Can everyone tell from the record on the board how many objects you must be holding now? Record the number in an appropriate place. Do the same thing with the other column. Repeat this activity using different combinations of objects, if necessary; then try just pretending to pass the objects to the end of a row or column and finding what the total would be. When the pupils can do this, they are ready for page 20.

Be sure to review the accompanying booklet, *Games and Enrichment Activities*, for suggestions you may wish to incorporate in your daily lesson plan.

Headline Story: There are 7 days in a week. (Pupils extend the story.)

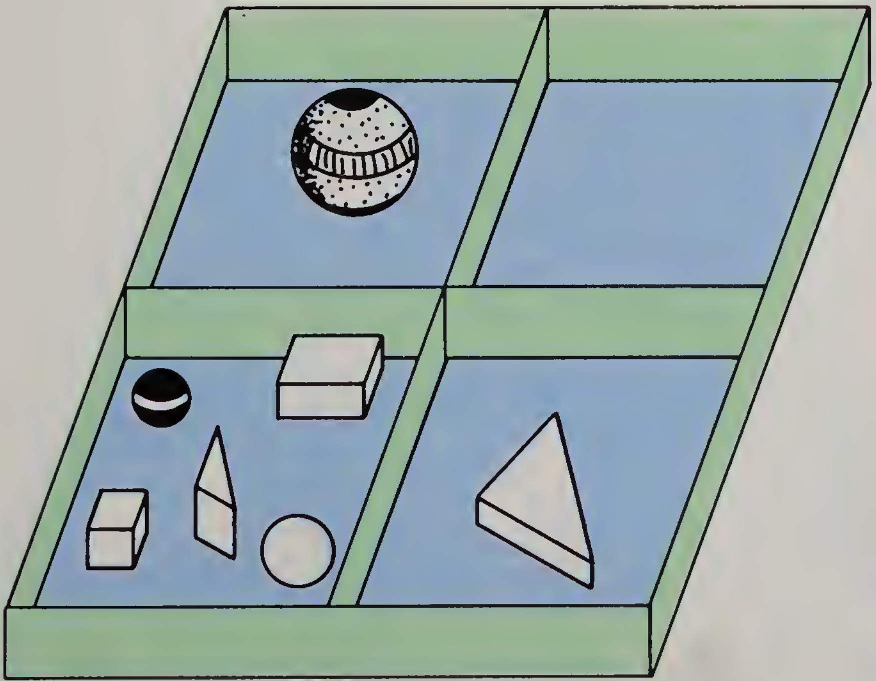


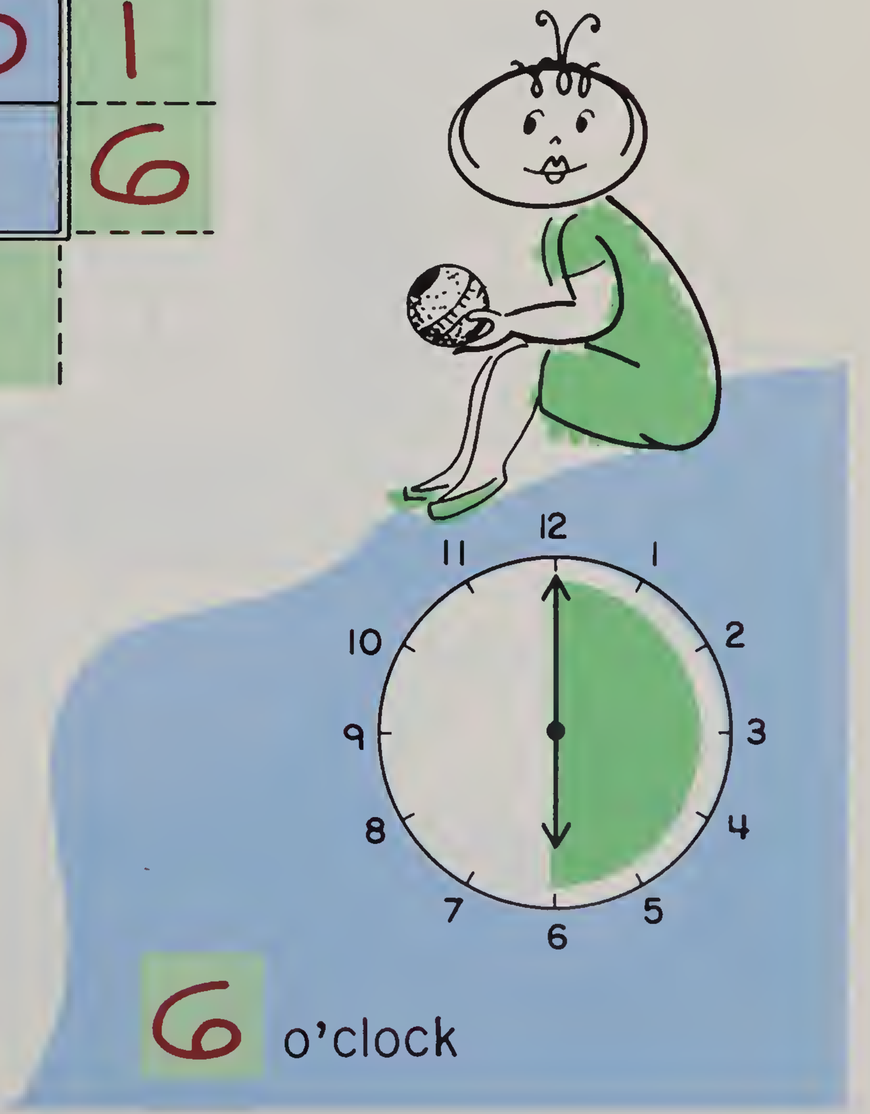


3	0	3
1	3	4
4	3	

2	3	5
0	2	2
2	5	

1	0	1
5	1	6
6	1	




12
1
2
3
4
5
6
7
8
9
10
11

6 o'clock

Notes on Pupil Page 21

Tasks: *Pupils record the number of objects in each compartment and the number of objects in each row and each column. Pupils are also asked to record the correct time on the clock.*

Purpose: To introduce the class to the number 10 via the Cross Number Puzzle.

Unifying Ideas: Number; counting.

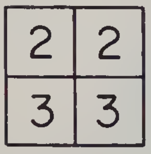
Materials for Teacher: A demonstration Cross Number Puzzle box and an assortment of ten objects to place in the puzzle box compartments.

Materials for Pupils: Oak tag sheets with an outline of the Cross Number Puzzle on each sheet and an assortment of ten counters for each pupil.

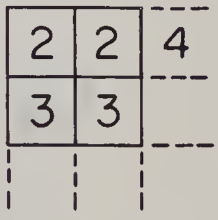
The Lesson:

Without showing the class that you have ten objects, place the objects at random in the compartments of the Cross Number Puzzle box. Sketch a diagram of the compartments on the board, but do not fill in the cells Ask pupils to count the number of objects in each compartment. You record the count. Now ask a pupil to come to the front and empty out the top row and tell the class how many in all he has in his hands. You record the count to the right of the top row; e.g., suppose diagram A, below, is your first written record:

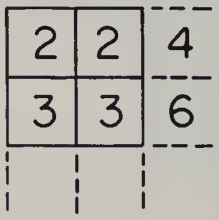
A



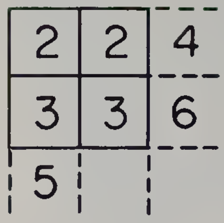
B



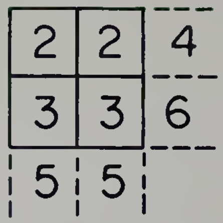
C



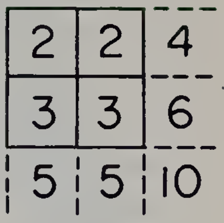
D



E



F

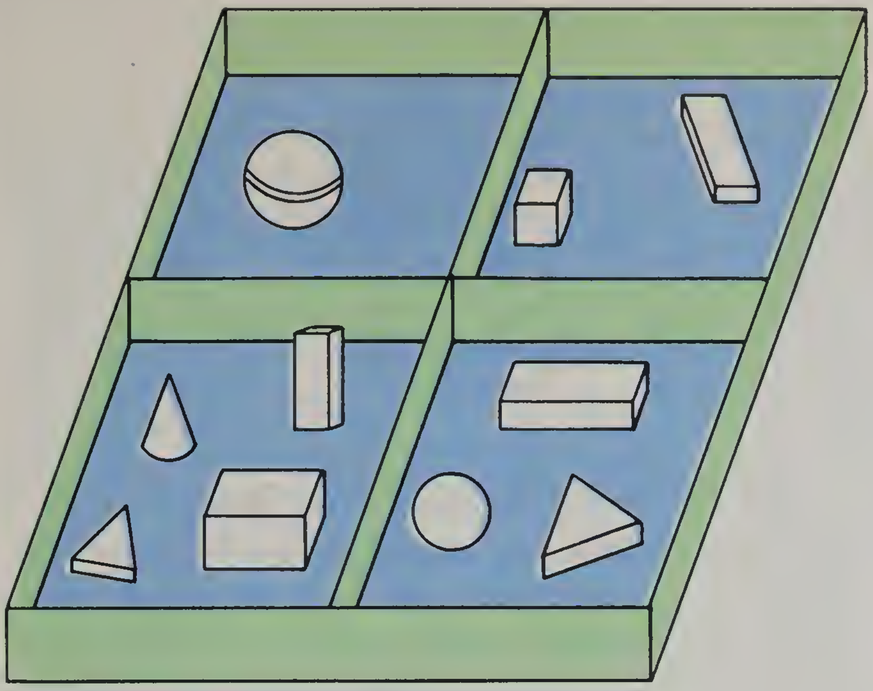


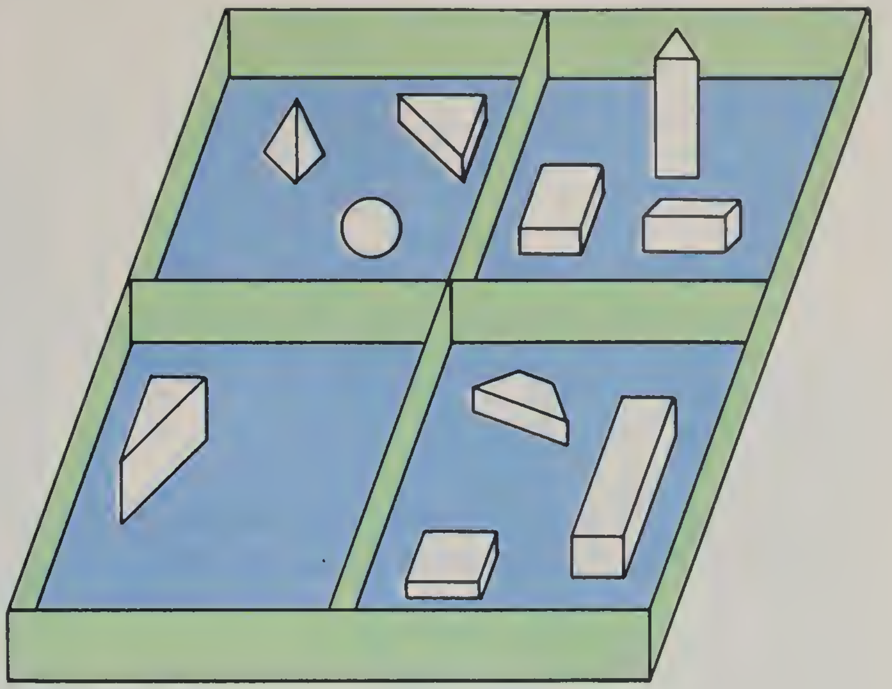
Your record now looks like diagram B above. Ask the pupil to replace the objects in the compartments in the original order. He now selects another classmate to remove the objects from the second row, count, and tell the class how many in all he has in his hands. Your record now looks like diagram C. After the pupil returns the objects to their original compartments, he selects someone to empty the first column and count how many in all he has in his hands. After another classmate checks his count, your record looks like diagram D. He returns his objects to their original compartments and selects someone to empty the second column, count the objects, and tell the class how many he has in all. Someone else checks his count. Your record now looks like diagram E.

Each pupil now uses his own desk counters to reconstruct the original arrangement of the ten counters. Ask half the class to sweep the counters in the two columns to the bottom row. Now they count all counters in the bottom rows. The other pupils sweep the counters to the right column and count the total.

Construct several different situations similar to the above pattern of activities. At this point the class is ready to complete page 21 of their work-text.

Headline Story: Tell a story that fits the headline: 2 + 1 = 3

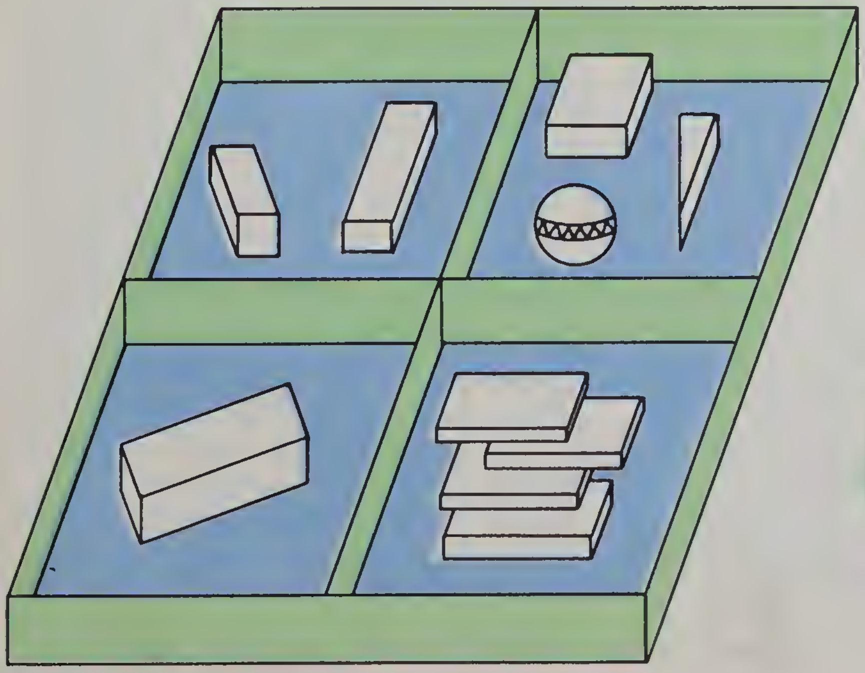


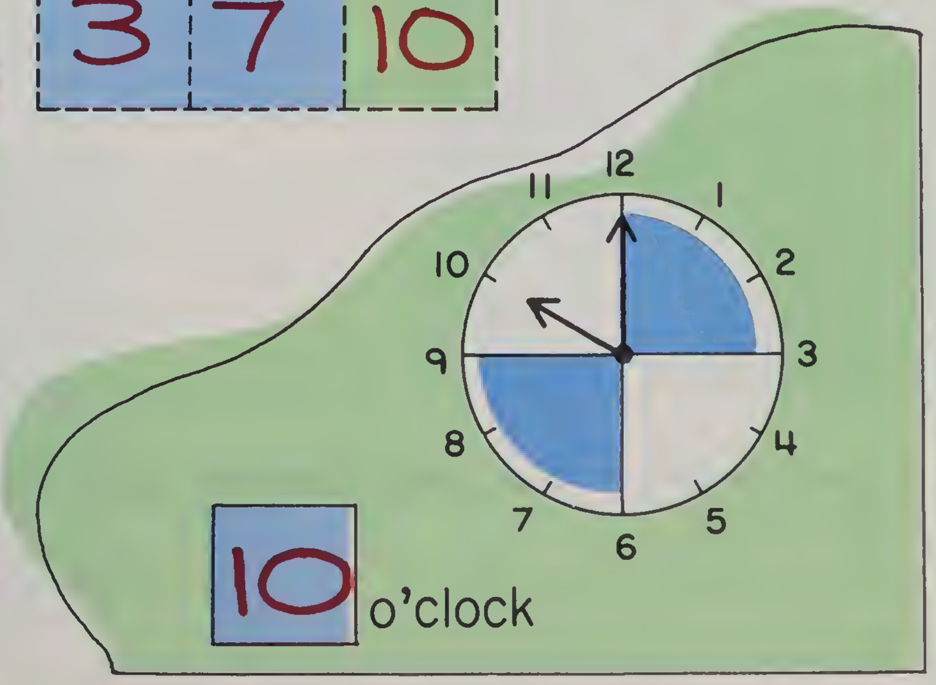


1	2	3
4	3	7
5	5	10

3	3	6
1	3	4
4	6	10

2	3	5
1	4	5
3	7	10




12
1
2
3
4
5
6
7
8
9
10
11

10 o'clock

Notes on Pupil Page 22

Tasks: *Pupils supply the numbers missing in the records at the top of the page and supply numbers and pictures in the records at the bottom of the page.*

Purpose: To provide practice in addition and subtraction through the use of a Cross Number Puzzle.

Unifying Ideas: Sets; number and counting; addition and subtraction; structure.

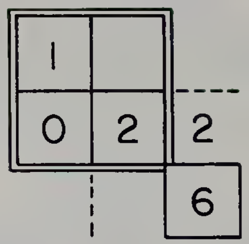
Materials for Teacher: No special materials are needed.

Materials for Pupils: Copies of the blank puzzle in the lower left-hand corner of this page, prepared on a Ditto master.

The Lesson:

The top half of this page is easy practice on what your pupils have just been doing. The bottom half is harder in that the pupils have to work backward, in a sense. They must start with the record, find the missing items in it, then draw pictures in the blank puzzle to go with the record. The activity described below is designed to help them with this.

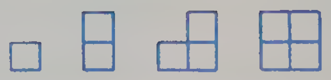
The pupils have copies of blank puzzles. Put the following drawing on the board and ask the pupils to copy it in one of the puzzles on their paper.

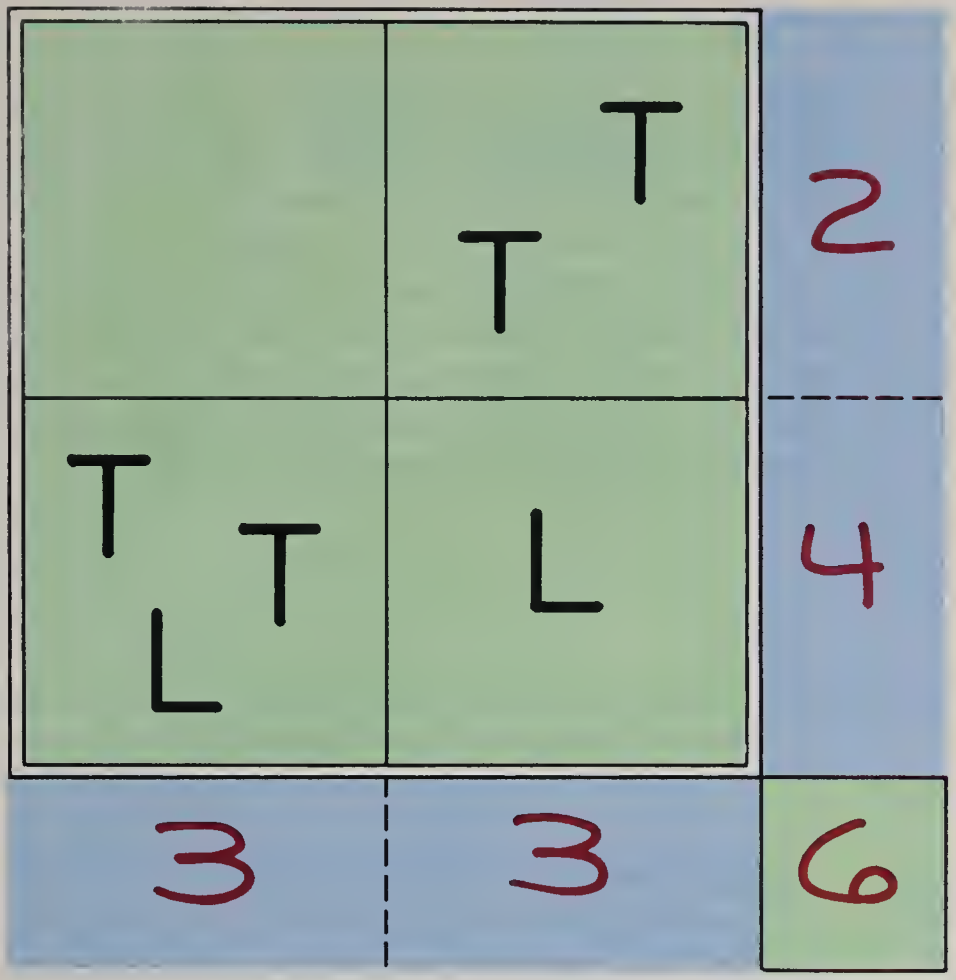


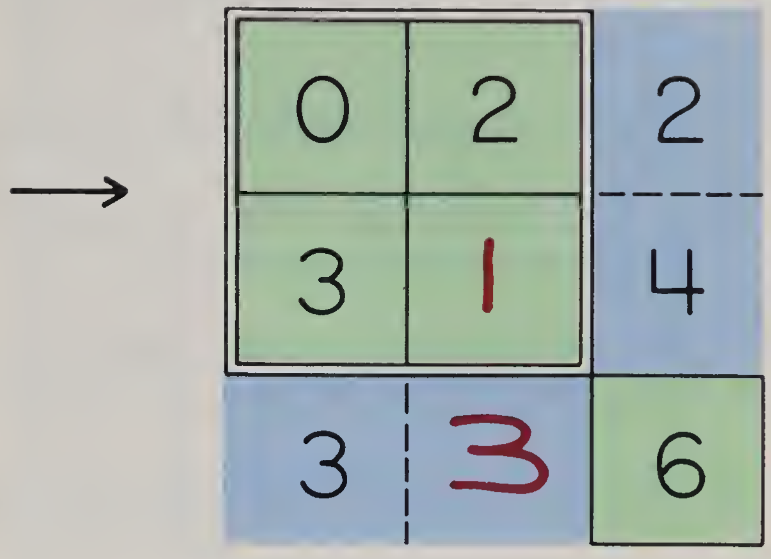
Can everyone fill in the blank spaces? Discuss how this is done, and complete your work on the board so that each pupil can check his paper. Then ask the pupils to use another puzzle on their papers to draw pictures (in the large square part of the puzzle) to fit their record. Suggest that they use something quick and easy to make, letters of the alphabet, for instance, but let them draw something else if they wish. When they have done this satisfactorily, they may write the total counts of objects in rows, in columns, and all together. Repeat this activity, if necessary; then assign page 22 for individual study.

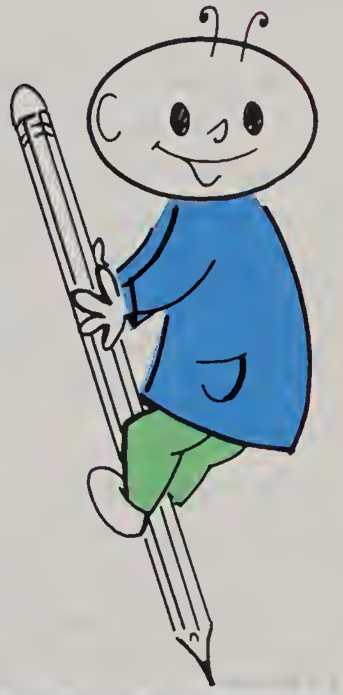
Discuss the completed page with the class. Do they think it an accident that the sum for the bottom row and the sum for the right-hand column of each puzzle are the same? Let them use some of their blank puzzles to try other combinations of numbers in the four basic boxes and see if they can find some which will not work out this way. (Carefully check this activity to keep the numbers small enough so that the totals will be numbers they can handle.) They will never find combinations of numbers which do not behave like this in this context (assuming no mistakes) and some pupils may be able to tell you, or show you by combining corresponding numbers of objects, why this must be so.

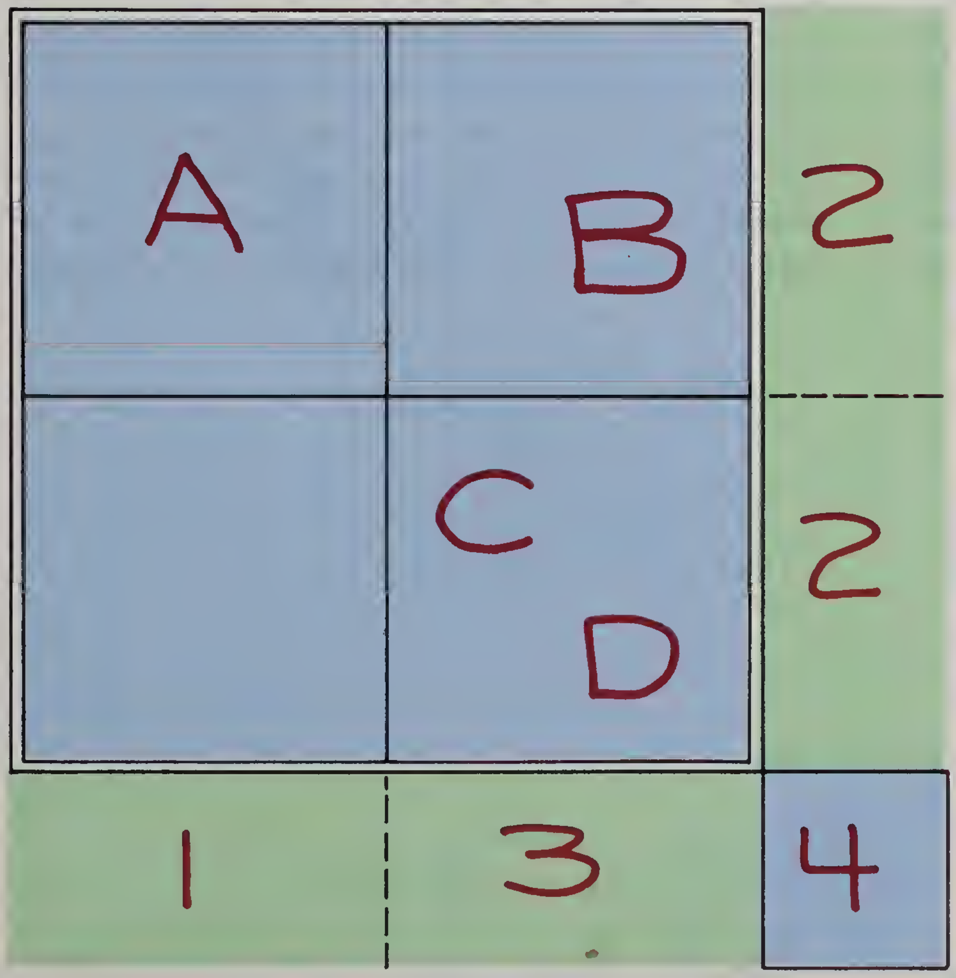
Headline Story: Tell a story suggested by the sketch:

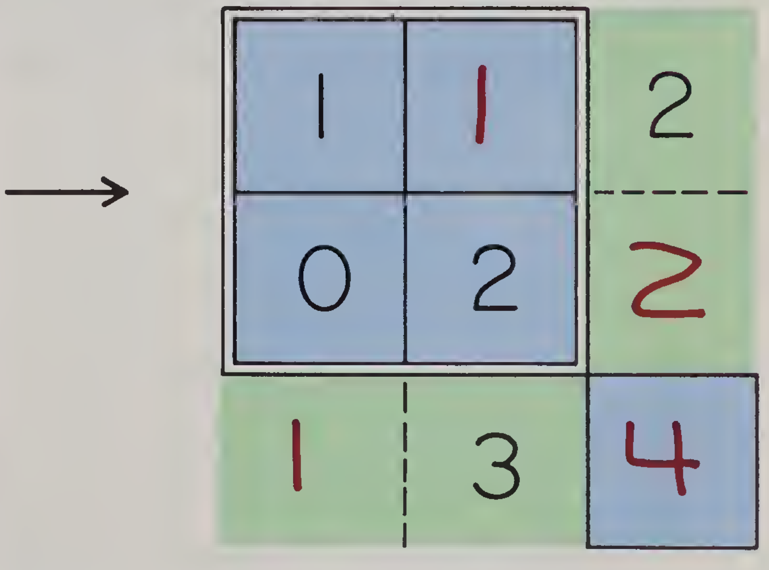



T
T
T
T
L
L
2
4
3
3
6


0
2
2
3
1
4
3
3
6




A
B
2
C
D
2
1
3
4


1
1
2
0
2
2
1
3
4

Notes on Pupil Page 23

Tasks: *Pupils count the number of like objects in a set, count the total number of objects in the set, record the counts, and write appropriate equations.*

Purpose: To teach addition facts with sums of 7.

Unifying Ideas: Sets; number and counting; addition and subtraction; structure.

Materials for Teacher: Seven objects of each of two kinds which can be lined up on the chalkrail.

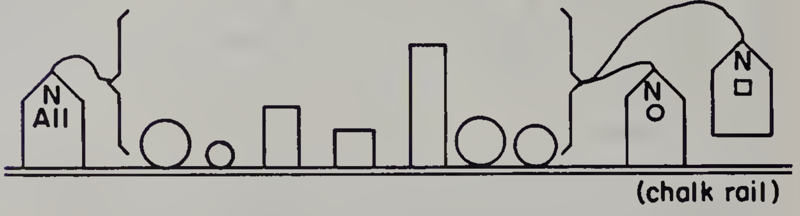
Materials for Pupils: Seven counters.

The Lesson:

This lesson is similar to several others your pupils have had, and there should be little difficulty with it. It may be well to begin the lesson by reviewing addition facts with sums of 1 to 6. Some of your pupils may be able to recall these facts immediately. Others may need to work them out with counters, and the slower ones might be helped by having two boxes to use in acting out the stories: 2 in one box, 3 in the other, dump them all together, 5 in all.

Have each pupil take 7 counters and find out how many ways he can arrange them in two groups (or put them into two boxes). Keep a record of what the pupils are discovering. Guide the activity, if necessary, so that you have a total of eight facts listed on the board. Study the list. Can the pupils find pairs of facts in which the very same numbers are used? What is alike about the two facts in each pair? What is different? Can someone show you with counters or boxes, or both, why 5 + 2 must equal 2 + 5, for instance?

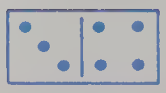
Arrange 7 objects, along with brackets and tags, on the chalkrail. Have a pupil fill in the tags. Ask another pupil to rearrange the objects. Do any of the tags need correcting now? Use a different combination of objects and repeat the activity.

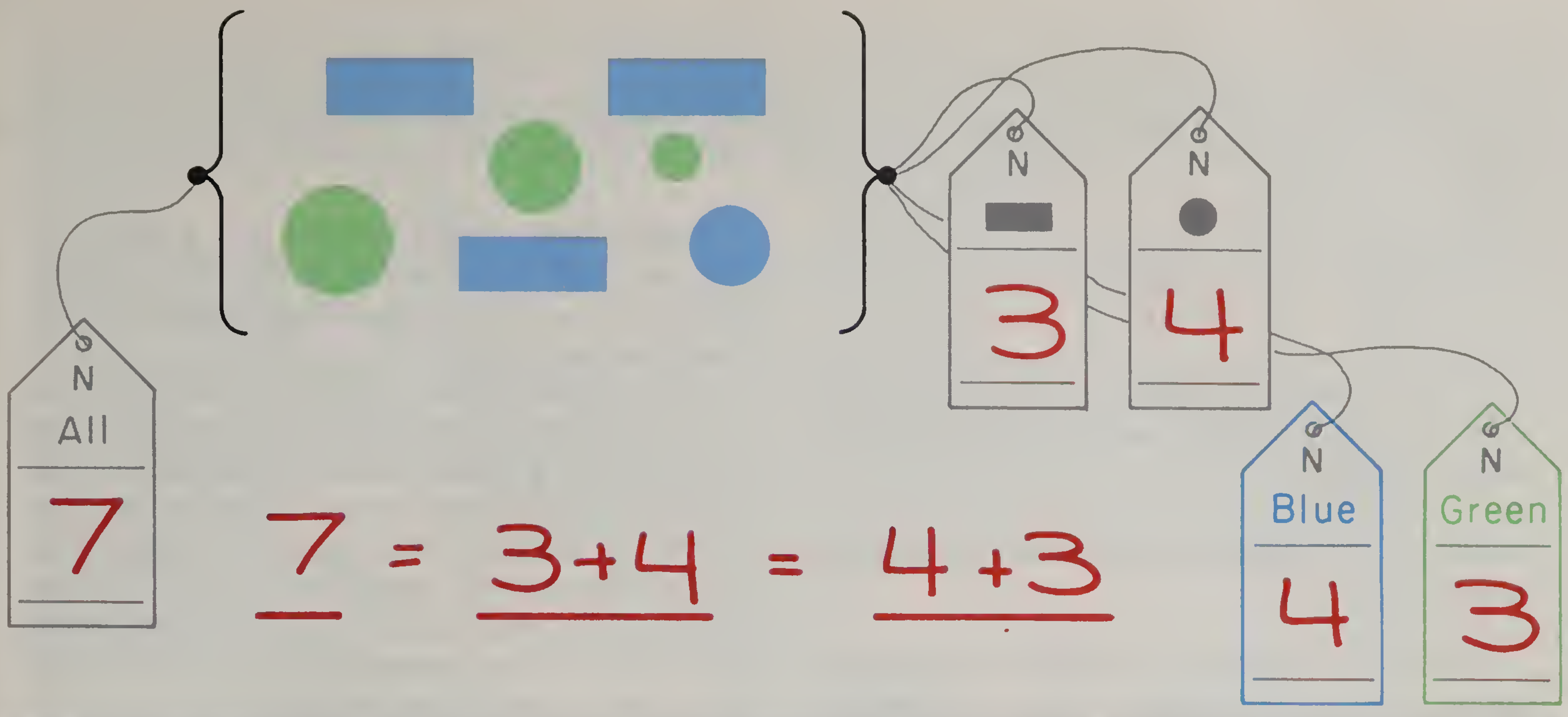


Assign page 23 for individual study.

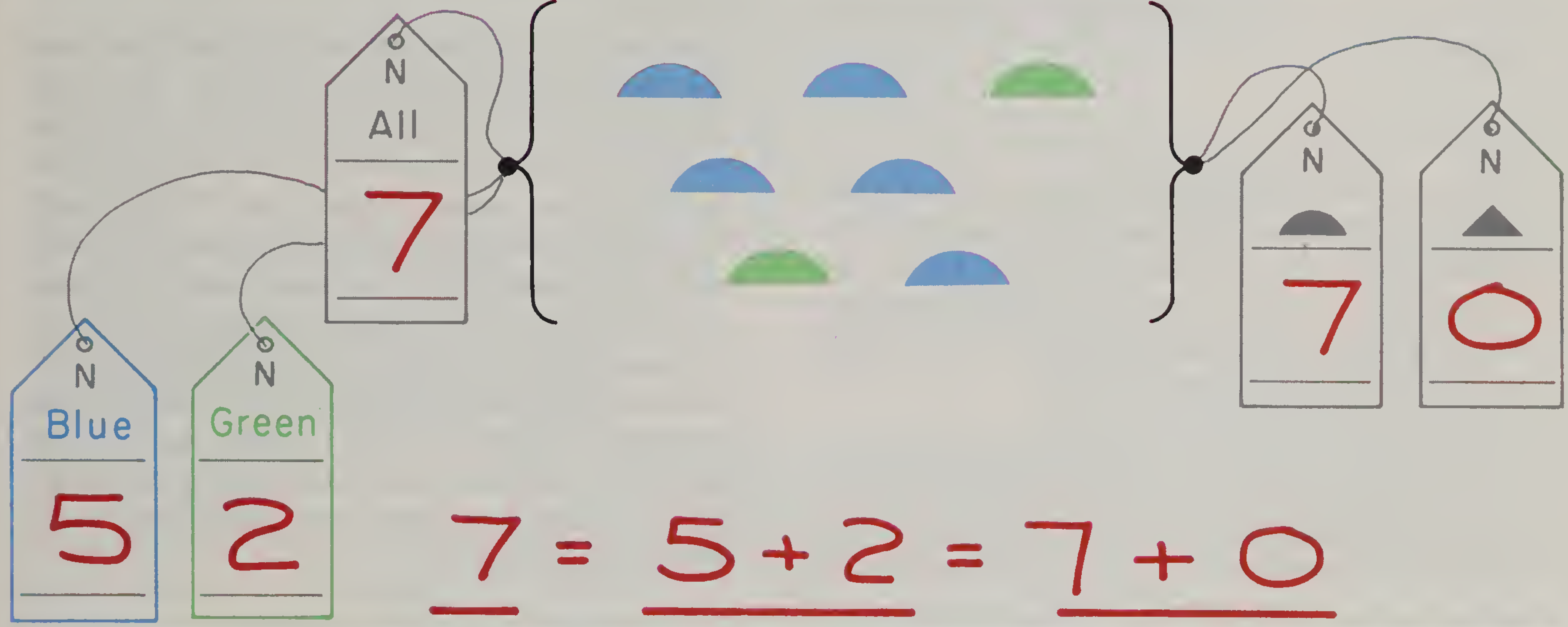
If some pupil is having difficulty in doing the work in this text because he has trouble discriminating among the various shapes and colors he has to identify, cut out cardboard shapes, use all the wooden and plastic shapes you can find, and let him handle them. Such kinesthetic-tactile experiences are helpful in increasing perceptual efficiency. See if he can eventually draw pictures of the shapes he has handled. See if he can choose a shape to match a picture. If the above activities seem not to produce significant changes in pupil behavior, consider consulting the school nurse for a thorough physical examination. (There are tragic instances of pupils who have been considered slow learners for several years until someone discovered that their eyesight or their hearing was defective.)

Headline Story: Tell a story suggested by the sketch:

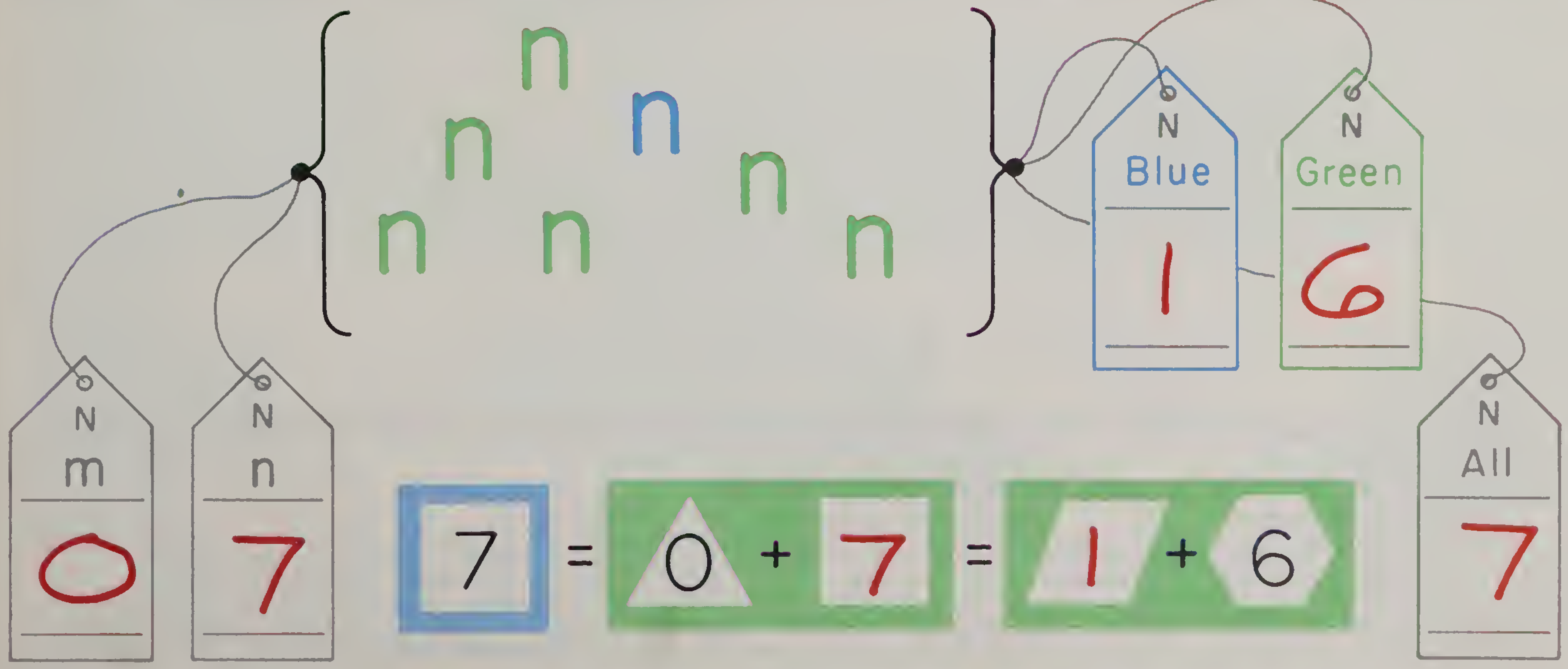



N
All
7
N
▬
3
N
●
4
N
Blue
4
N
Green
3

7 = 3+4 = 4+3


N
All
7
N
Blue
5
N
Green
2
N
7
N
▲
0

7 = 5+2 = 7+0


n n n n n n n
N
Blue
1
N
Green
6
N
m
0
N
n
7
N
All
7

7 = 0 + 7 = 1 + 6

Notes on Pupil Page 24

Tasks: *Pupils find the number of bricks which are missing in each of the "holes in the wall" and write the appropriate number in each "hole."*

Purpose: To furnish experience with a particular kind of spatial relation and to give practice in simple counting.

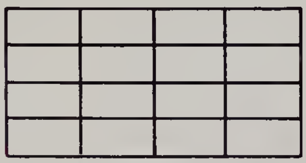
Unifying Ideas: Number and counting; measurement.

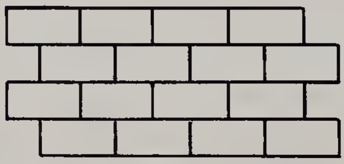
Materials for Teacher: Cuisenaire® Rods, Stern Blocks, or carefully measured strips of cardboard or tag to use as bricks.

Materials for Pupils: Duplicates of the teacher's material.

The Lesson:

Use concrete materials to build "walls" on a flat surface to illustrate the idea of this page. The red Cuisenaire® Rods will just fit the "bricks" in the picture. Other similar rods, or carefully measured strips of paper, may also be used. The walls may look like either of these drawings:





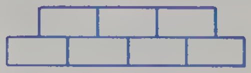
Having built a wall, you can play a game. All but one pupil close their eyes. The one pupil removes one or more bricks. Other pupils then tell how many are missing.

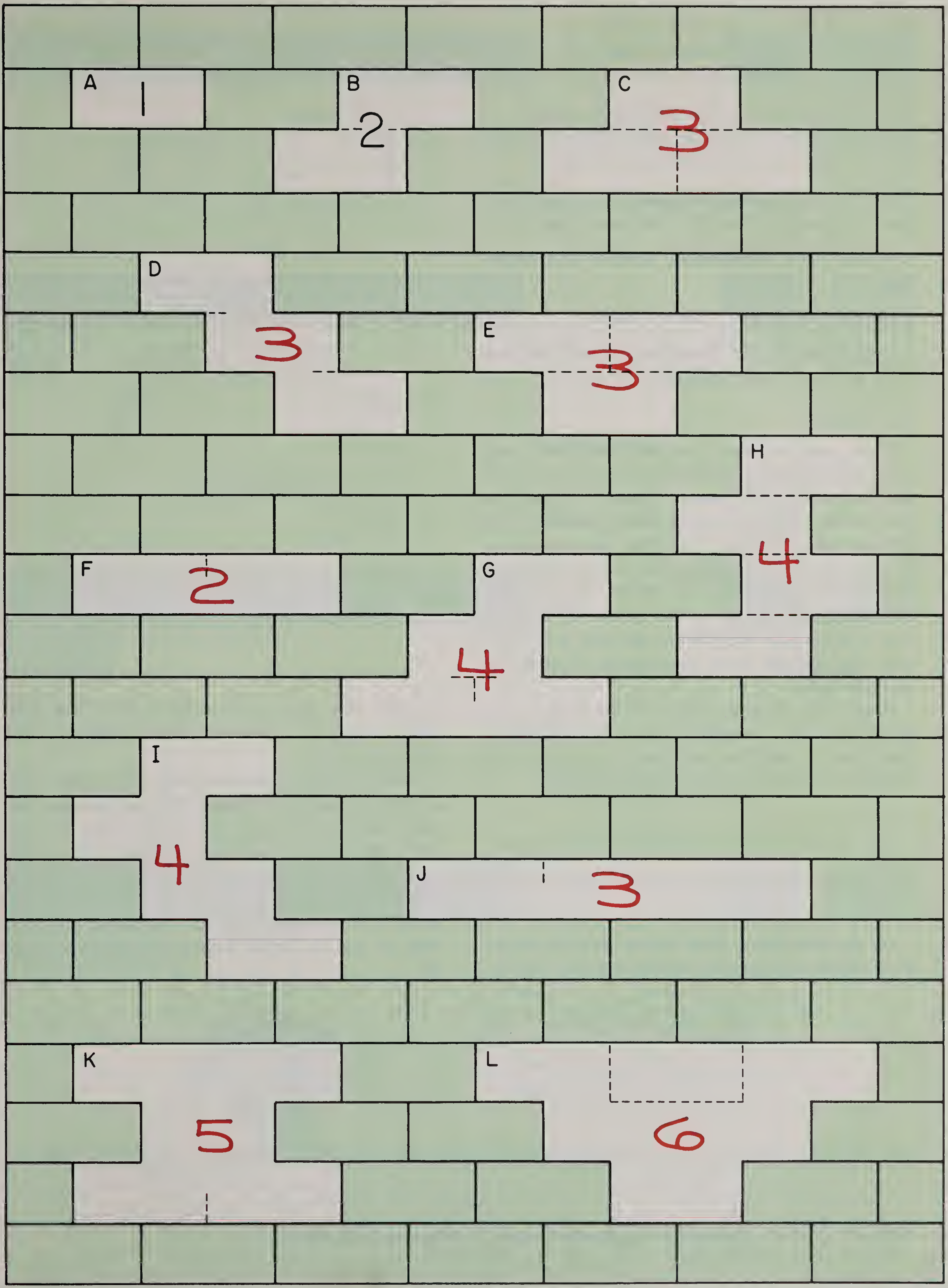
Build a small section of wall using not more than 12 bricks. Cover this with a piece of paper. Lift the paper for a moment and have the pupils study what they see. Cover the wall again and ask how many bricks were used. Record the different results. Then lift the paper and count to check results. Note the various ways the pupils count the bricks. Are any counting by 2's or 3's or 5's? If so, they should be encouraged.

Assign page 24 for individual study. Allow pupils who have difficulty to use their concrete materials for help.

Pupils who are good at handling number ideas and seeing spatial relations may be asked the following questions: How many bricks in the top row? In the bottom row? In the row above the one which has the letters K and L in it? How many bricks are missing from the second row? The third row? The fourth row? How many rows are made with all whole bricks? Has any row two half bricks? Are there any rows with just one half brick? Are there any with three half bricks? Are there enough half bricks along one side to make up a whole row? (No) Are there enough half bricks on the two sides to make up a whole row? (Yes) How many would be left over?

Headline Story: Tell a story suggested by the sketch:





A
1
B
2
C
3
D
3
E
3
H
4
F
2
G
4
I
4
J
3
K
5
L
6

Notes on Pupil Page 25

Tasks: *Pupils write either the missing addends or sums or both and complete the labeling of the number line.*

Purpose: To introduce the class to the "teen numbers" 10 through 13 as "10 and some more."

Unifying Ideas: Numeration; number and counting.

Materials for Teacher: Colored chalk.

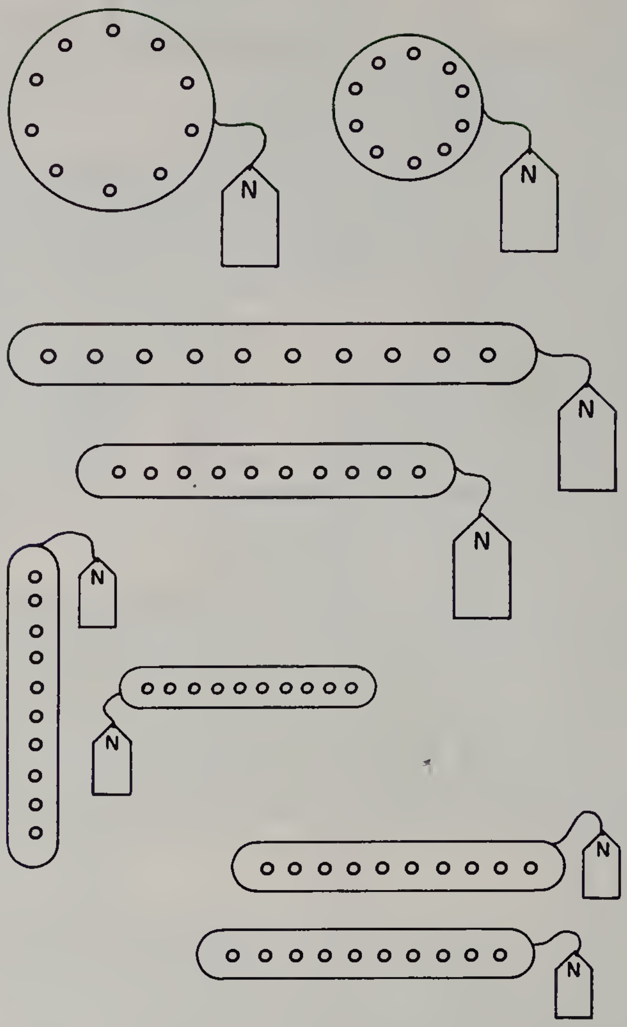
Materials for Pupils: Desk counters—ten of one color and six of another color.

The Lesson:

This lesson is one of several concerned with showing graphically the "10 and some more" aspect of the so-called "teen numbers." (It might be nice if the numeral 11 were read "oneteen" and 12 were read "twoteen," but they are not; so we have to contend with this situation as we find it.) Spend several days on this lesson, if necessary. The place-value nature of our system of numeration is of primary importance, and this is the first time the children have encountered it explicitly in their books.

Begin with the strip with 10 unit squares described on the preceding page. Have the children place an object on each unit, then remove the objects, count them, and record the results.

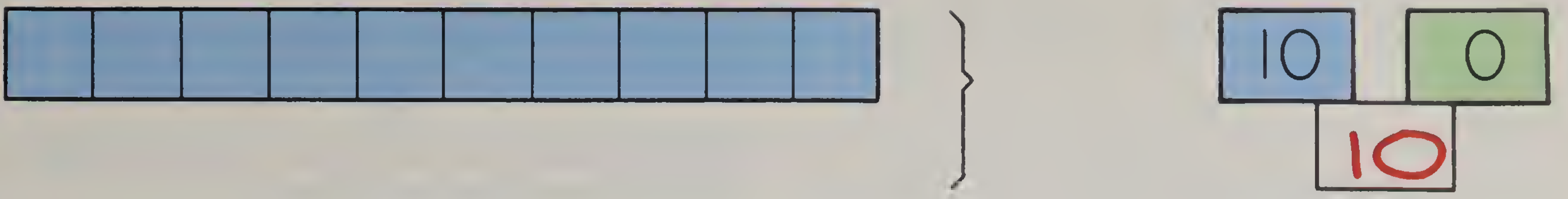
On the chalkboard, draw several pairs of groups of 10 objects arranged in a variety of ways, such as:

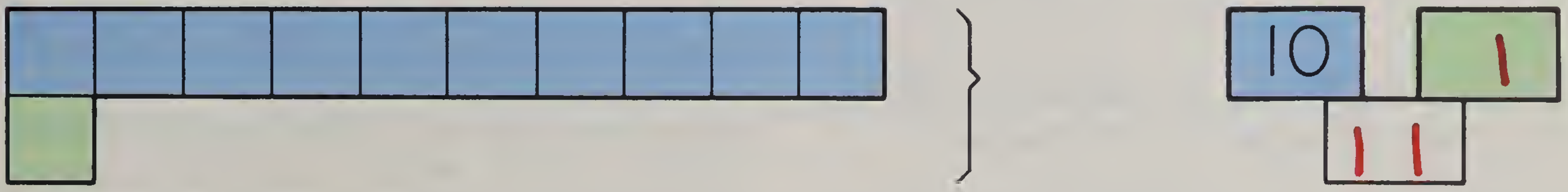


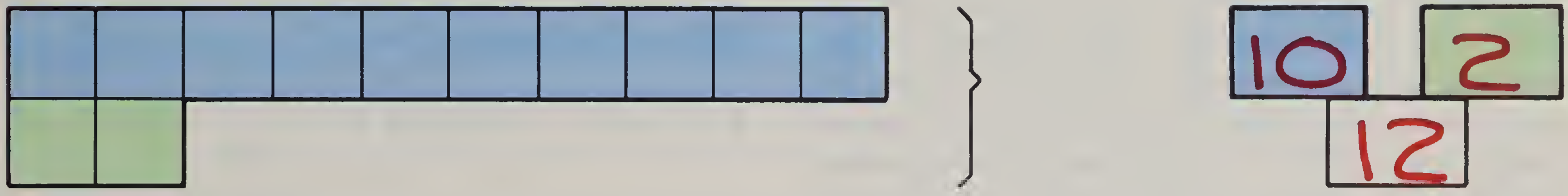
In each case, ask the children in which arrangement they think there are more things in the picture and why.

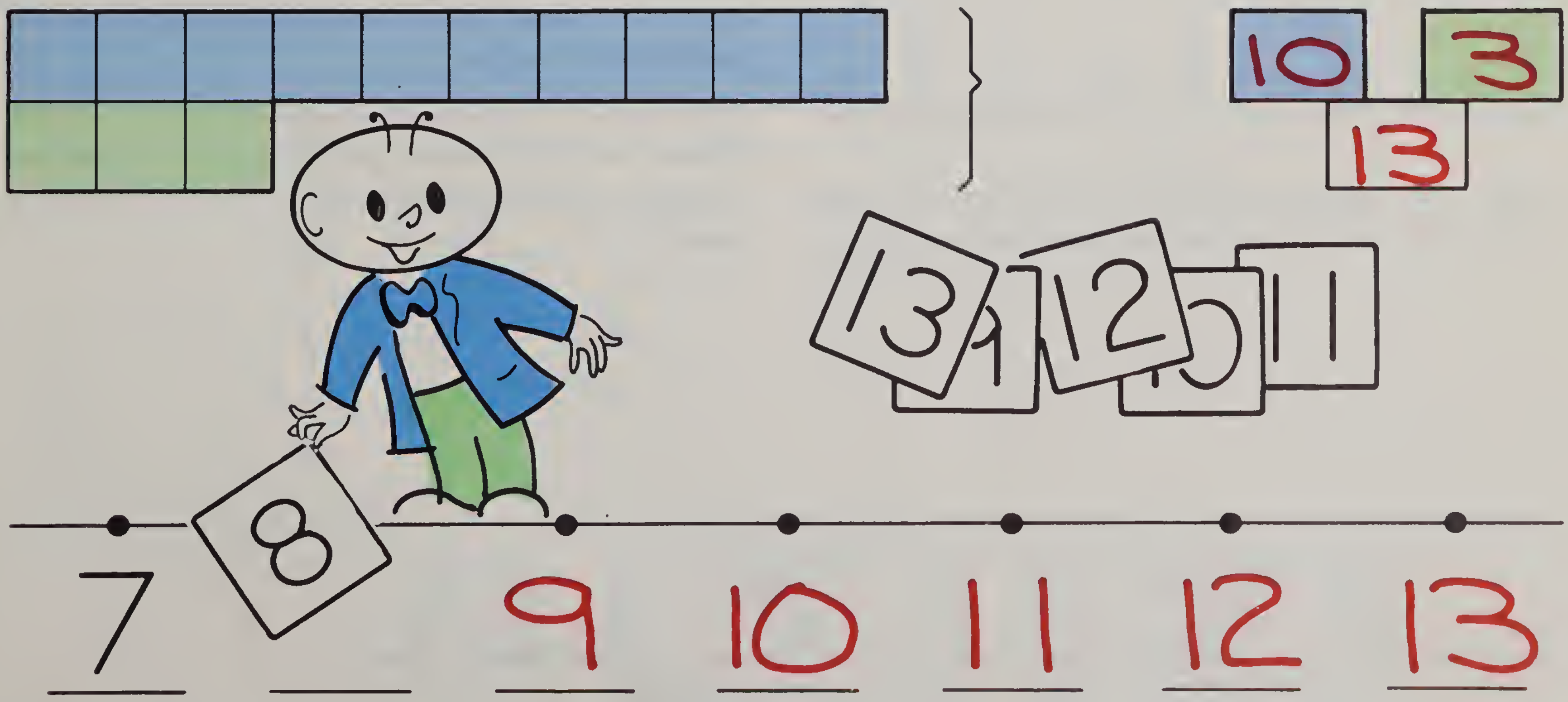
Add one more picture to each group. Ask how each group has been changed. "Should we erase the 10 in the first tag now? Should we erase all the 10's? Should the same new number go in all the tags now? What number is it?" Count each group and record the result. To the right of the tags in which 11 is recorded, write 10 + 1. Proceed similarly with 12 and 13. Pupils may now proceed to page 25.

Headline Story: There were 10 girls and 3 boys. (Pupils extend the story.)


10
0
10


10
1
11


10
2
12


10
3
13
13
12
11
7
8
9
10
11
12
13

Notes on Pupil Page 26

Tasks: *Pupils count the number of like objects in a set, the total number of objects in the set, record the counts, and write appropriate equations.*

Purpose: To teach addition facts with sums of 8.

Unifying Ideas: Sets; number and counting; structure.

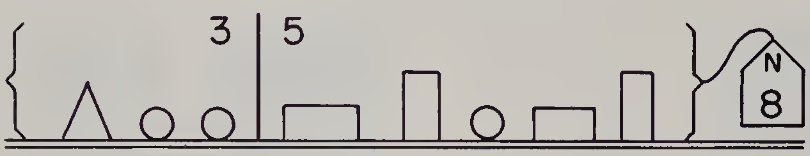
Materials for Teacher: A supply of at least 8 objects which can be placed on the chalkrail.

Materials for Pupils: No special materials needed.

The Lesson:

One way to introduce the lesson is to get out a supply of objects (cards of different shapes, perhaps, or anything else which is easy to handle) and ask a pupil to place 8 of these objects on the chalkrail. Have the other pupils check to see whether there are exactly 8 objects on the chalkrail. Draw some braces and a number tag on the board, and put in a vertical line which separates the objects into two groups. Count to find the total number of objects and the number in each group. Record the results in two ways, as shown below:

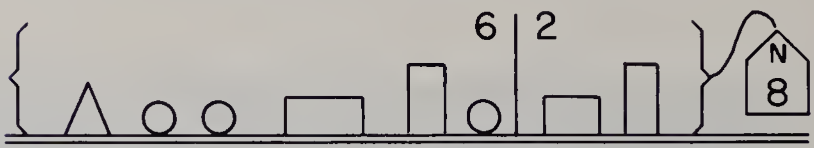
$8 = 3 + 5$



Erase the vertical line and the records of the counts. Do not erase the mathematical sentence.

Draw another line indicating another grouping. Again, count and record results.

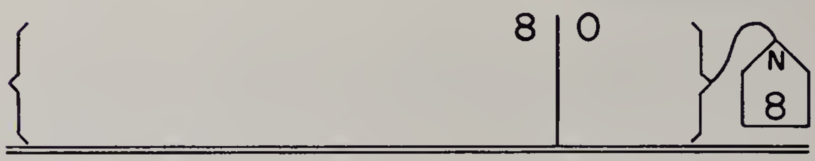
$8 = 3 + 5$ $8 = 6 + 2$



Erase the vertical line and the numbers on either side of it. Ask, "If we draw a new line to separate the objects into different groups, do we need to erase the number in the tag?" Rearrange the objects on the chalkrail. "Should we now change the number in the tag? Why (not)?"

Do similar exercises yielding the other facts with sums of 8. At some point, remove the objects from the chalkrail. See if the pupils can put the objects back so that what is written on the board will not have to be changed.

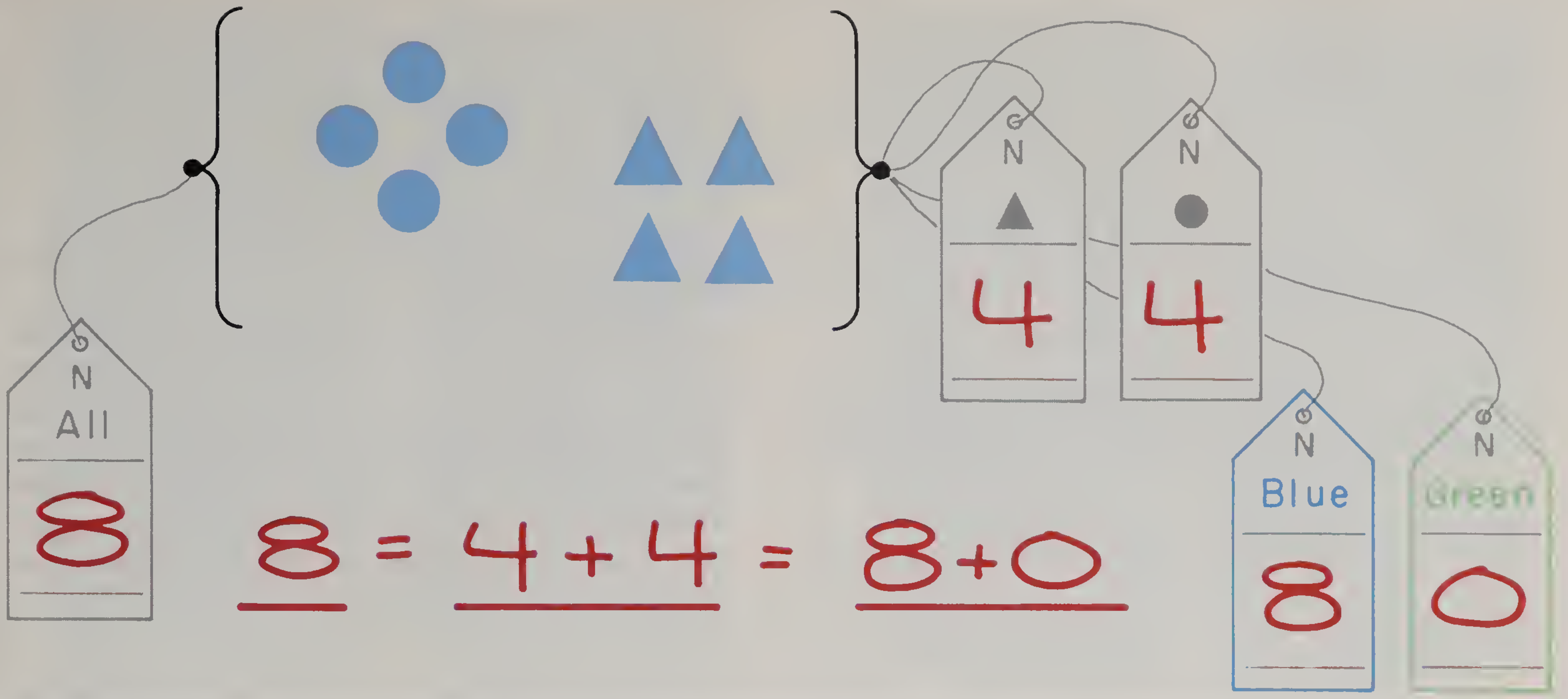
$8 = 3 + 5$ $8 = 6 + 2$ ------ $8 = 8 + 0$



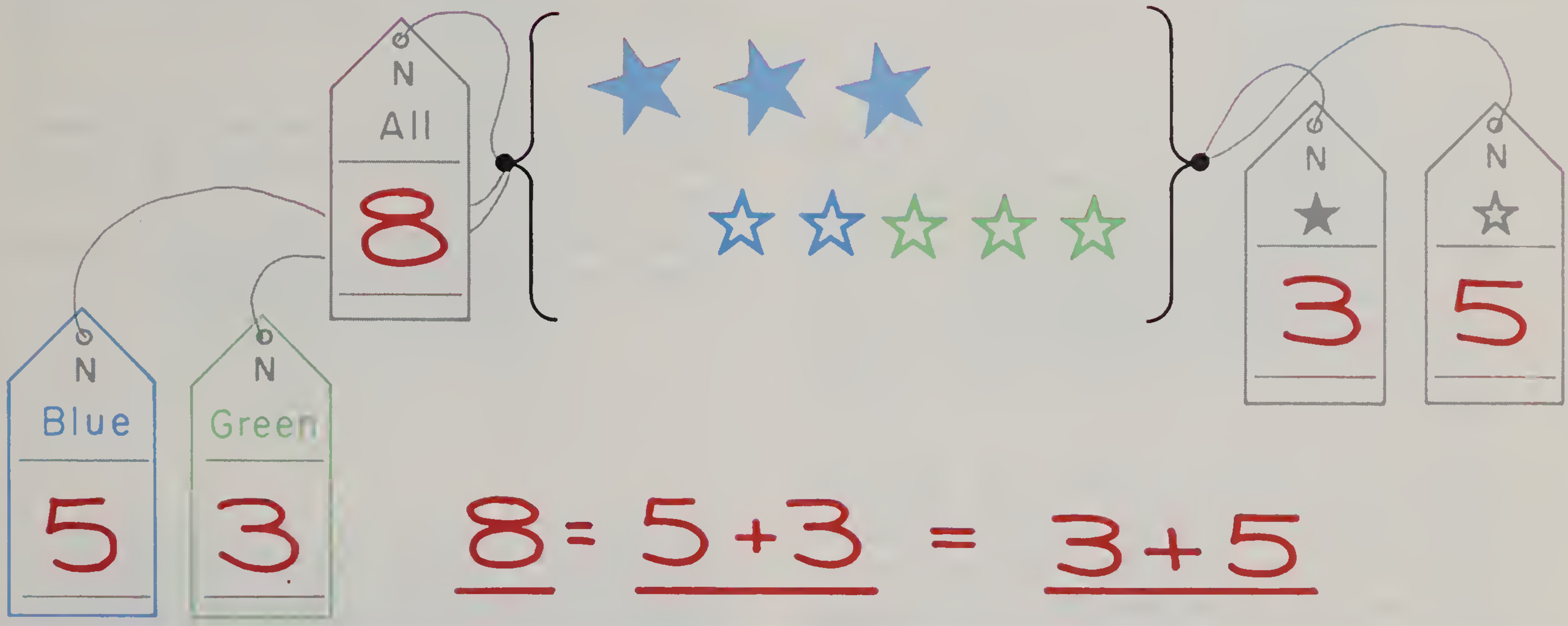
When you get through with all of this, you should have all the addition facts with sums of 8 written in a row. Decide whether or not you want to erase the board before having the pupils do the page in their books.

Be sure to review the accompanying booklet, *Games and Enrichment Activities,* for suggestions you may wish to incorporate in your daily lesson plan.

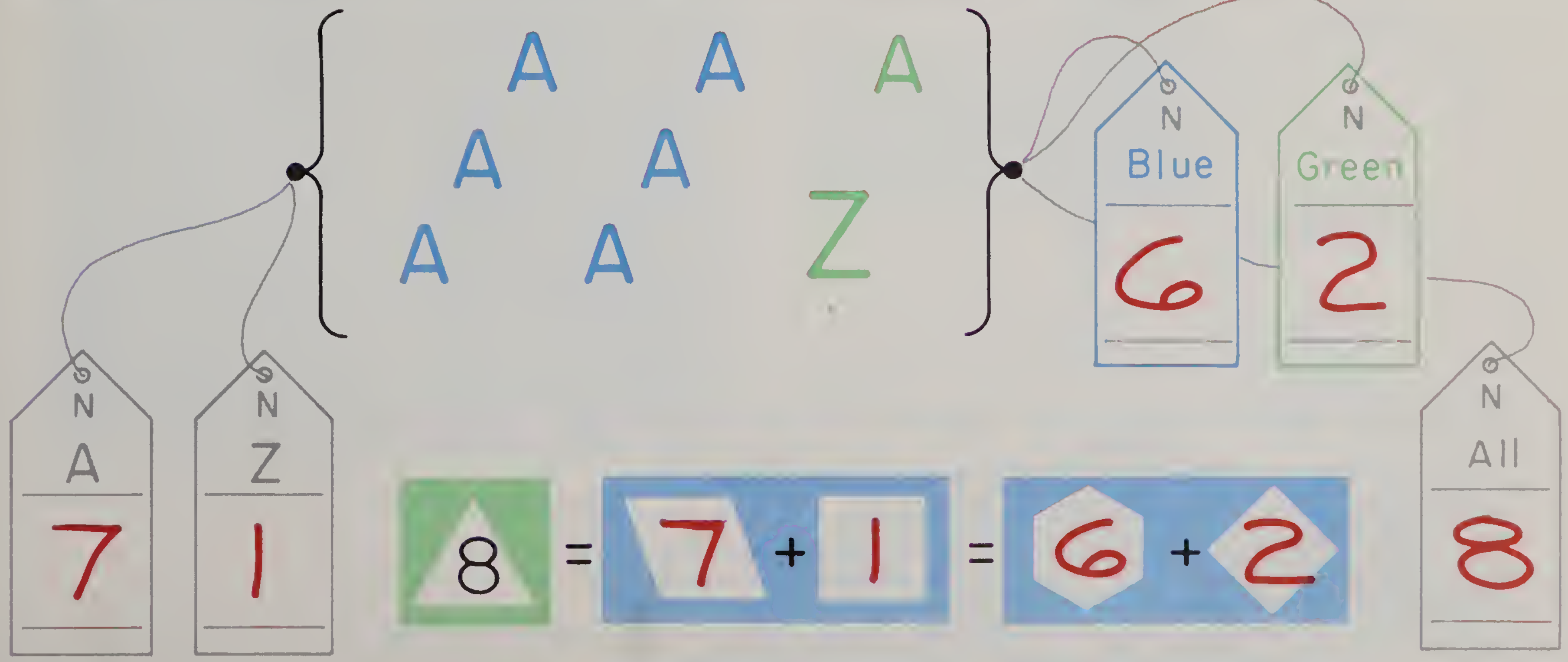
Headline Story: A wheel on Jane's tricycle was broken. (Pupils extend story.)


N
All
8
N
▲
4
N
●
4
N
Blue
8
N
Green
0

8 = 4 + 4 = 8 + 0


N
All
8
N
★
3
N
☆
5
N
Blue
5
N
Green
3

8 = 5 + 3 = 3 + 5


A A A
A A
A A Z
N
Blue
6
N
Green
2
N
A
7
N
Z
1
N
All
8

8 = 7 + 1 = 6 + 2

Notes on Pupil Page 27

Tasks: Pupils fill in the empty cells in the Cross Number Puzzles and record the count of the books pictured at the bottom of the page.

Purpose: To review some addition facts and the idea that changing the order or grouping of numbers to be added does not affect the sum.

Unifying Ideas: Sets; number and counting; structure.

Materials for Teacher: Domino cards.

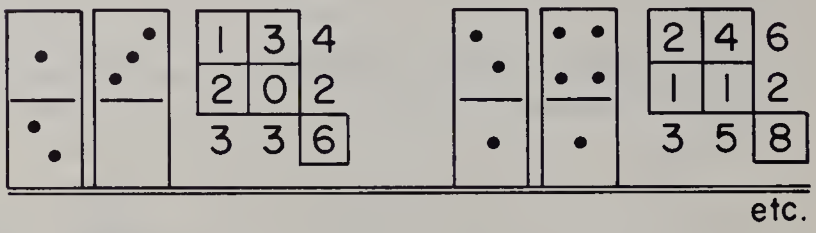
Materials for Pupils: Number lines on paper or on the chalkboard.

The Lesson:

Page 27 combines domino cards with the Cross Number Puzzle form as a record of particular arrangements. The few places on this page which may appear to involve addition facts not yet learned should be treated as simple counting exercises. The pupils should always be able to count to find out how many whenever that situation arises.

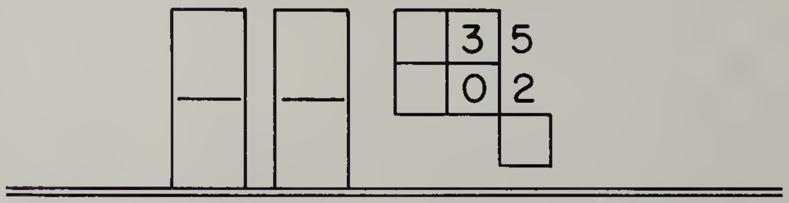
It is well to provide frequent short periods of practice with number sequences. On the chalkboard, or on a Ditto master, draw several number lines and have the pupils label them. For some of these, use numbers less than 10 (old information) and on others, use numbers greater than 10, with particular attention to the interval 10-20 (relatively new information).

Having made sure that all of your pupils know that 9, 10, and 11 follow 8, in that order, you may wish to set the stage for the culmination of this lesson by using the domino cards described in your *Games and Enrichment Activities* manual. Arrange several pairs of domino cards on the chalkrail and fill in the puzzle forms so that they reflect the situation in each pair.



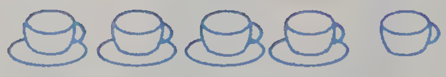
Remove the cards and put them back in the stack. Can the pupils find the cards which were used in each example?

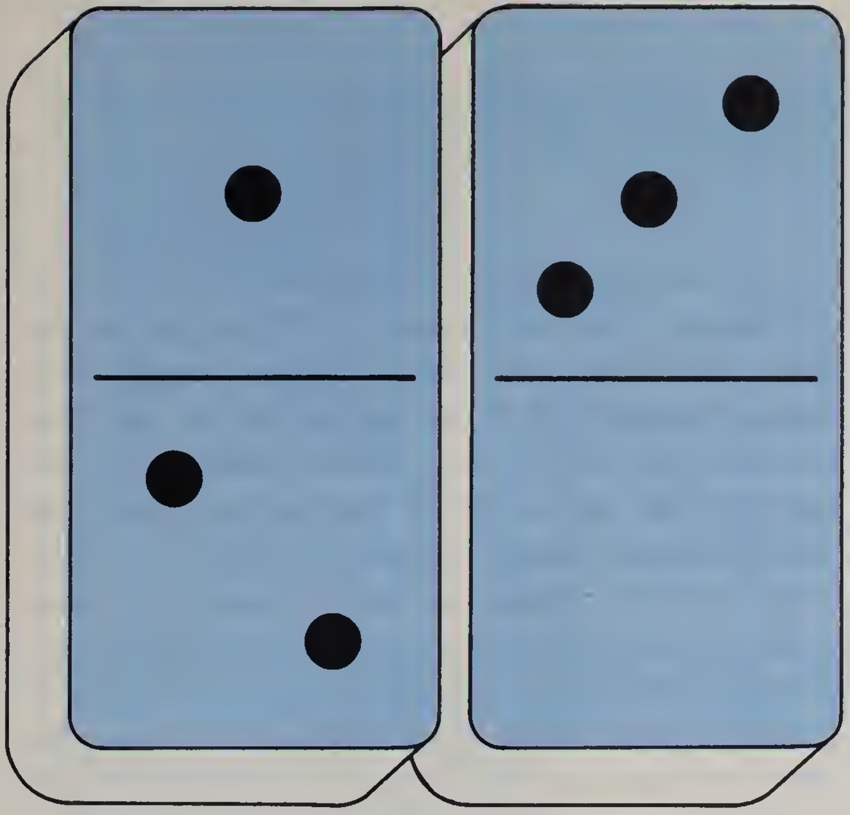
Remove the cards and erase part of the records so that one example might look like this:

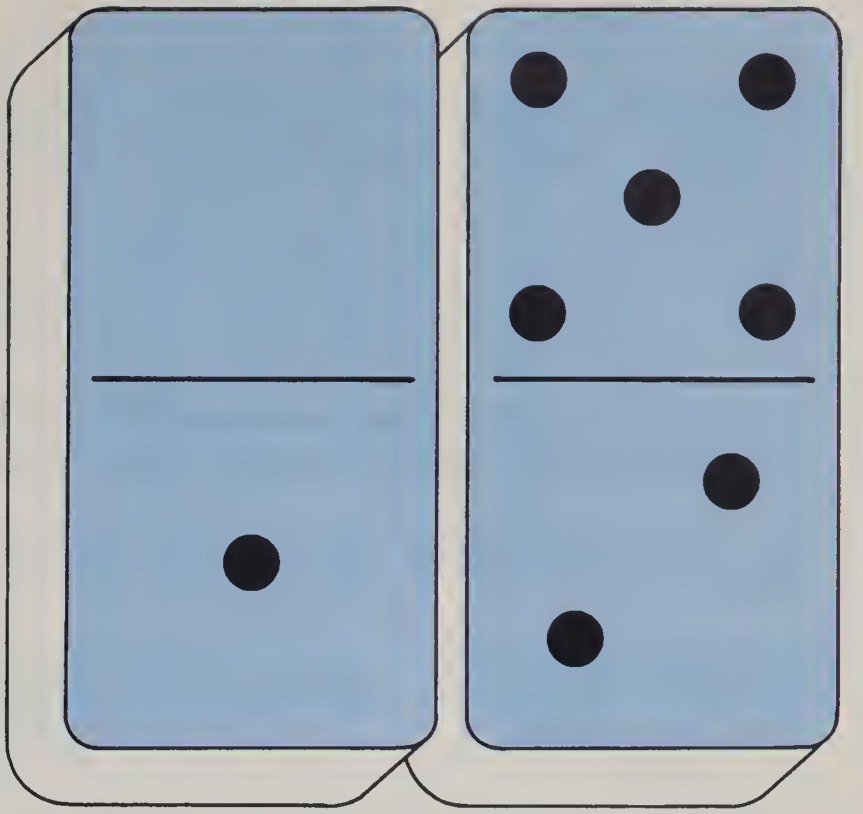


Can the pupils find the cards indicated by the partially completed record?

Headline Story: Tell a story suggested by the sketch:

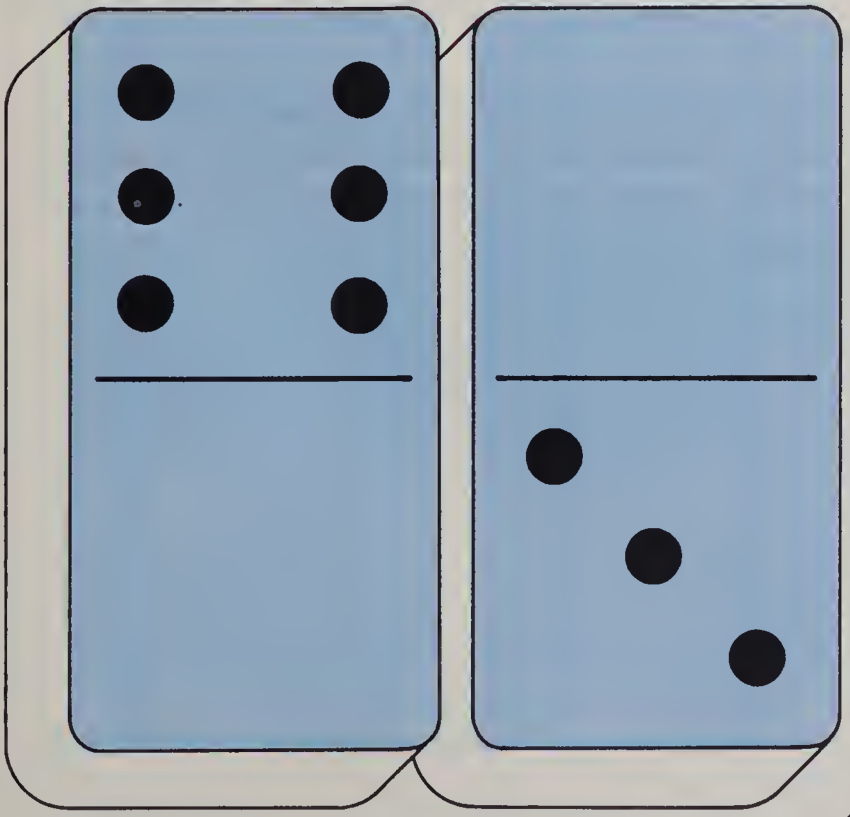




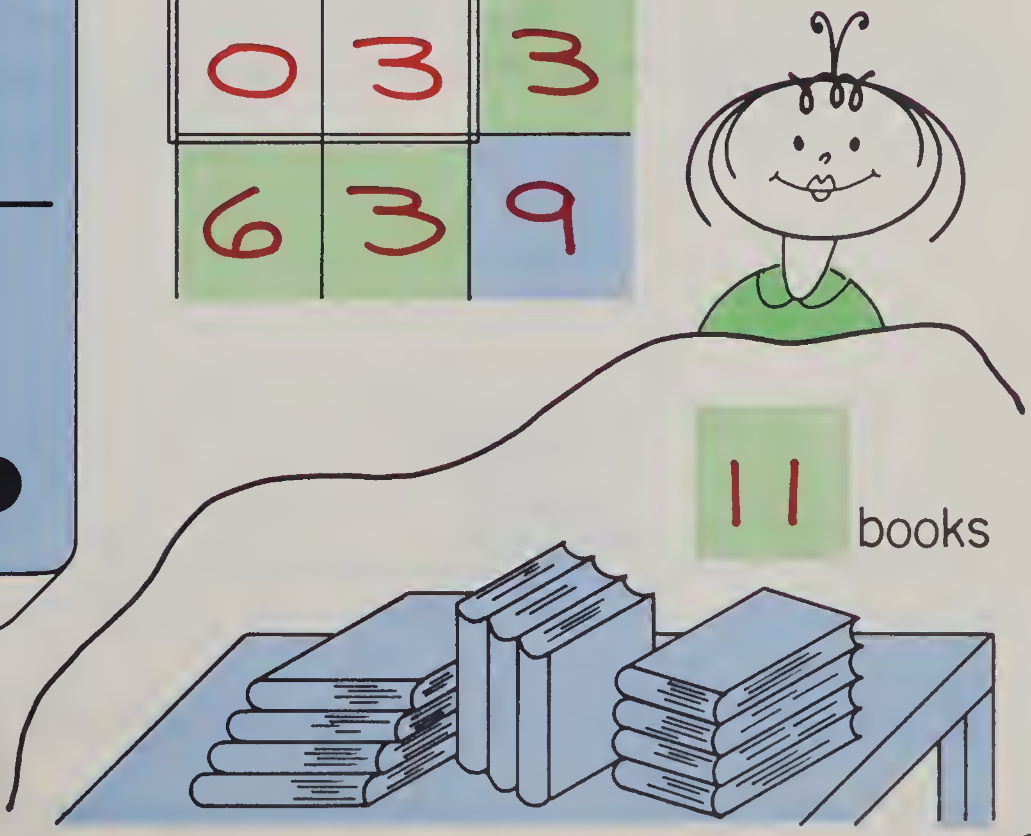


1	3	4
2	0	2
3	3	6

0	5	5
1	2	3
1	7	8



6	0	6
0	3	3
6	3	9



11 books

Notes on Pupil Page 28

Tasks: *Pupils supply the missing addends and sums and complete the labeling of the number line.*

Purpose: To introduce the numbers 14-17 as "10 and some more,"—another lesson on place value.

Unifying Ideas: Number and counting; numeration.

Materials for Teacher: A supply of at least 17 objects for counting.

Materials for Pupils: No special materials needed.

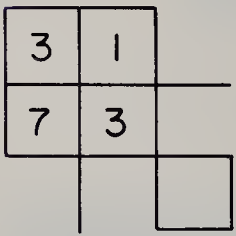
The Lesson:

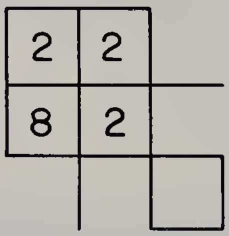
You might begin the lesson by having the pupils fill in the missing labels for the number line at the bottom of the page. Check to see that everyone has the correct idea of the sequence of the numbers in this particular range. For someone who may not, ask questions such as, "What number is 1 more than 6? Then what number should be 1 more than 16?" Even your fastest learners can profit from discussions of this kind.

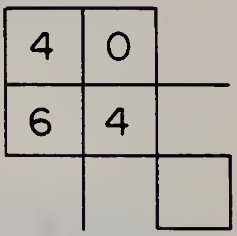
Now call attention to the long strip at the top of the page. How many squares are in this strip? Where should we record this number? Look at the short strip just below this long strip. How many squares in this short strip? Where should we record this number? How many squares in both strips together? (Does anyone know the answer without counting? Good for him.) Where should we record this number? Now, what do we know? Fourteen is ten and four more. Write on the board: $14 = 10 + 4$.

Proceed in the same manner with the rest of the page. Notice whether there are any pupils who, having counted one of the ten-strips, do not have to count the rest. This is mature performance. Be sure to point out explicitly that any number word which ends in "teen" means "10 and . . ." Ask what *fourteen* means; *seventeen; sixteen; fifteen.*

Having completed the page, ask a pupil to get you a stated number of counters, 11-17. What follows is a description of an activity if the number you have chosen is 14. You can adapt it to other numbers. Arrange four desks in two rows of two, and draw a Cross Number Puzzle form on the board. Arrange the objects on the four desks yourself, because you wish to come out with totals of 10 and 4 each time. Shown below are three appropriate arrangements. If you interchange the rows, you will get three more, and if you interchange the columns, you will get three more. For each arrangement, make appropriate entries in the record on the board. Complete each puzzle.

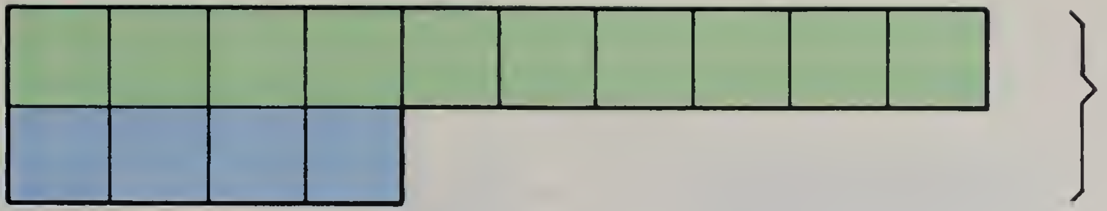






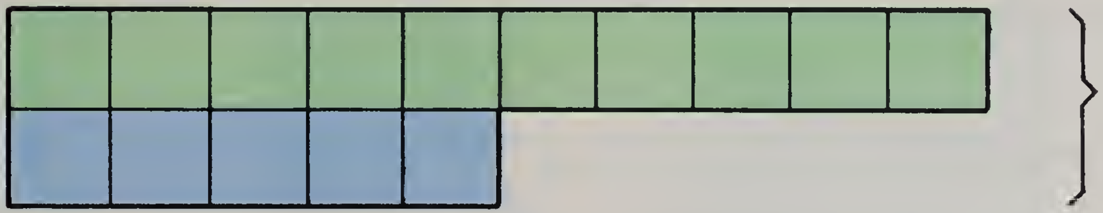
Be sure that every pupil sees that you were always working with 14 counters in all and that each of these arrangements showed 14 as either 10 and 4 or 4 and 10.

Headline Story: Tell a story with 2 and 10 in it. Another with 12 in it.



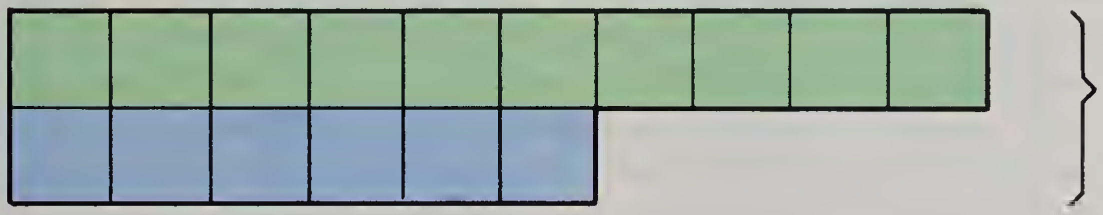
10 4

14



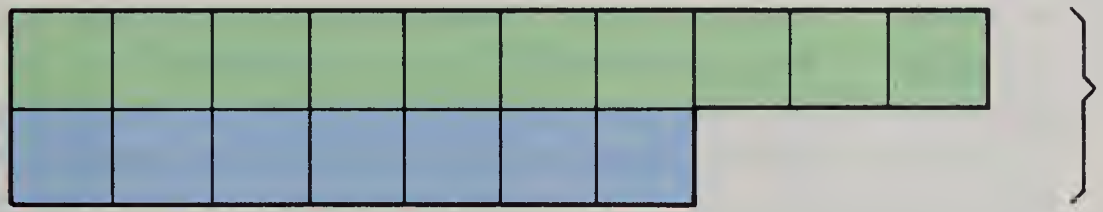
10 5

15



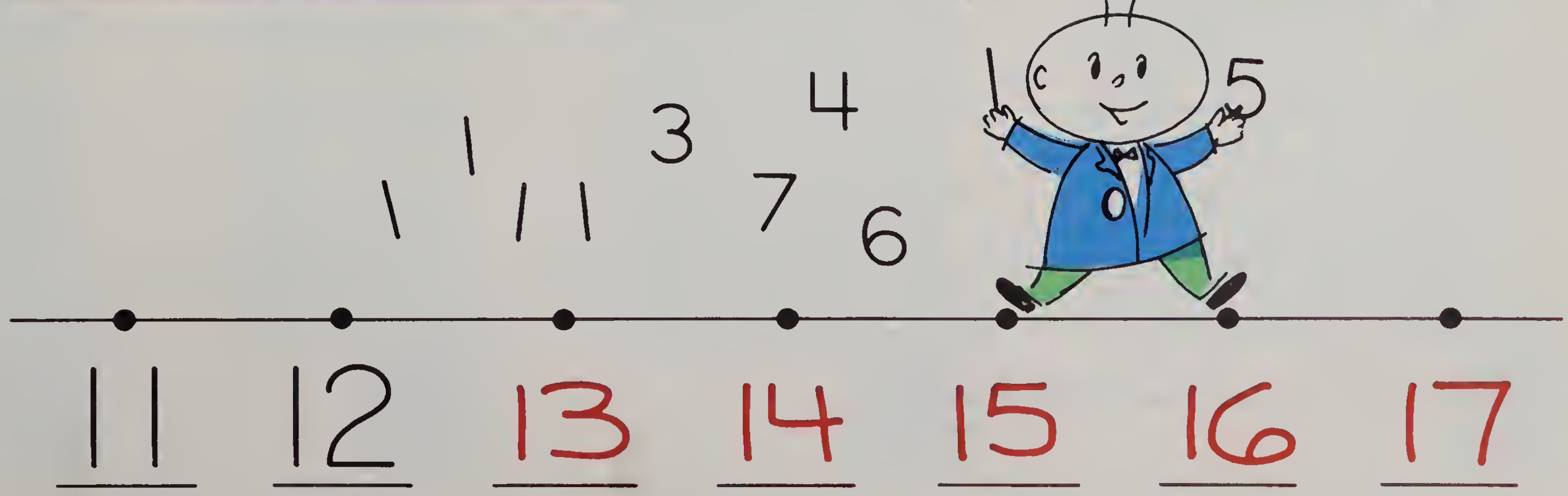
10 6

16



10 7

17


1
1 1 1
3
4
7
6
5
0
11
12
13
14
15
16
17

Notes on Pupil Page 29

Tasks: *Pupils count the squares and record the counts.*

Purpose: To furnish a counting experience structured to provide readiness for multiplication.

Unifying Ideas: Number and counting; geometry.

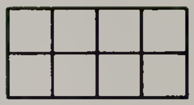
Materials for Teacher: No special materials are needed.

Materials for Pupils: No special materials needed.

The Lesson:

Let the pupils do the page in their own way. Check their results. (An easy way to do this is to choose a paper which is all correct and put it on an opaque projector. The rest of the pupils can check their own answers against this.) Then discuss how different pupils arrived at their answers. Was anyone counting by 4's in the last row, for instance? Did anyone get the 12 at the end of the third row by adding 3 to the 9 he had just before it? Are there any numbers which appear in two places on the page? (2, 4, 6, 8, 12.) What is alike and what is different about the two places they appear?

If you wish to foreshadow multiplication in a much more pointed way with a selected group of children, begin by calling their attention to certain blocks of tiles pictured on the page, doing three or four in random order. In each case, ask, "How many rows? How many tiles in each row? How many all together?" As they answer, write casually on the board the appropriate version of the following:



2 of 4 = 8

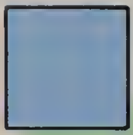
Then pick a clean spot on the board, and work with one row of pictures at a time. For the second row, you will have written:

2 of 1 = 2
2 of 2 = 4
2 of 3 = 6
2 of 4 = 8

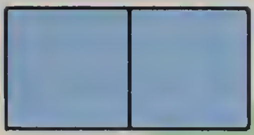
Now study the numbers listed on the right-hand side of your = signs. They "go by 2's"; each number is 2 more than the number just above it. How do the pictures in the book show this? Have a similar discussion about the results obtained from the other rows.

An interesting and productive extension of this lesson for all pupils can be provided if you make a set of cardboard pieces resembling the squares and rectangles on page 29. Have the pupils arrange these pieces on the chalkrail from the smallest to largest. Count the blocks in each piece and record the results on the board. Problems arise. Two different shapes have the same number of blocks—1 by 4 and 2 by 2. Two cards have the same shape and the same number of blocks, but they can be placed in two different ways. Certain numbers are not suggested by any of the pieces. (Where is the the piece for 5? 7? etc.) Can it be that there are some numbers which you cannot suggest by a rectangular piece other than a piece just one square wide? (There are. These are the prime numbers.)

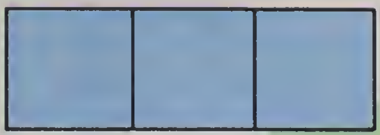
Headline Story: Tell a story with 16 in it.



1



2



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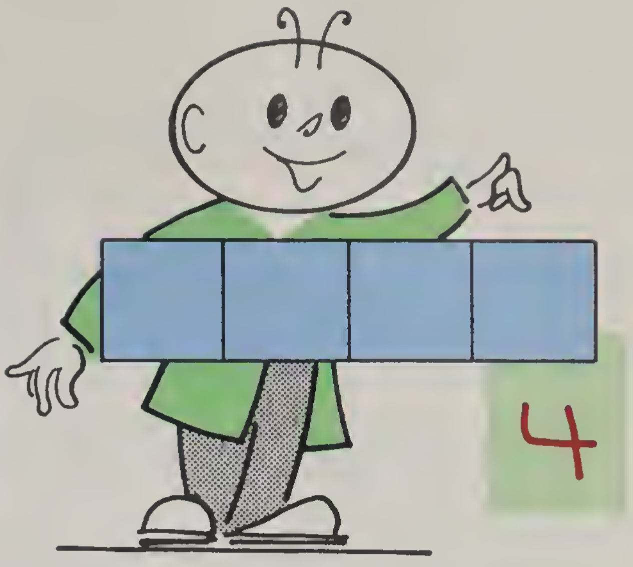
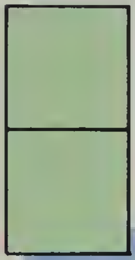


4



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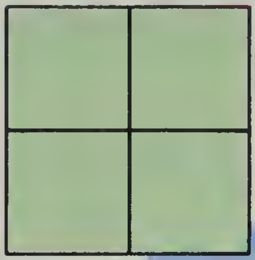
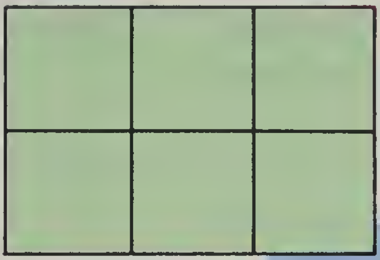


4



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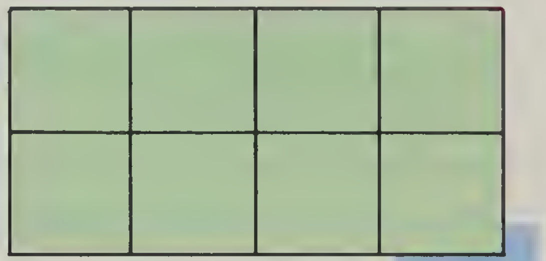

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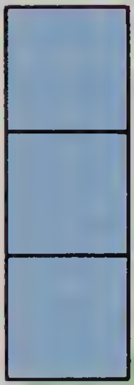
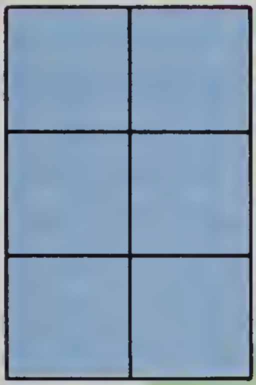
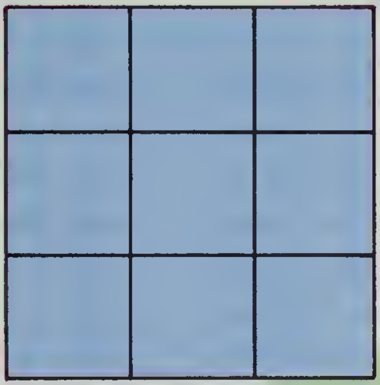


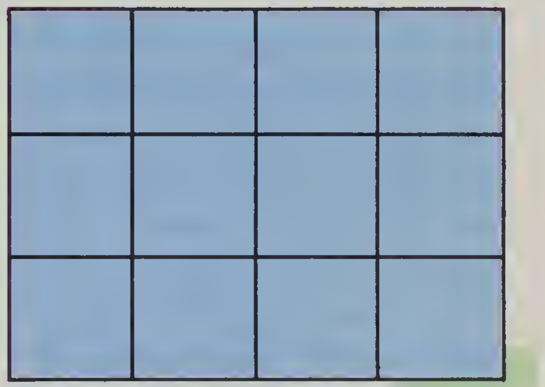
3



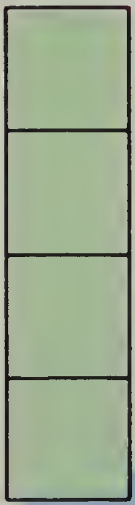
6



9



12



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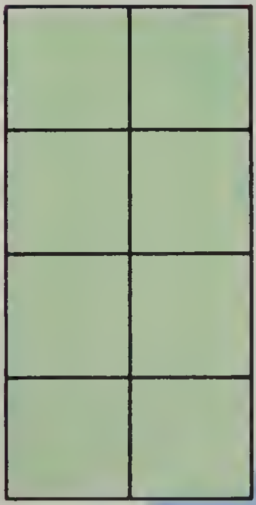

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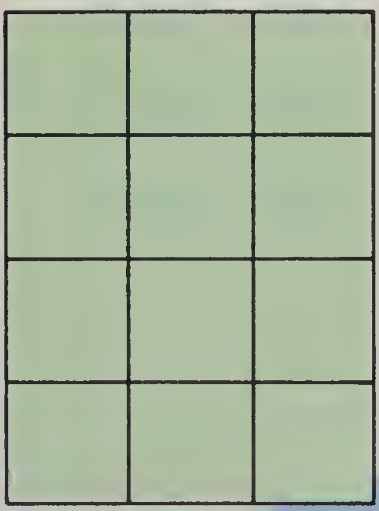

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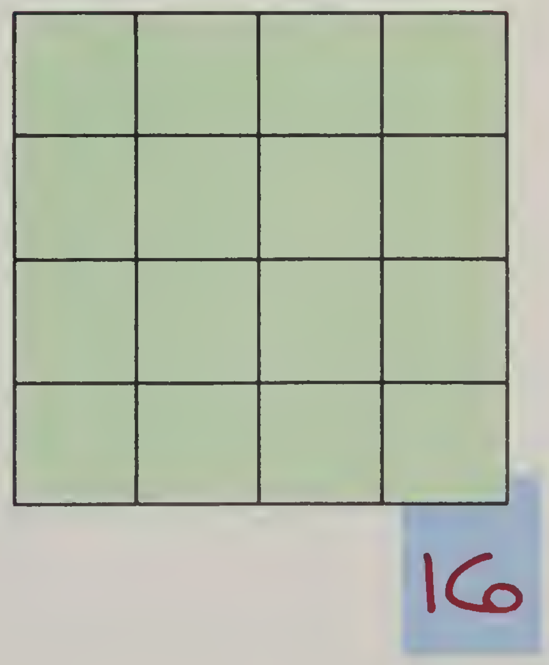


16

Notes on Pupil Page 30

Tasks: Pupils count the number of like objects in a set, the total number of objects in the set, record the counts, and write appropriate equations.

Purpose: To teach addition facts with sums of 9.

Unifying Ideas: Sets; number and counting; addition and subtraction; structure.

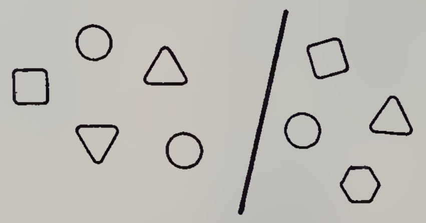
Materials for Teacher: Nine objects for counting; a pointer, stick, or piece of string; number line, labeled at least to 10.

Materials for Pupils: A supply of things to count.

The Lesson:

First, take steps to see that everyone knows how many objects make 9 of that object. Choose pupils who may not know and ask them to bring you 9 pieces of chalk, 9 square counters, 9 crayons, or the like. Call the names of 9 pupils and ask them to stand. Have a pupil count to find out how many are standing. Place a counter on each unit point of a number line, beginning with 1. Be sure everyone notices the position of the last counter, then remove the counters and have a pupil count them. If he does not count correctly, have him lay the counters on the number line, one at a time, the first counter on 1, and say a number for each counter as he lays it down. Ask what the last number he said tells him. Pupils who still cannot count correctly at least up to 10 should be grouped for immediate special attention.

One way to introduce addition facts with sums of 9 is to place 9 objects on a table in any arrangement you please, but far enough apart so that a stick (pointer, yardstick, even a piece of string) can be used to separate the 9 objects into two groups. Have the objects counted and be sure everyone agrees there are 9 in all. Then lay the stick down in some convenient position. Ask, "How many on the left side of the stick? How many on the right side? How many in all?" Write the appropriate addition fact on the board.



$5 + 4 = 9$

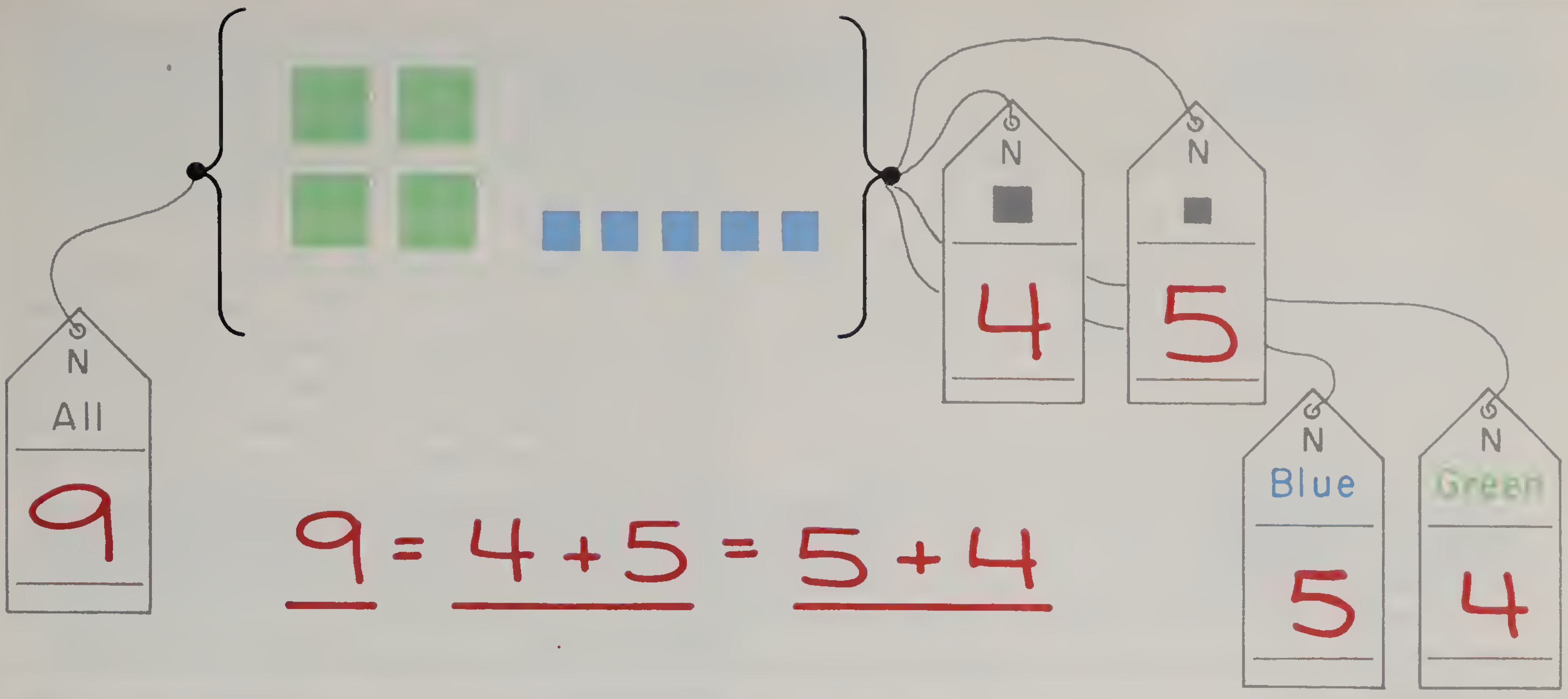
Continue in this way until all the facts are recorded. There are ten of them. You get the facts which have 0 as an addend by laying the stick at one side or the other of the whole group.

Direct the pupils' attention to the list of facts. Ask them to find pairs of facts which use the same numbers. How many of your pupils realize now that the order in which two numbers are added does not affect their sum? (This is the commutative principle of addition.) Do your pupils realize, also, that $7 + 2 = 9$ and $9 = 7 + 2$ convey precisely the same information?

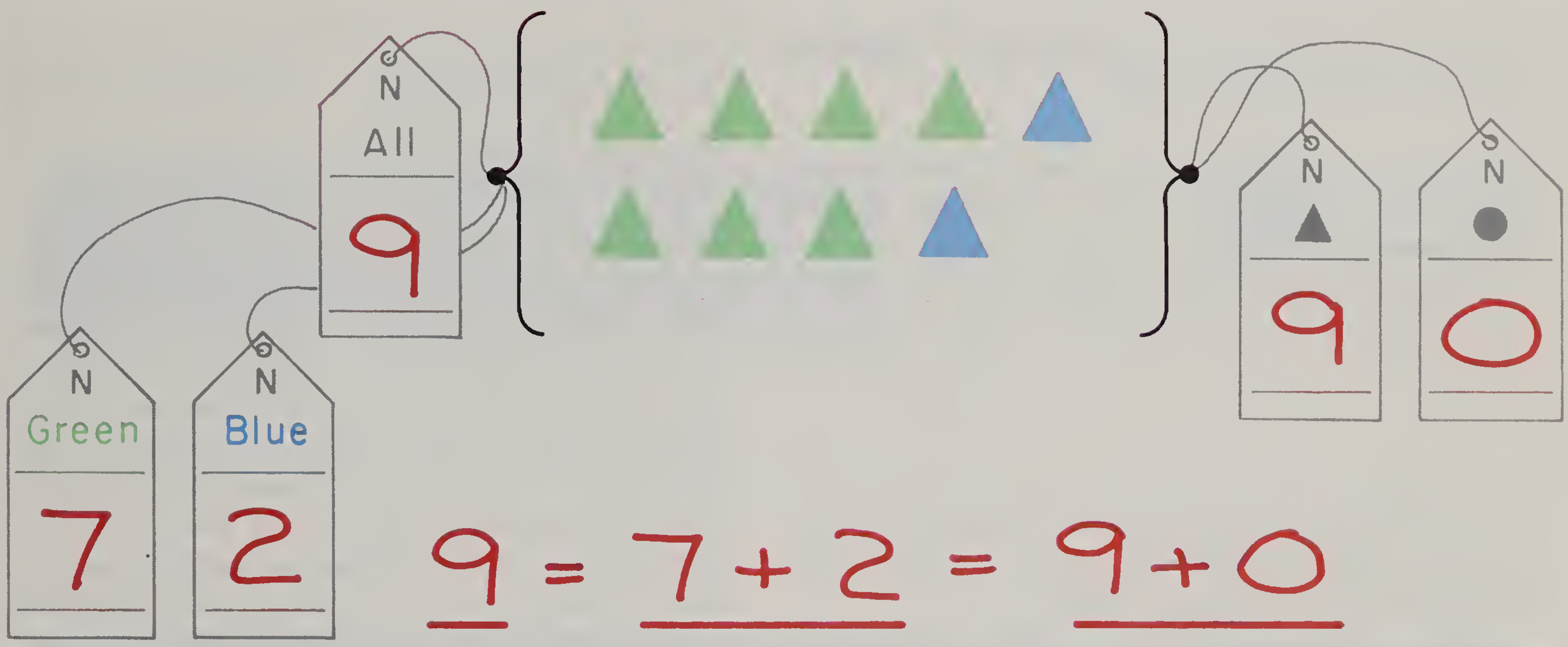
Be sure to review the accompanying booklet, *Games and Enrichment Activities*, for suggestions you may wish to incorporate in your daily lesson plan.

Headline Story: Complete the headline and tell a story that fits:

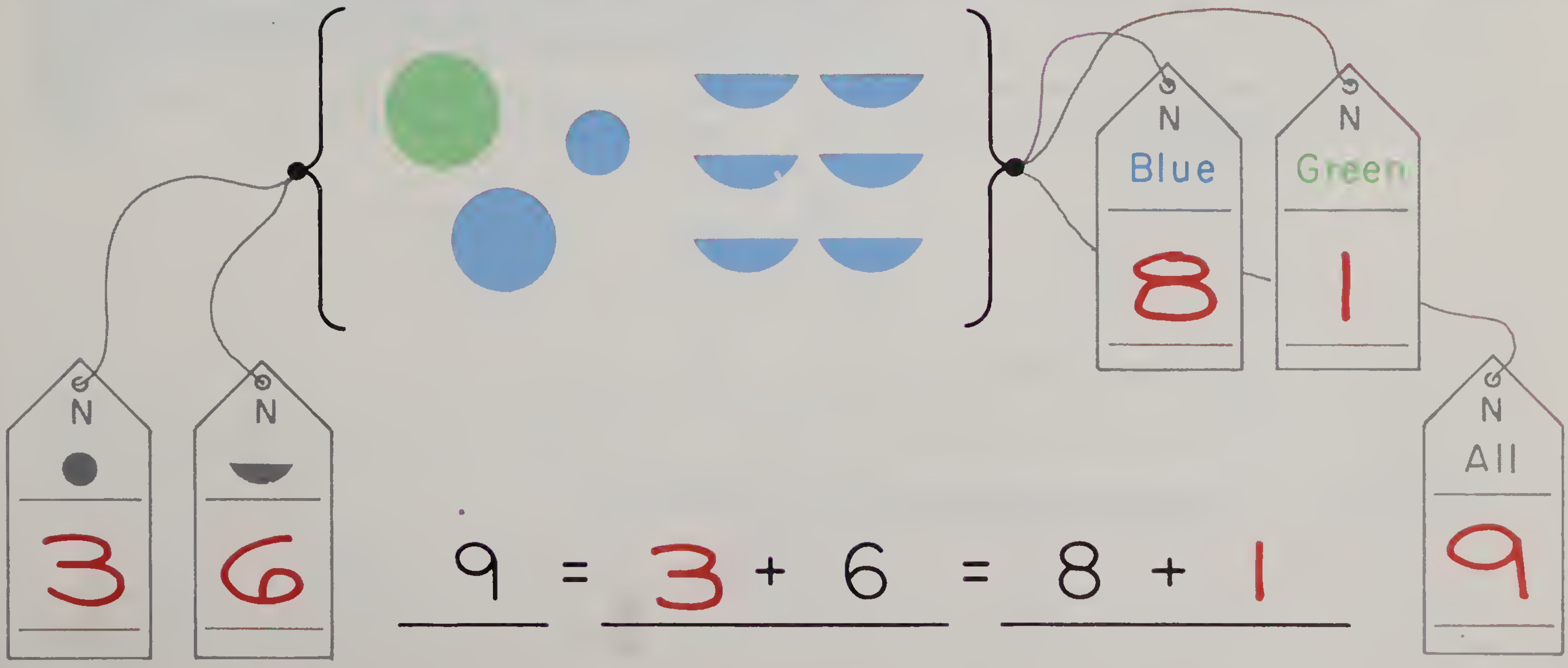
$2 + 1 = \square$


N
All
9
N
4
N
5
N
Blue
5
N
Green
4

9 = 4 + 5 = 5 + 4


N
All
9
N
9
N
0
N
Green
7
N
Blue
2

9 = 7 + 2 = 9 + 0


N
Blue
8
N
Green
1
N
3
N
6
N
All
9

9 = 3 + 6 = 8 + 1

Notes on Pupil Page 31

Tasks: *Pupils fill in boxes to show how many blue blocks, how many green blocks, and how many in all; then they complete the labeling of points on the number line.*

Purpose: (a) To help pupils see 18 as 10 and 8 more, 19 as 10 and 9 more, 20 as 2 tens and no more, and 21 as 2 tens and 1 more; (b) to show the sequence of the numbers 15-21 and to emphasize the idea that when arranged consecutively each number is 1 more than the number which precedes it.

Unifying Ideas: Numeration; structure.

Materials for Teacher: Two dimes and 9 pennies in real money; colored chalk.

Materials for Pupils: Two dimes and 9 pennies in play money, or two kinds of counters the pupils can pretend are dimes and pennies, or white and orange Cuisenaire® Rods (white rods can represent pennies, while orange rods can represent dimes).

The Lesson:

To review the number names in sequence, have several pupils count from 1 to 25; then have the class as a whole do it once or twice.

Have the pupils take out their pennies and dimes. Keep your supply of real money at hand for the use of any pupil who needs help. (It works wonders.) Ask the pupils to show 1 cent with their counters, 4 cents, 5 cents, 8 cents, 9 cents. Check to see whether all the pupils know how many cents a dime is worth. Ask how they can show 10 cents with their money when they have only 9 pennies. Now, build up past 20 cents, 1 cent at a time, having the pupils show successive amounts with their counters while you keep a record on the board. If possible, use white chalk for dimes and brown chalk for pennies. As you progress from 11 to 12, 12 to 13, and so on, emphasize the "1 more" relation.

Ⓓ Ⓟ 11 = 10 + 1
Ⓓ ⓅⓅ 12 = 11 + 1 = 10 + 2
Ⓓ ⓅⓅⓅ 13 = 12 + 1 = 10 + 3

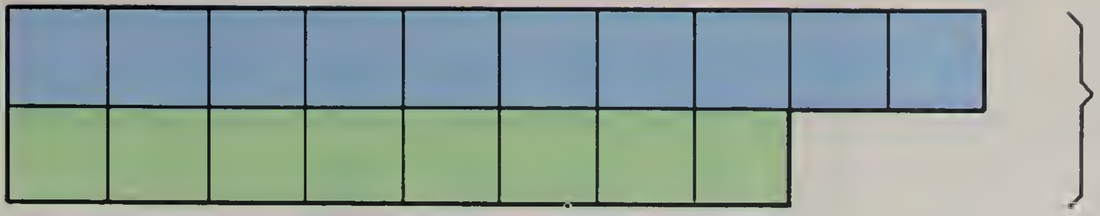
Ⓓ ⓅⓅⓅⓅⓅⓅ 16 = 15 + 1 = 10 + 6
Ⓓ ⓅⓅⓅⓅⓅⓅⓅ 17 = 16 + 1 = 10 + 7

Choose a pupil to hold a dime in one hand and 5 pennies in the other. Have another say, "10 and 5, 15." Have another pupil show 18 in the same way. These pupils should stand facing the class with 18 at the right of 15, as the class sees them. Choose another pupil to show 17. Ask where he should stand. Cover all the numbers 15-22 in this way, and when the children are all lined up in the correct order, ask the appropriate pupils to say, in turn, "10 and 5, 15; . . . 10 and 9, 19; 2 tens and 0, 20; 2 tens and 1, 21; 2 tens and 2, 22."

As the pupils do page 31 in their books, watch to see whether any of the pupils can tell the number of squares in the rows after the first row without counting. If they can do so, praise them. When there is an opportunity, have them discuss their methods with the rest of the class.

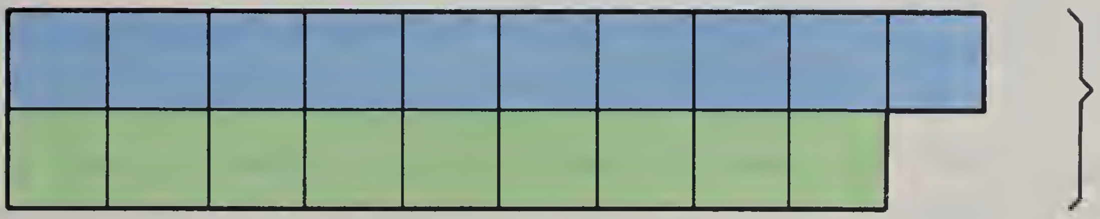
Headline Story: Complete the headline and tell a story that fits:

$$3 + \triangle = 5$$



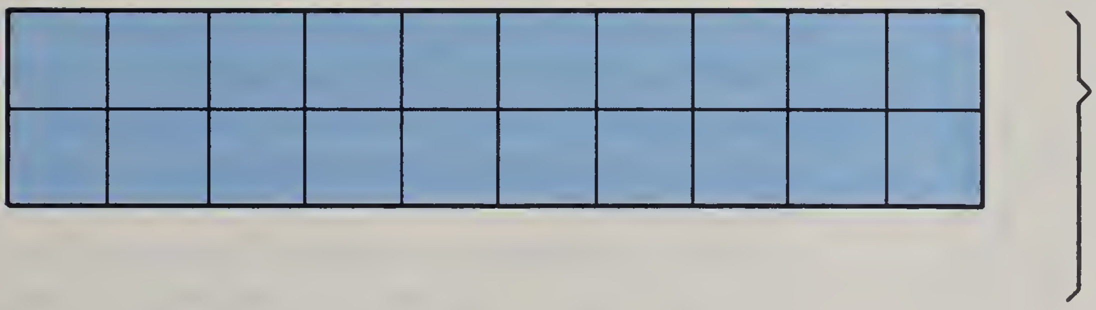
10 8

18



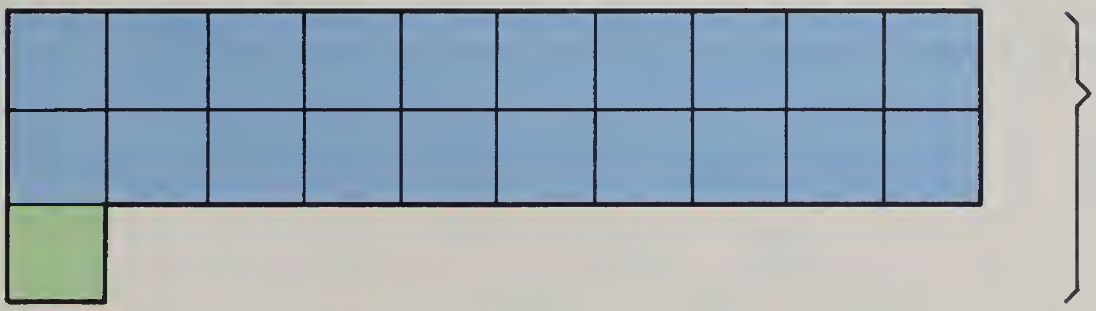
10 9

19



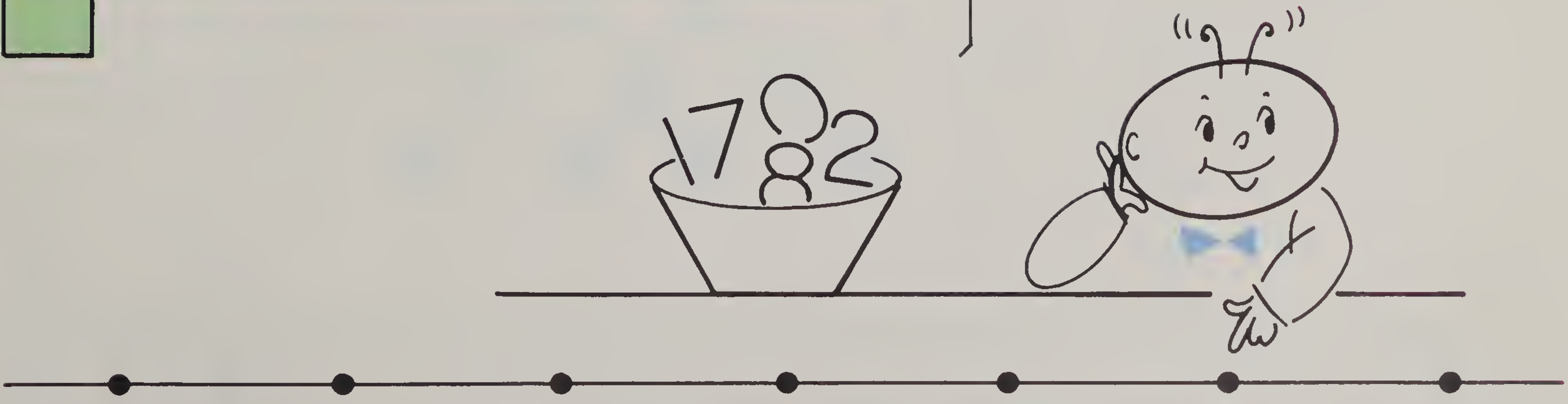
20 0

20



20 1

21



15 16 17 18 19 20 21

Notes on Pupil Page 32

Tasks: Pupils fill in the number tags according to the codes on these tags, then fill in the blanks in the open sentences to make true statements.

Purpose: To teach addition facts with sums of 10.

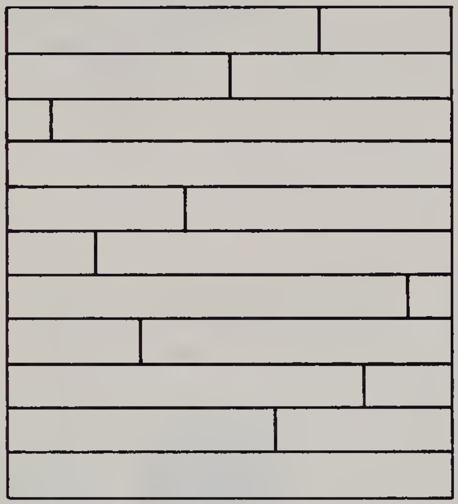
Unifying Ideas: Addition and subtraction; sets.

Materials for Teacher: Demonstration rods cut in lengths from 1 to 10 units, or strips of oak tag cut accordingly.

Materials for Pupils: Rods cut in lengths from 1 to 10 units, or strips of graph paper cut accordingly; paper on which to keep records.

The Lesson:

Use your demonstration materials to help the pupils build strips 10 units long out of no more than two strips or rods. Do not suggest any particular order; rather, encourage each pupil to keep a record of each row he has constructed.



10 = 7 + 3
10 = 5 + 5
10 = 1 + 9
10 = 10 + 0
10 = 4 + 6
10 = 2 + 8
10 = 9 + 1
10 = 3 + 7
10 = 8 + 2
10 = 6 + 4
10 = 0 + 10

When a pupil claims he is finished, check to see whether he has 11 facts in all. If he has not, ask him to try to find some other ways to make 10. When all have finished, have one pupil at a time read a fact from his paper while you keep a record on the board. Have the pupils check their lists against the list you have on the board. The 10 strip, of course, can be thought of as representing 10 + 0 or 0 + 10.

Have pairs of pupils come to the front of the room. The first pupil in each pair is to pick up one of your demonstration rods. The second pupil is to find the rod which goes with that to make 10. The rest of the class is to check the performance in each case and a pupil who finds an error may tell or demonstrate how to correct it.

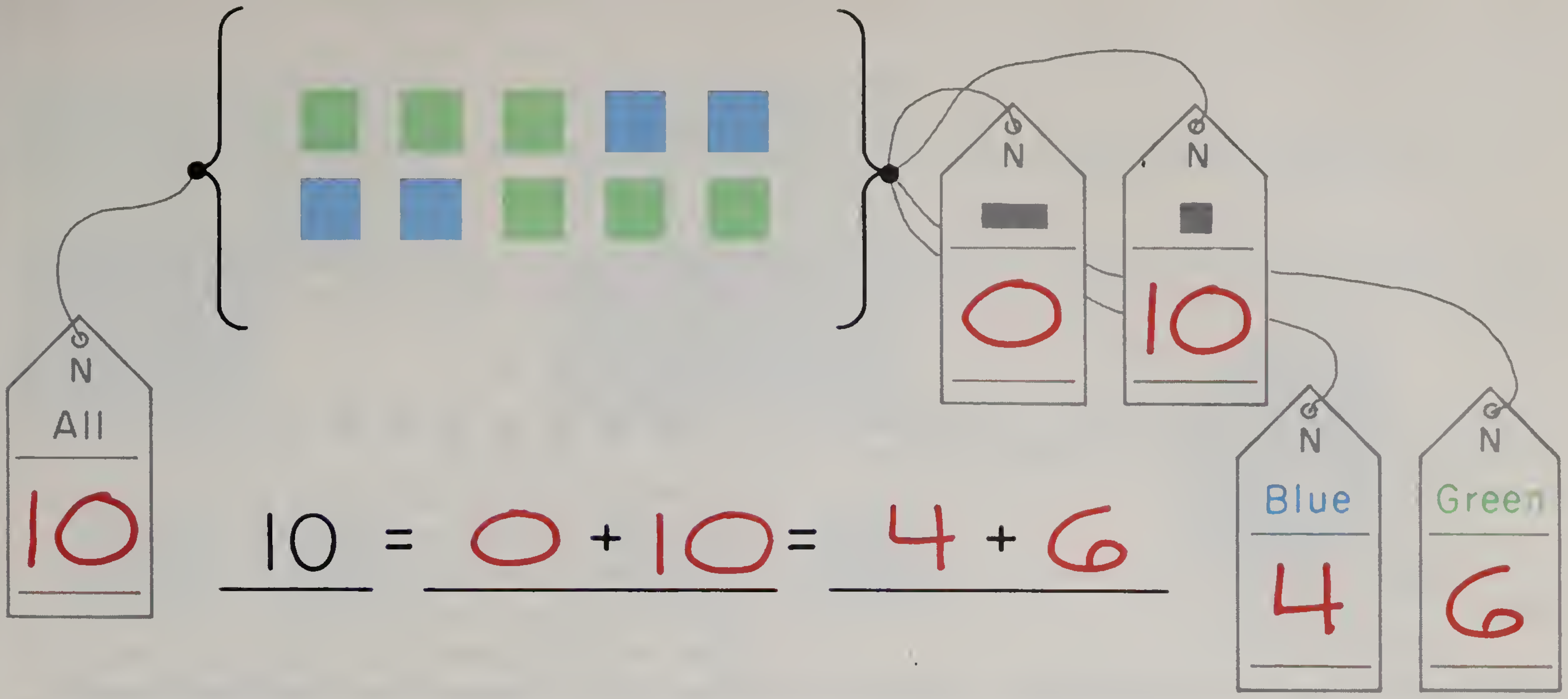
When the pupils turn to the work in their books, help them get started on the top row. Suggest that in each section they use the numbers on the tags in that section to complete the open sentences. If a pupil does not follow your suggestion to use the numbers on the tags, but fills in the sentences to make true sentences in some other way, accept his work as correct.

Put the following examples on the board and have the pupils complete the statements, using counters or rods or the pages in their books to help them, if necessary:

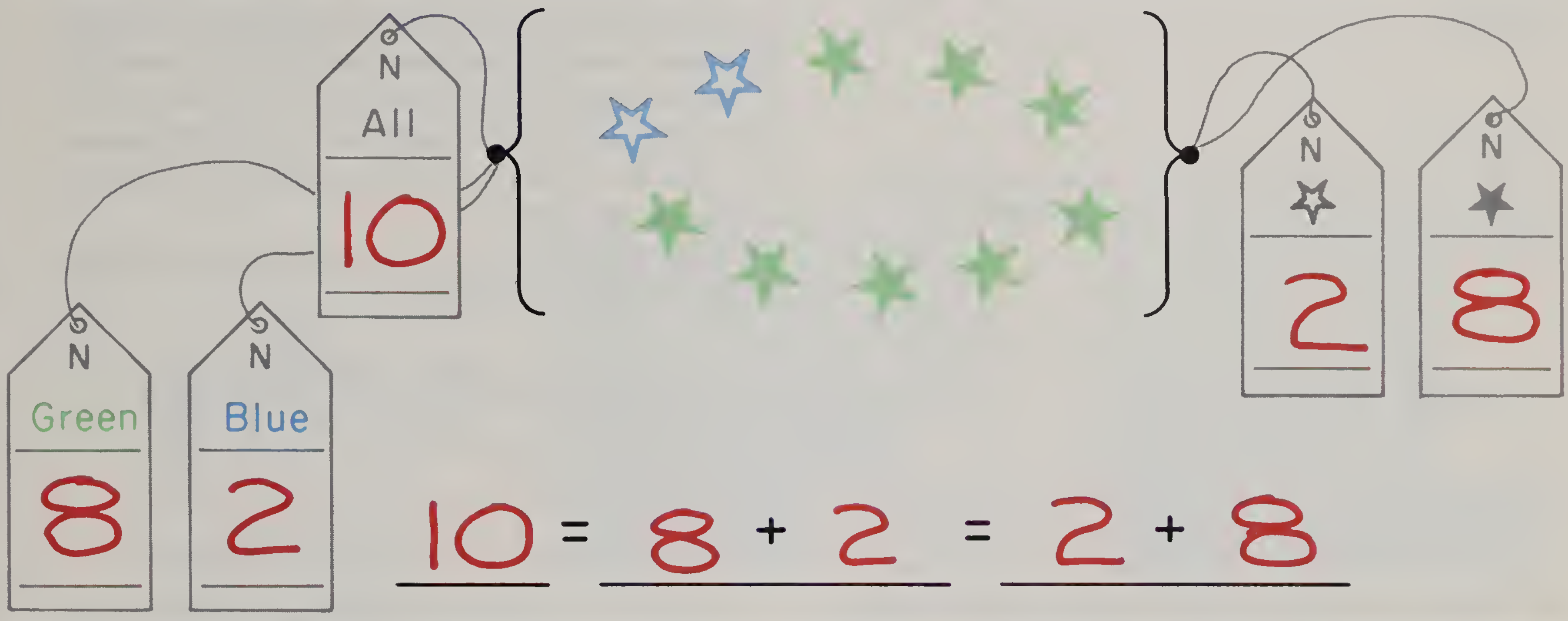
4	7	5	2	9	3	6	8	1	10	0
+6	+3	+5	+8	+1	+7	+4	+2	+9	+0	+10
__	__	__	__	__	__	__	__	__	__	__

Headline Story: Complete the headline and tell a story that fits:

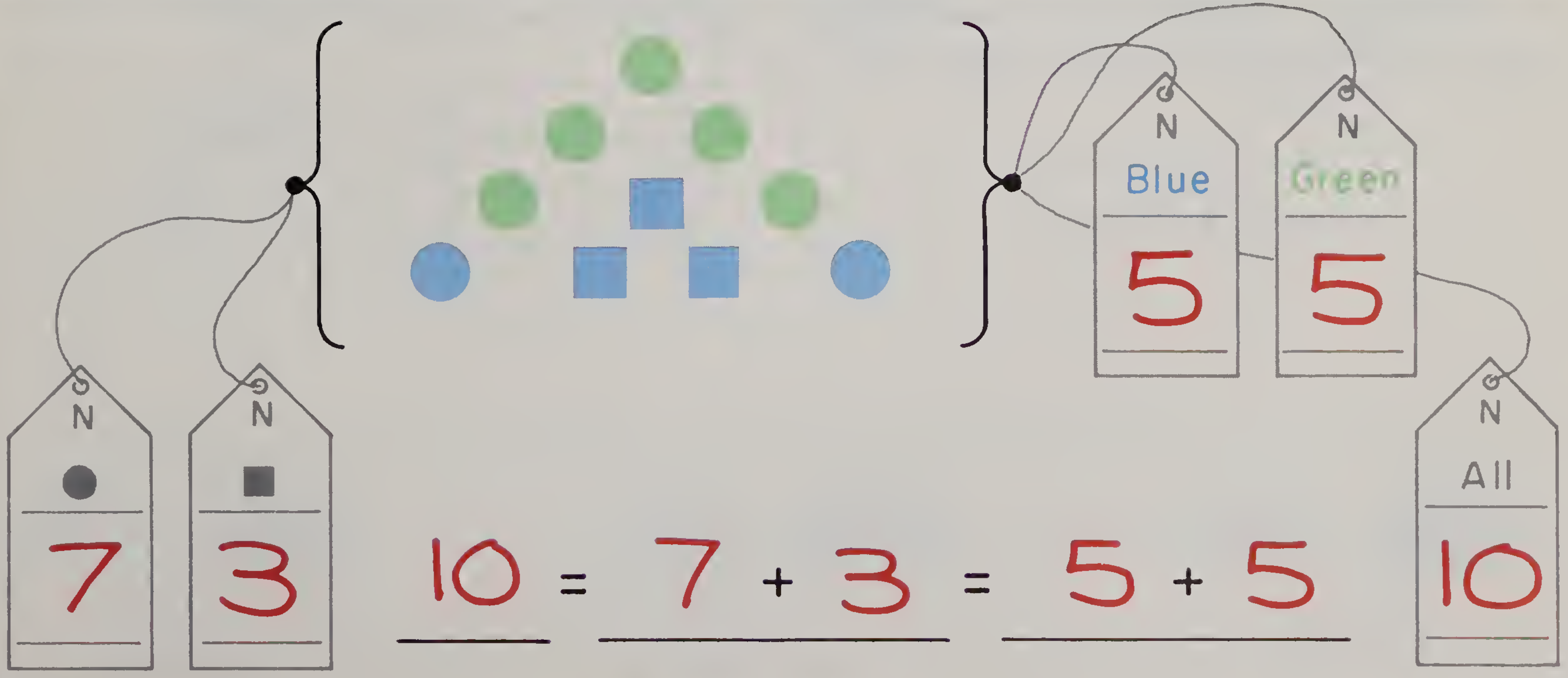
⬡ + 4 = 7


N
All
10
N
▬
0
N
■
10
N
Blue
4
N
Green
6

10 = 0 + 10 = 4 + 6


N
All
10
N
Green
8
N
Blue
2
N
☆
2
N
★
8

10 = 8 + 2 = 2 + 8


N
Blue
5
N
Green
5
N
●
7
N
■
3
N
All
10

10 = 7 + 3 = 5 + 5

Notes on Pupil Page 33

Tasks: *Pupils fill in the boxes to tell, in some cases, how many of each kind, and in all cases, how many in all.*

Purpose: To have the pupils observe a connection between the meaning of the teen numbers and sums in the teens.

Unifying Ideas: Addition and subtraction; numeration.

Materials for Teacher: Demonstration counters for use with a flannel board or cork board.

Materials for Pupils: Ten each of two kinds of counters.

The Lesson:

Put on your flannel board 10 of one kind of counter in one group and 4 of the other kind in another group.

○ ○ ○ ○ ○ ○ ○ ○ ○ ○
● ● ● ●

Have the pupils tell you how many in each group. Then ask, "What number is 10 and 4?" Have the pupils duplicate this arrangement on their desks. Ask them how many counters they have in each group, and how many in all.

Then have the pupils put 7 counters of each kind on their desks while you do the corresponding thing at the flannel board (or have a pupil work at the flannel board).

○ ○ ○ ○ ○ ○ ○
● ● ● ● ● ● ●

Ask them to move enough counters from one group to the other so there will be 10 in the larger group.

○ ○ ○ ○
● ● ● ● ● ● ● ○ ○ ○

Ask how many are in each group now, and how many in all. Do they have the same number of counters now as they had before they moved some of them? How many are 7 and 7?

Have the pupils start again, this time with 4 counters and 8 counters. Have them move enough counters from the group of 8 so there are 10 in the other group. How many are in each group now? How many in all? How many are 4 and 8? Have them put back the counters in groups of 4 and 8, and move enough from the group of 4 to make 10 in the other group. How many in each group now? How many in all? How many are 4 and 8? Repeat this exercise with other teen numbers, as long as it appears necessary.

Pupils might complete the following records of the four pictures on the page:

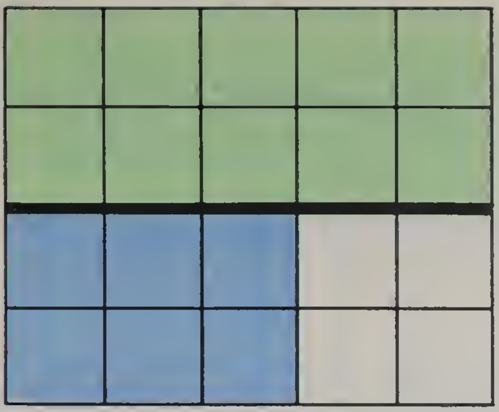
	blue	green	white	total
a.	6	10	4	20
b.				
c.				
d.				

Pupils might extend their tables showing other ways to color. Some possibilities are:

e.	10	10	0	20
f.	1	18	1	20
g.	10	0	10	20

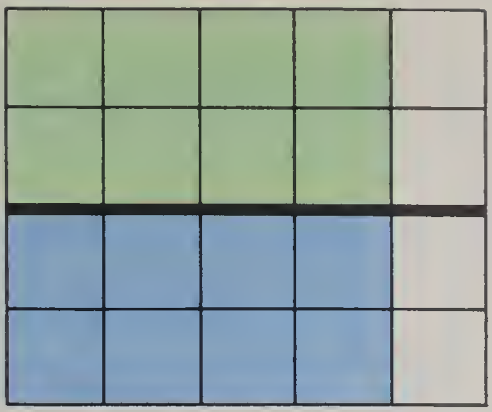
Headline Story: Complete headlines and tell stories to fit:

□ + △ = 10 ○ + ⬡ = 20



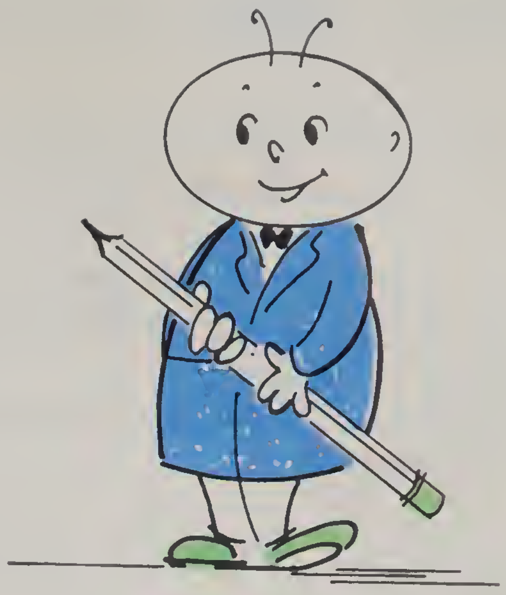
10 6

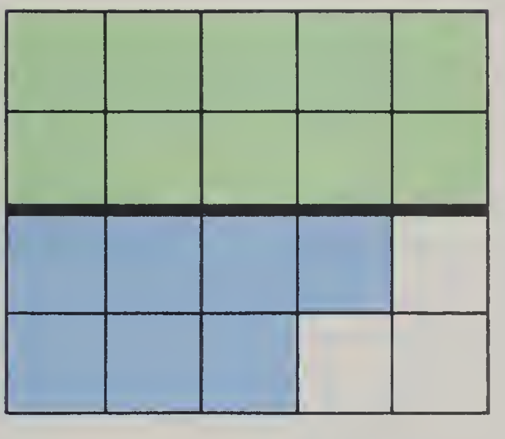
16



8 8

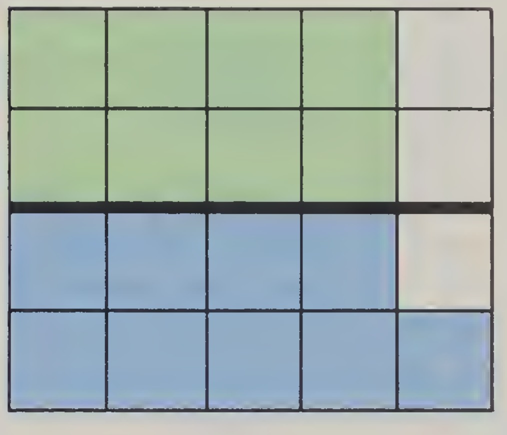
16





10 7

17



8 9

17

10 4

14

9 3

12

4 8

12

Notes on Pupil Page 34

Tasks: *Pupils fill in the boxes to tell how many of each kind and how many in all.*

Purpose: To show the usefulness of "building a ten" in finding sums greater than 9.

Unifying Ideas: Addition and subtraction; numeration; structure.

Materials for Teacher: Demonstration rods or strips.

Materials for Pupils: Individual rods or strips, or square containers which can be put side by side to look like the drawings on this page.

The Lesson:

Have the pupils turn directly to page 34 and use their materials to help them do the work. Some of them may need to be shown how to get started, and you should keep your demonstration materials handy for use with any pupils who need further help.

When the pupils have finished their work on the page, work with the class as a whole in summarizing what has been shown. (The associative principle for addition has been applied, but these words need not be used with your pupils.) As you write the following number sentences on the board, have pupils point out how the drawings in their book illustrate what you are writing. Write each line as you discuss each exercise in turn.

A. 10 + 5 = 15

B. 15 + 5 = (10 + 5) + 5 = 10 + (5 + 5) = 10 + 10 = 20

C. 6 + 7 = (3 + 3) + 7 = 3 + (3 + 7) = 3 + 10 = 13

D. 9 + 8 = 9 + (1 + 7) = (9 + 1) + 7 = 10 + 7 = 17

E. 8 + 6 = (4 + 4) + 6 = 4 + (4 + 6) = 4 + 10 = 14

F. 9 + 10 = 19

G. 9 + 9 = 9 + (1 + 8) = (9 + 1) + 8 = 10 + 8 = 18

Have the pupils refer to page 34 for this next set of counting and matching activities.

In exercises A-G, have pupils estimate how many more squares they would need to make two complete rows of 10. Write each line as you discuss each exercise in turn.

A. 10 + 5 + ____ = 20
B. 15 + 5 + ____ = 20
C. 6 + 7 + ____ = 20
D. 9 + 8 + ____ = 20
E. 8 + 6 + ____ = 20
F. 9 + 10 + ____ = 20
G. 9 + 9 + ____ = 20

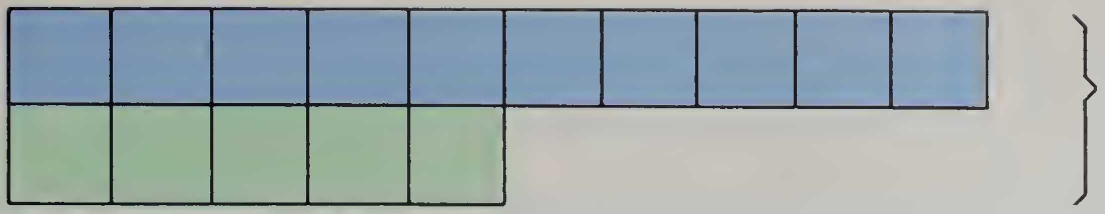
Leave the completed sentences on the board. Write the following sentences nearby:

19 + ____ = 20
13 + ____ = 20
15 + ____ = 20
18 + ____ = 20
17 + ____ = 20
20 + ____ = 20
14 + ____ = 20

Have pupils come to the board and draw a line from a sentence in the first list to the one which best matches it in the second list.

Headline Story: Mary had a dime, a nickel, and 2 pennies. She had ___¢ altogether.

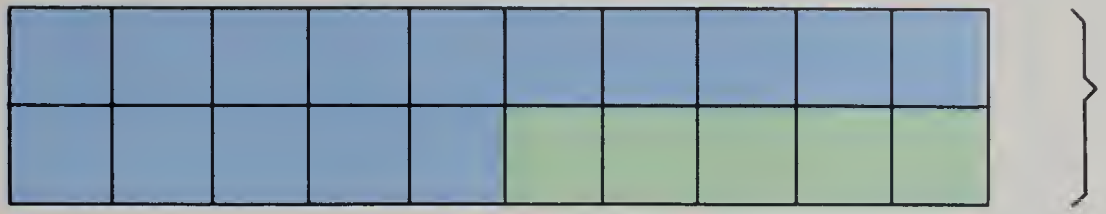
A



10 5

15

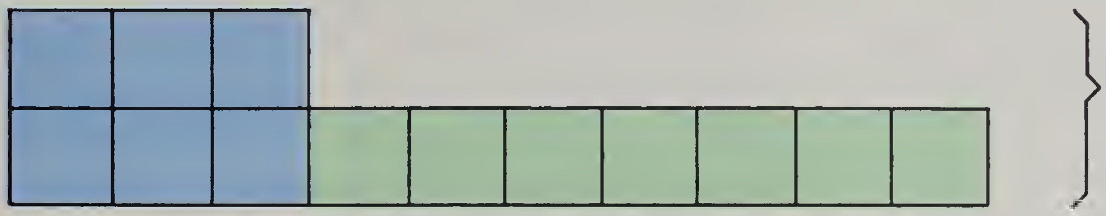
B



15 5

20

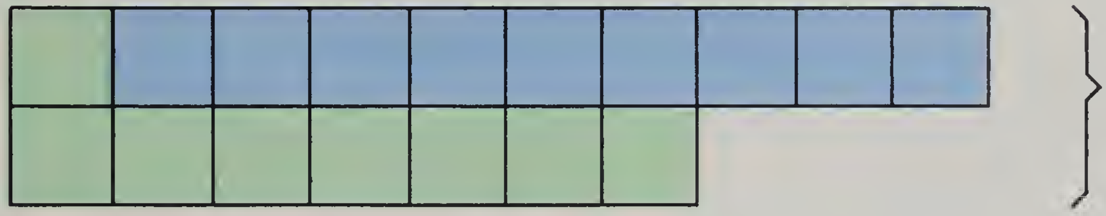
C



6 7

13

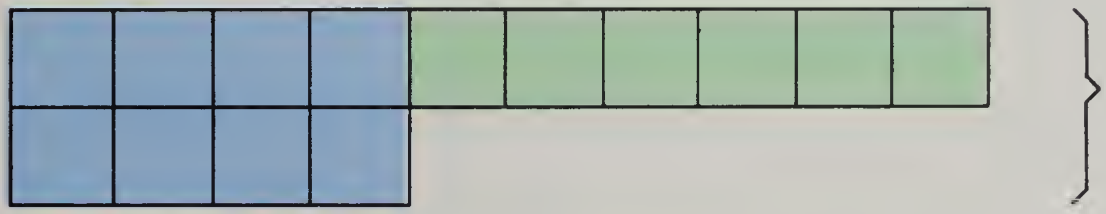
D



9 8

17

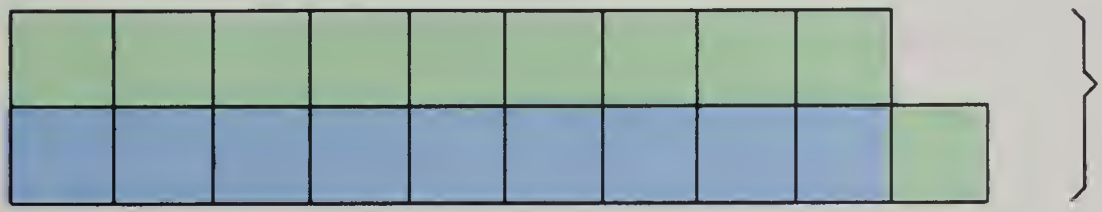
E



8 6

14

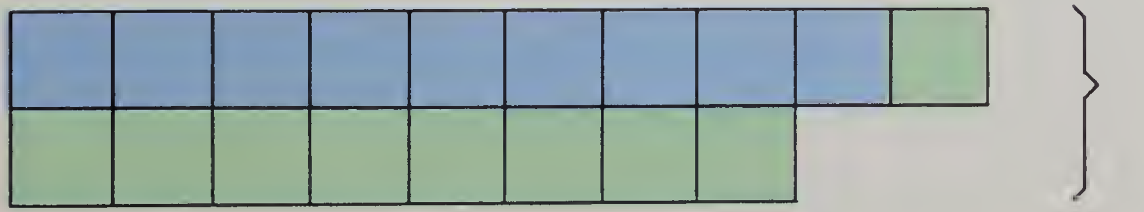
F



9 10

19

G



9 9

18

Notes on Pupil Page 35

Tasks: *Pupils draw arrows from starting points to landing points, as indicated, circle each landing point number, and write the number inside the white ring. They write labels for points on the number line where labels are missing.*

Purpose: (a) To familiarize the pupils with the use of purposeful jumps on the number line; (b) to reinforce the ideas of "the next number" and "adding 1."

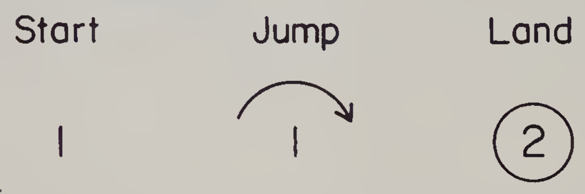
Unifying Ideas: Functions and relations.

Materials for Teacher: A number line on the floor with points labeled beginning with 0; a similar number line on the chalkboard.

Materials for Pupils: No special materials needed.

The Lesson:

Have a pupil come and stand at the point labeled 1 on the number line on the floor. Tell him to jump forward one space. Ask where he landed. Have another pupil picture this action on the number line on the board by drawing a curved arrow with its tail on 1 and its head on 2. Have him circle the number which labels the landing place. Summarize this yourself by writing and explaining how to read the words:



Do several similar examples using jumps of 1 and 2 until pupils understand the idea.

When pupils have finished page 35, find out how far they can extend the activity. The character of your class will determine whether you can have them "think" the number line up to 100 or above or whether you will have to use a model of it. Ask questions such as: If we start at 13 and jump (forward) 1, where do we land? If we start at 31? 27? 19? 44? 59? 70? 89? 98? etc.

Then see if they can do combinations of jumps. Start at 0, jump 2, and 1. Start at 3, jump 1, then 1 more. Start at 8, jump 2, then 1, etc. In each case, be sure to ask, "Where do we land?"

When you check this page in the pupils' books, count as complete and correct a page in which all the white rings are filled in correctly regardless of whether or not arrows and other numbers have been written. Pupils who successfully replace actual actions upon their models by mental ones should be encouraged to function at this more efficient level of activity.

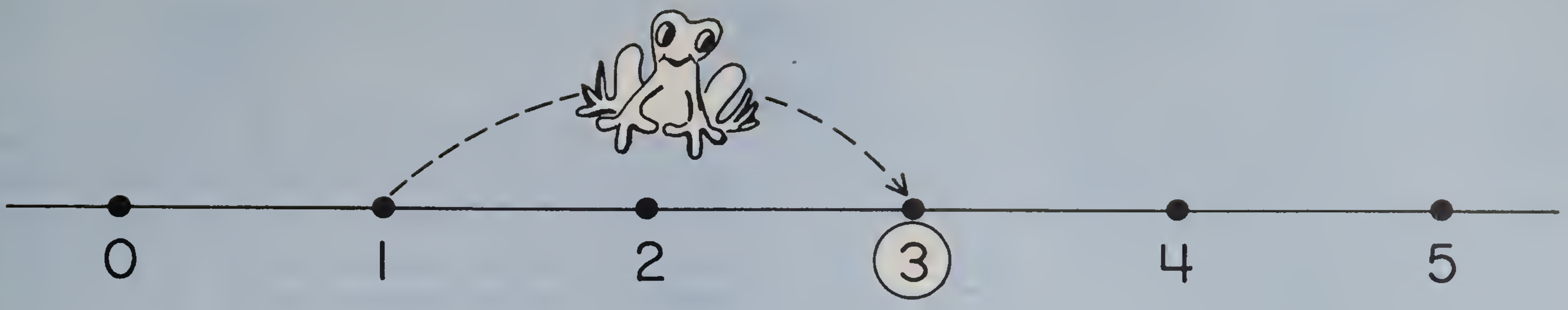
Some pupils will be interested in extending patterns in such tables as follows:

start	jump	land
0	2	2
2	2	4
4	2	
etc.		

or in recording random instances of such jumps:

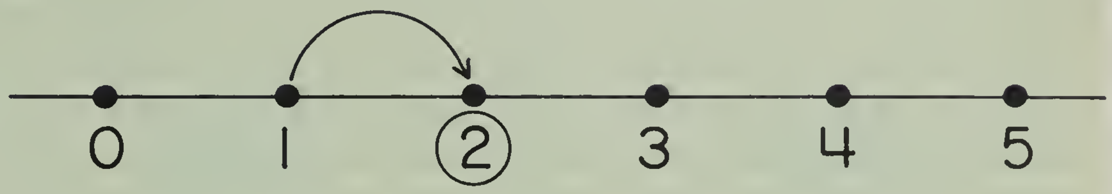
start	jump	land
10	5	15
22	7	29
89	4	
etc.		

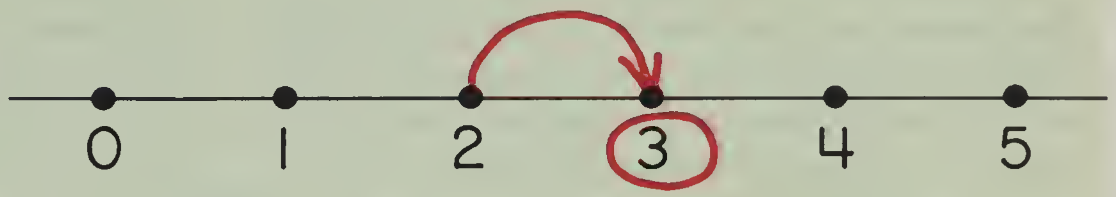
Headline Story: Harry had one more book than Ruth. (Pupils extend story.)

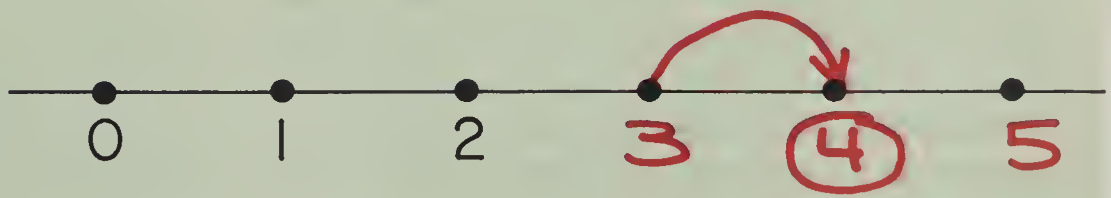


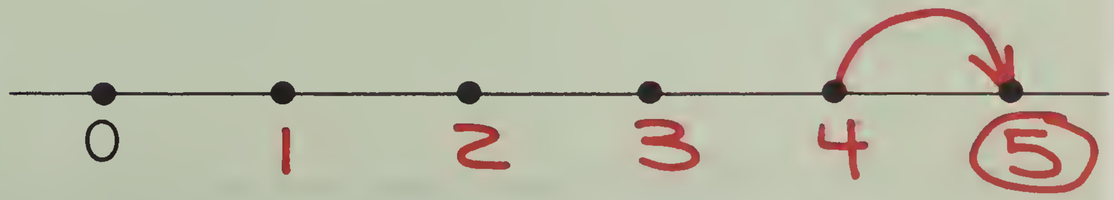
start	jump		land
1	2	goes to	3

start	jump		land
1	1	→	2
2	1	→	3
3	1	→	4
4	1	→	5









Notes on Pupil Page 36

Tasks: *Pupils supply missing arrows, or parts of arrows, to show indicated jumps on the number line, and fill in the missing numbers in the mathematical sentences.*

Purpose: To show an additive situation in terms of successive jumps on the number line.

Unifying Ideas: Functions and relations; addition and subtraction; structure.

Materials for Teacher: A demonstration number line.

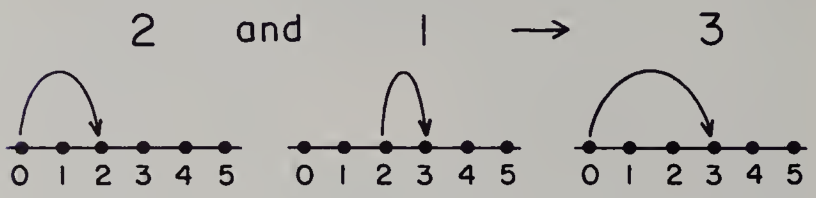
Materials for Pupils: No special materials needed.

The Lesson:

This lesson is an extension of the lesson on the preceding page. In this lesson, the pupils are finding the size of a single jump which would have the same effect as two specified jumps, always starting at zero. They should engage in some large muscle activity which can be pictured by drawings like those on page 36.

On the chalkboard, draw sections of number lines, three in a row, like those on page 36. Use them for keeping records. Use a number line on the floor, a number line on the chalkrail, or both, in directing activities like those described below. In using the chalkrail number line in this context, it is useful to have a toy animal of some kind to do the jumping, propelled by the pupils, of course.

Have a pupil come up to the floor number line and stand on 0. Have him jump two spaces. Ask another pupil how to draw a picture on the board of the jump which has just been taken. Have the first pupil jump one more space. How will we picture this? Where is he now? How could he have gotten to the same place with just one jump? How can we picture that? Now, write a shorthand record of what has been done. By this time, you should have the following entries. "2 and 1 ⟶ 3" is read, "Two and one goes to three."



Repeat the same activity using different combinations of jumps. Include at least one example where the size of one jump is 0. (The pupil, or the toy animal, jumps up and lands in the same spot. The arrow which pictures this is a loop.)

Now, do some examples in which essentially this activity is reversed. On the board, above one of your rows of number lines, write: 2 and 2 ⟶ ☐ . Ask a pupil to do the jumps on the number line that this story describes. How should the box be filled in? How should we draw arrows with the number lines to picture all of this?

When the pupils have finished their work on the page, ask if they notice anything interesting. Some will observe that in each row, each pair of jumps has the same result as a single jump of 5, and that the order of two jumps doesn't change the result; e.g., a jump of 2 followed by a jump of 3 has the same result as a jump of 3 followed by a jump of 2.

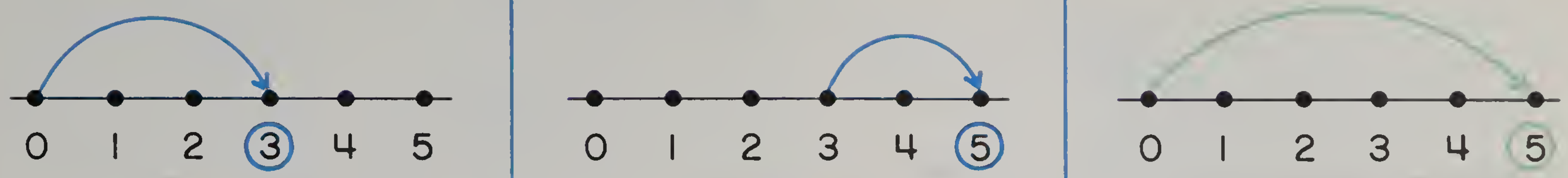
Further questions some pupils will wish to explore include:

How many other pairs of jumps land on 5 if we start at 0?

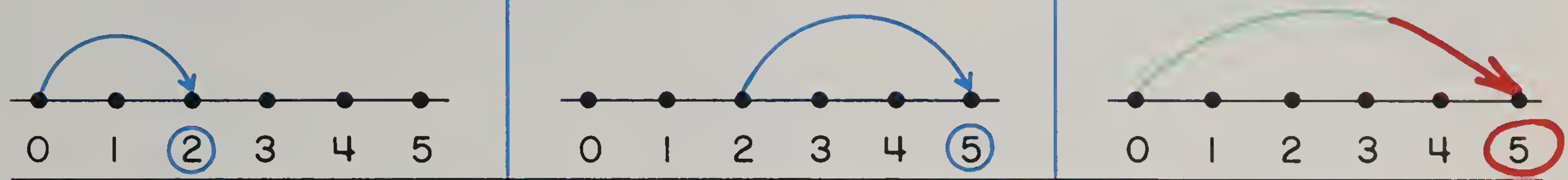
What triple jumps are possible if you start at 0 and end at 5 (or any number)? Does it matter in which order we make these jumps?

Headline Story: Bill lived 2 blocks farther from school than Betty. (Pupils extend story.)

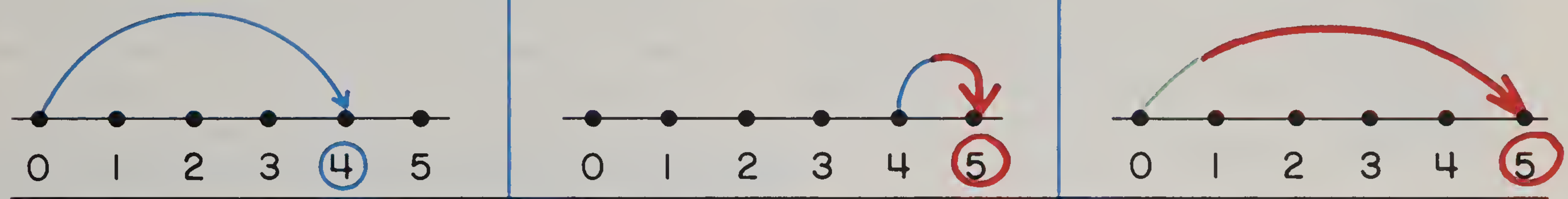
3 and 2 → 5


0 1 2 3 4 5
0 1 2 3 4 5
0 1 2 3 4 5

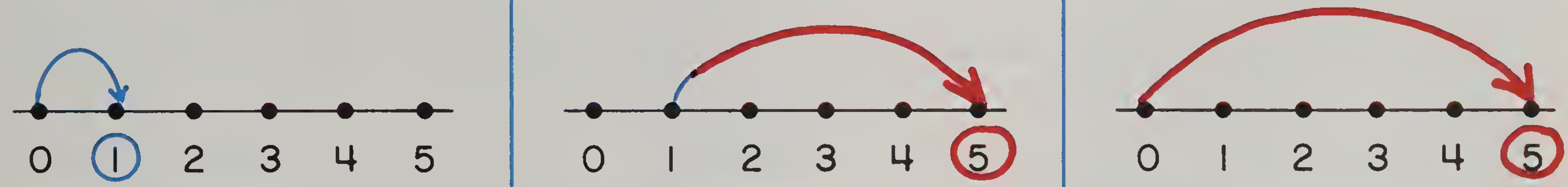
2 and 3 → 5


0 1 2 3 4 5
0 1 2 3 4 5
0 1 2 3 4 5

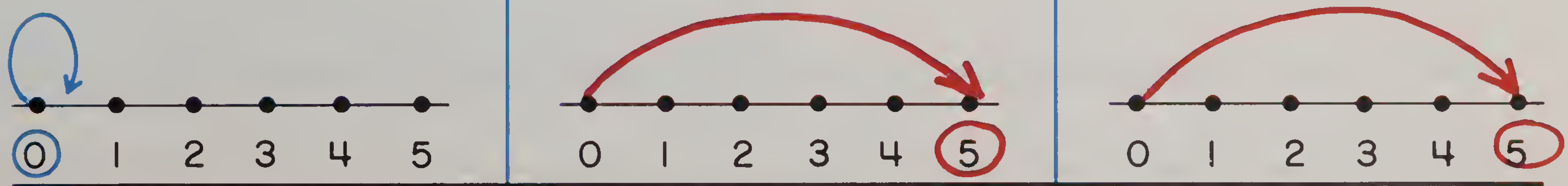
4 and 1 → 5


0 1 2 3 4 5
0 1 2 3 4 5
0 1 2 3 4 5

1 and 4 → 5


0 1 2 3 4 5
0 1 2 3 4 5
0 1 2 3 4 5

0 and 5 → 5


0 1 2 3 4 5
0 1 2 3 4 5
0 1 2 3 4 5

Notes on Pupil Page 37

Tasks: *Pupils fill in the missing numerals in the Cross Number Puzzles at the top of the page. They complete the record which has been started in the lower half of the page and draw pictures to fit the record in the squares at the lower left.*

Purpose: (a) To reinforce addition concepts with sums to 10; (b) to provide further experiences with the "in-any-order" principle for addition.

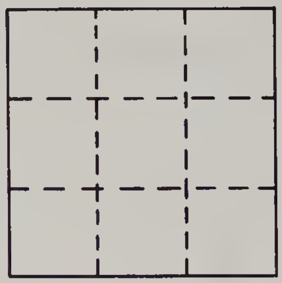
Unifying Ideas: Addition and subtraction; structure.

Materials for Teacher: Something to use as a demonstration Cross Number Puzzle—a magnetic board, a flannel board, the kind of large sketch pad used in art classes, a drawing on the chalkboard.

Materials for Pupils: A sheet of paper; 10 counters.

The Lesson:

Show the pupils how to fold a piece of paper twice each way to make 9 sections. Then have them open the paper and flatten it out on their desks.



Put the following drawing on the board and ask the pupils to arrange counters in corresponding boxes on their papers to go with this record. Use your demonstration puzzle if they need help in getting the idea.

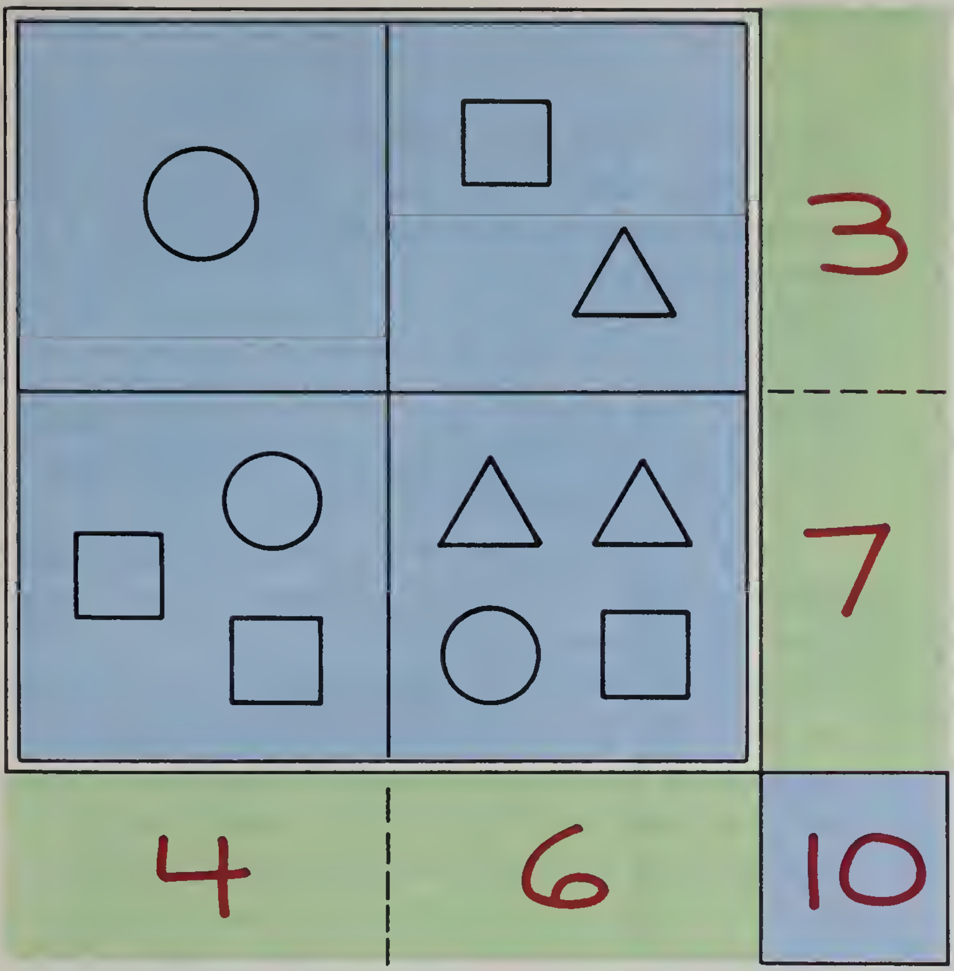
4	1	
2	3	

With suitable rearrangement, the first drawing on page 37 would be a picture of what they have on their desks now. Ask the pupils to sweep all the counters in the top row into the empty box in that row. Ask what you would write in your record to show this. (Write a 5 in that cell.) Do the same thing with the counters in the bottom row. Then sweep all of the counters down into the lower right-hand box. Make an appropriate entry in the record. Have the pupils copy on their papers what you now have on the board. Be sure they write the seven numbers in the proper boxes. Then have them put their counters back the way they were at first. (The record has served as a memory of this.)

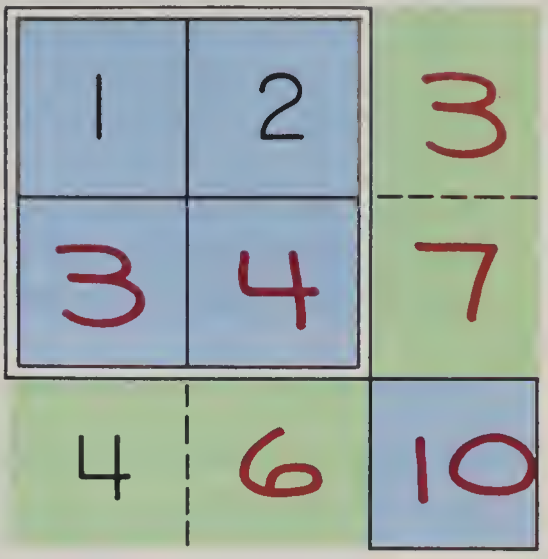
Now have them sweep all the counters in the left-hand column down into the lower box. Record. Do the same thing with the counters in the next column. Then sweep all the counters into the lower right-hand corner. Have pupils noticed that the record already tells how many objects there are in this box?

Explore the Cross Number Puzzle by having pupils keep records of various arrangements, and by having them draw or make models from partial or complete records.

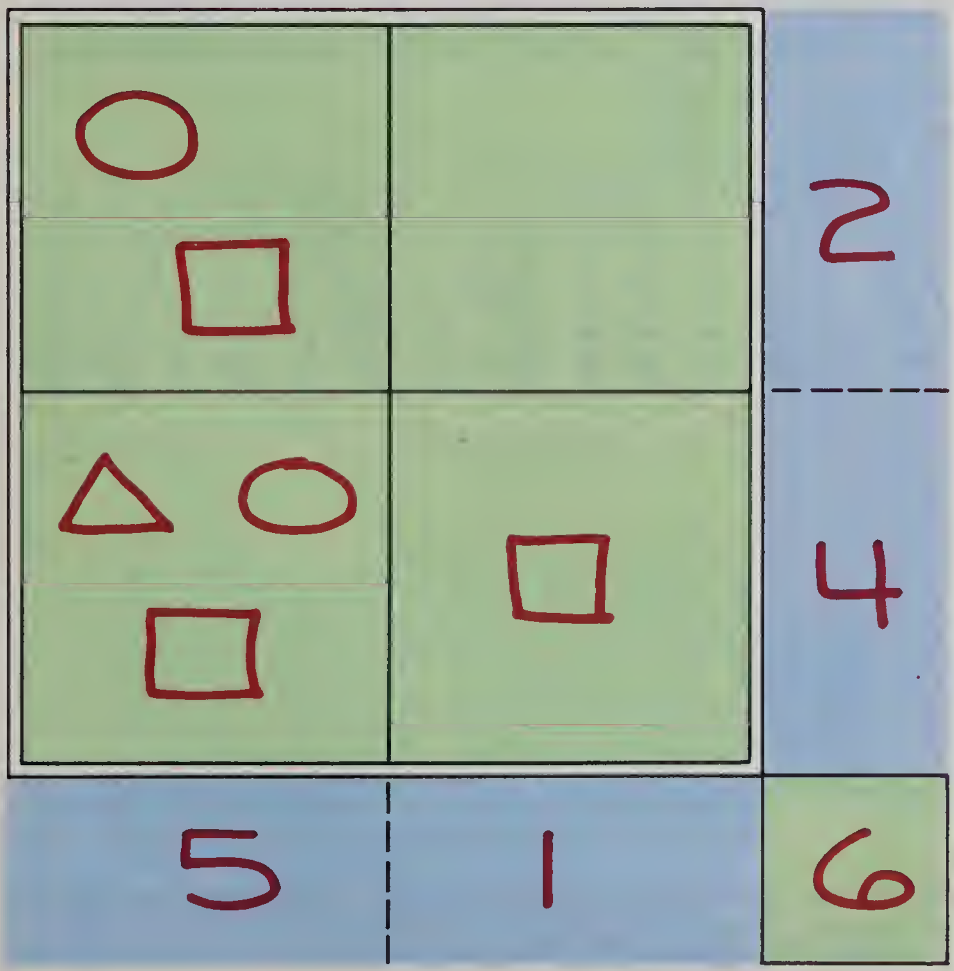
Headline Story: Carl had 10 marbles, some in each of 4 pockets. (Act out one version.)



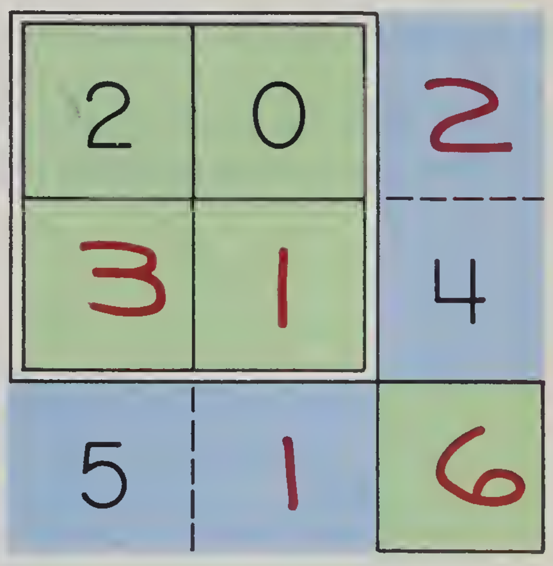
→



1	2	3
3	4	7
4	6	10



→



2	0	2
3	1	4
5	1	6

Tasks: *Pupils draw lines connecting points on the number line with objects in a one-to-one matching of the members of the sets, circle the numbers which tell how many in all, and fill in the boxes with numbers which summarize the activity.*

Purpose: (a) To reinforce the concept of counting; (b) to suggest the association of a number pair with a single number, their sum.

Unifying Ideas: Sets; addition.

Materials for Teacher: A number line on a table top or the chalk rail.

Materials for Pupils: Ten counters.

The Lesson:

Ask each pupil to hold two counters in his left hand and four in his right hand. Have one pupil come up to your number line and lay one counter from his left hand on the point labeled 1 and the other on 2. Have the other pupils lay their two counters side by side on their desks. Write a 2 followed by a comma on the chalkboard as a record of the situation so far.

2,

Ask the pupil how many counters he has in his right hand. Write a 4 followed by an arrow after the comma.

2, 4 ⟶

Then have the pupil lay those counters, one at a time, on successive points on the number line. Have the other pupils lay their 4 counters beside the 2 on their desks. Ask what number goes with the point where the last counter is now. Write a 6 after the arrow.

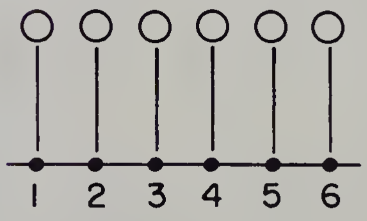
2, 4 ⟶ 6

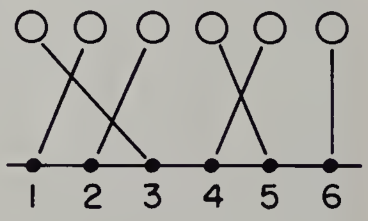
Have all the pupils count all the counters they have lined up now, touching each of the counters in turn as they say the number names.

Using different pairs of numbers and different pupils, repeat this exercise as many times as appears to be necessary.

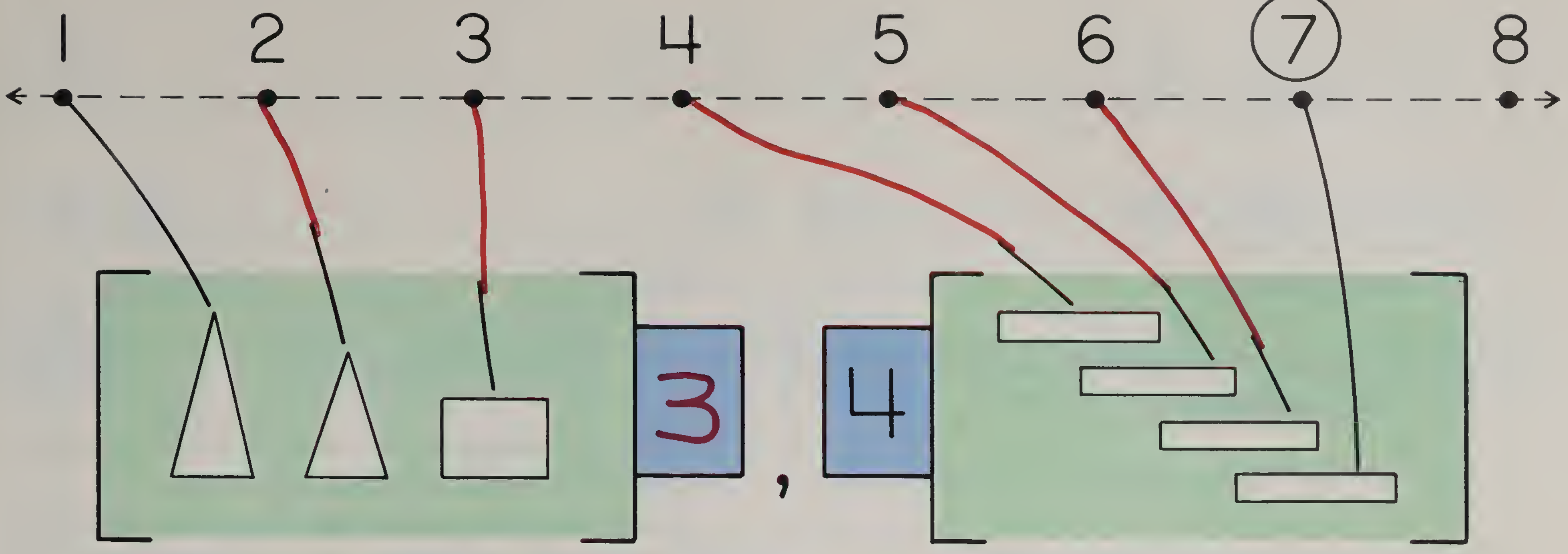
Now, use a chalk rail number line or one drawn on the board just above it and draw pictures of members of two sets of objects on the board. Ask the class what they could do to match these objects with the numbered points on the number line. If no one suggests drawing lines to show the matchings, suggest it yourself. Have a pupil show how to do this. Keep a record, as you did before.

Be sure everyone realizes that objects can be counted in any order, but numbers have to be used in one and only one way when we count. One way to do this is to use colored chalk to show matchings different from those a pupil has indicated. This will help pupils to see that the number of a given set is the same, regardless of the order in which the objects were matched to points on the number line.

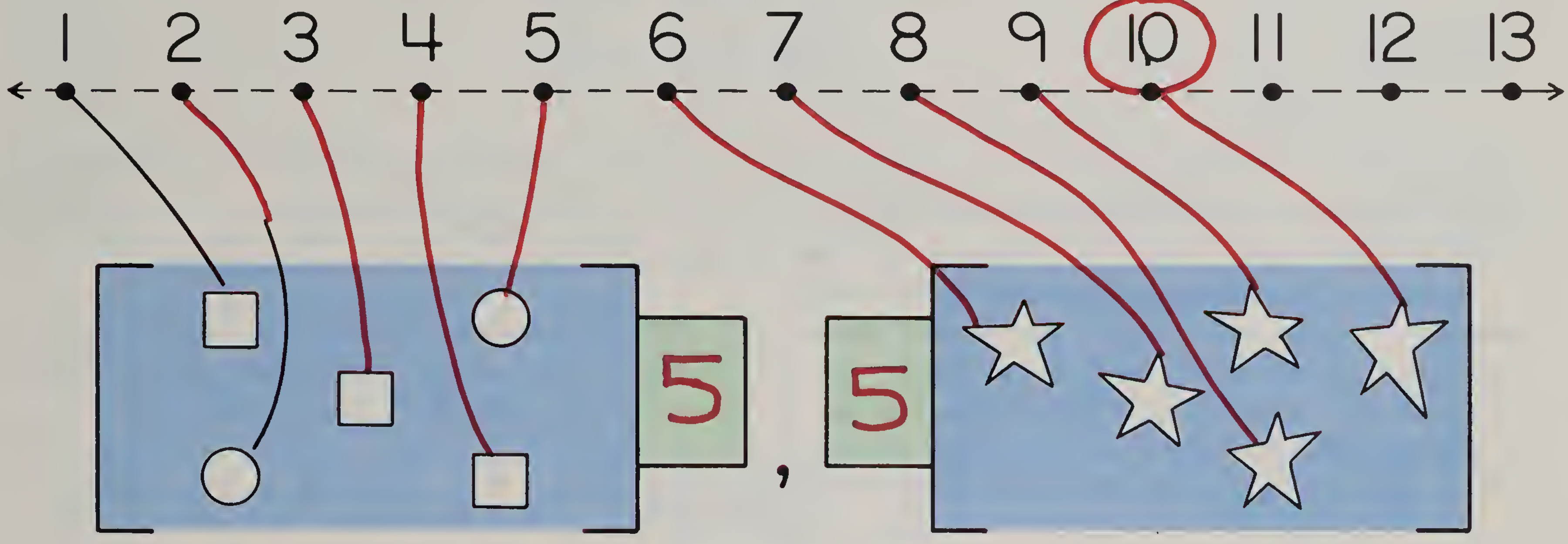




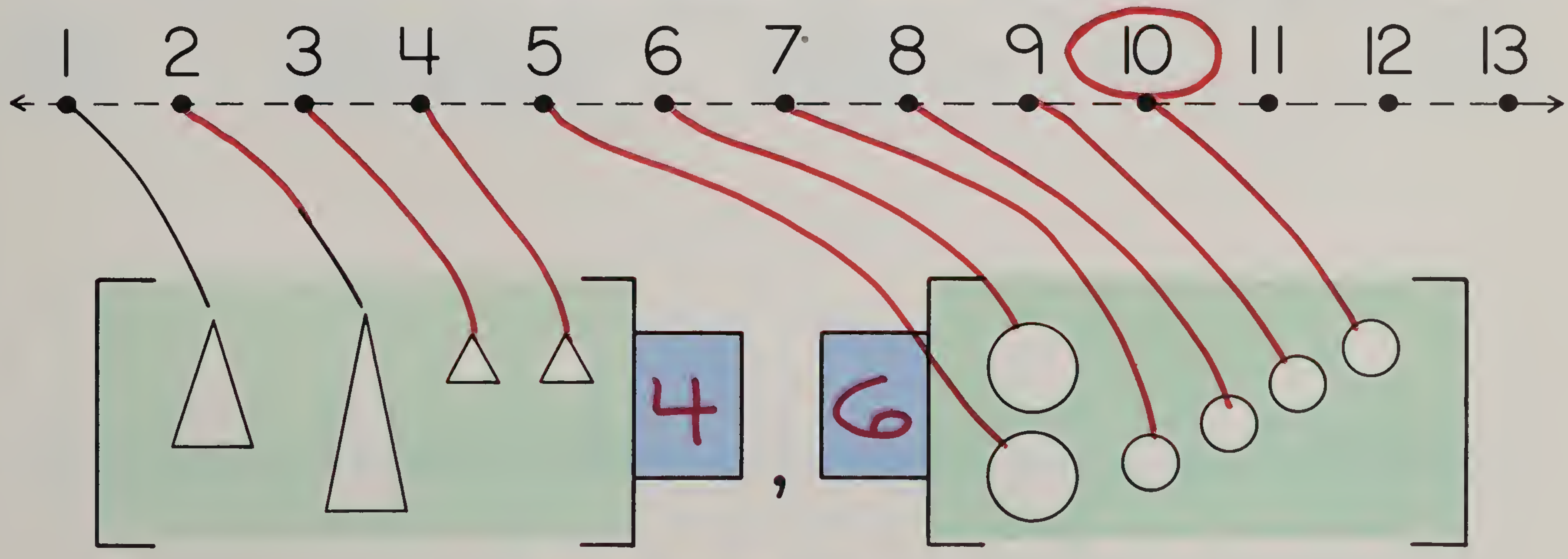
Headline Story: There were 3 large and 5 small chairs. How many children could be seated, one to a chair? . . . If large chairs hold 2 children and small chairs hold 1?


1
2
3
4
5
6
7
8
3
4

3, 4 → 7


1
2
3
4
5
6
7
8
9
10
11
12
13
5
5

5, 5 → 10


1
2
3
4
5
6
7
8
9
10
11
12
13
4
6

4, 6 → 10

Notes on Pupil Page 39

Tasks: *Pupils fill in the number tags to make true sentences. In the last row, they have to reconstruct the picture of one of the sets.*

Purpose: To show the analogy between working with sets and the addition of numbers.

Unifying Ideas: Sets; addition and subtraction.

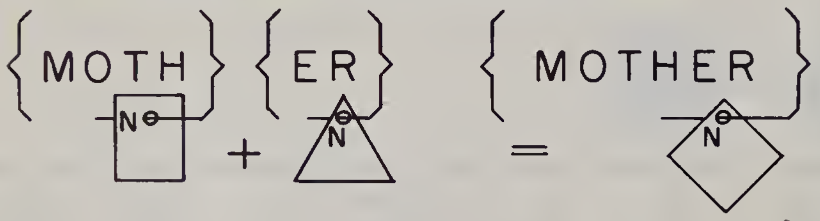
Materials for Teacher: Colored chalk.

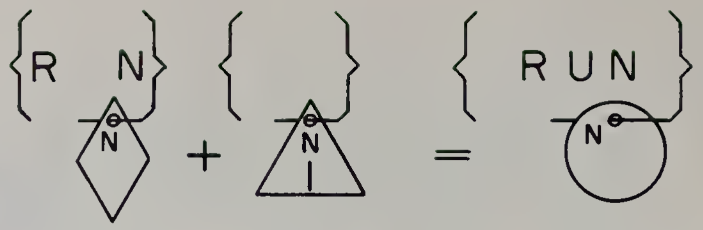
Materials for Pupils: Two kinds, or two colors, of counters.

The Lesson:

Have the pupils arrange counters in two groups on their desks to match the pictures they see in the first two sections on the top row of page 39. Then ask what the pupils could do with the counters on their desks to make an arrangement just like that in the picture on the right. They should see that moving either group of counters beside the other group would result in a "perfect fit." Have a pupil read the resulting number sentence.

Discuss such drawings as the following, if necessary, before assigning page 39 for individual study.





When all have finished, discuss with the pupils what they have learned from this lesson.

Then have the pupils copy on ruled paper the following exercises which you have written on the board. Tell them they may use their counters (or their rods, or their number lines) to help them find the missing numbers, if necessary.

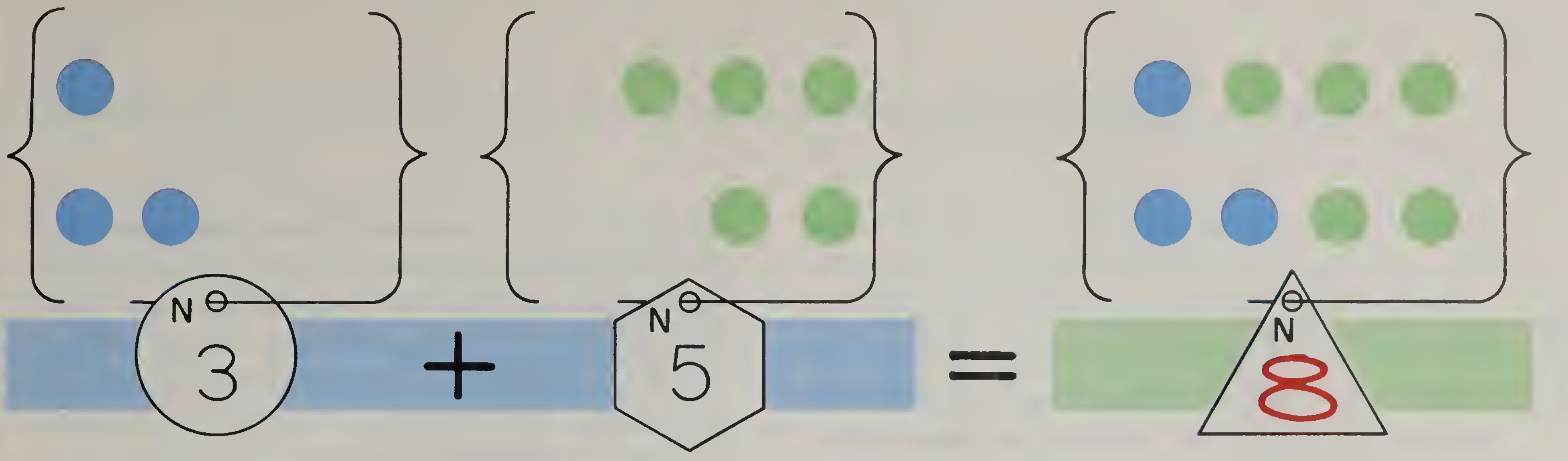
1+1=___ 3+1=___ 2+3=___ 0+5=___
2+1=___ 0+2=___ 4+1=___ 4+2=___
1+2=___ 4+0=___ 3+2=___ 5+1=___
3+0=___ 1+3=___ 1+4=___ 2+4=___

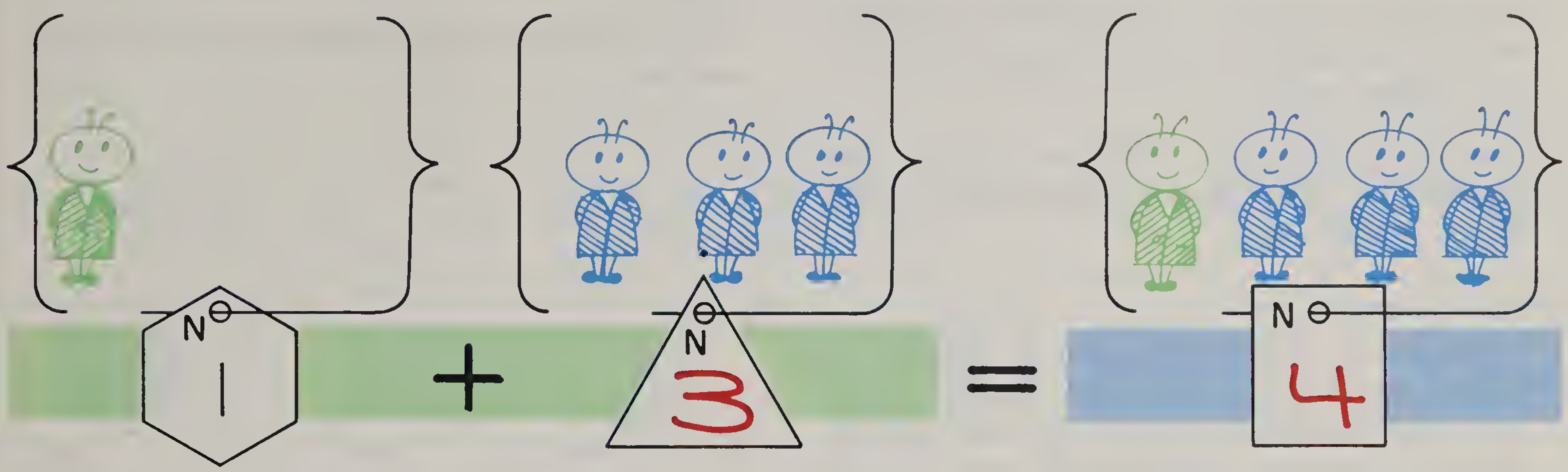
Completing such statements as the following by filling in the relation signs offers an enrichment experience for any pupil who wishes to try.

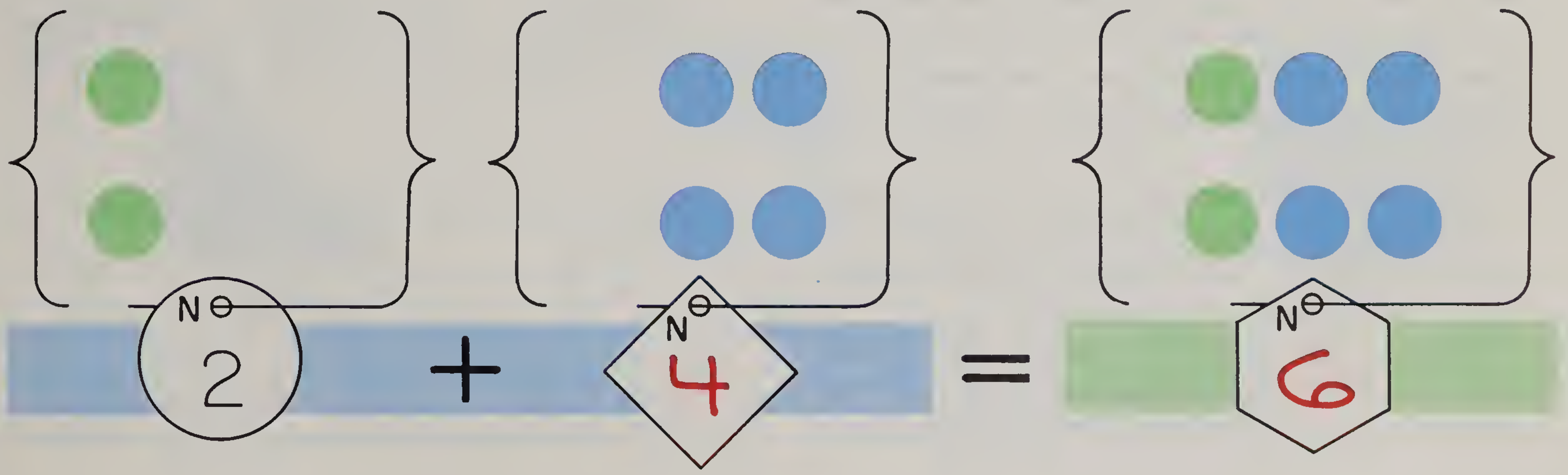
5 ___ 3 ___ 8
8 ___ 3 ___ 5
3 ___ 5 ___ 8
5 ___ 3 ___ 2

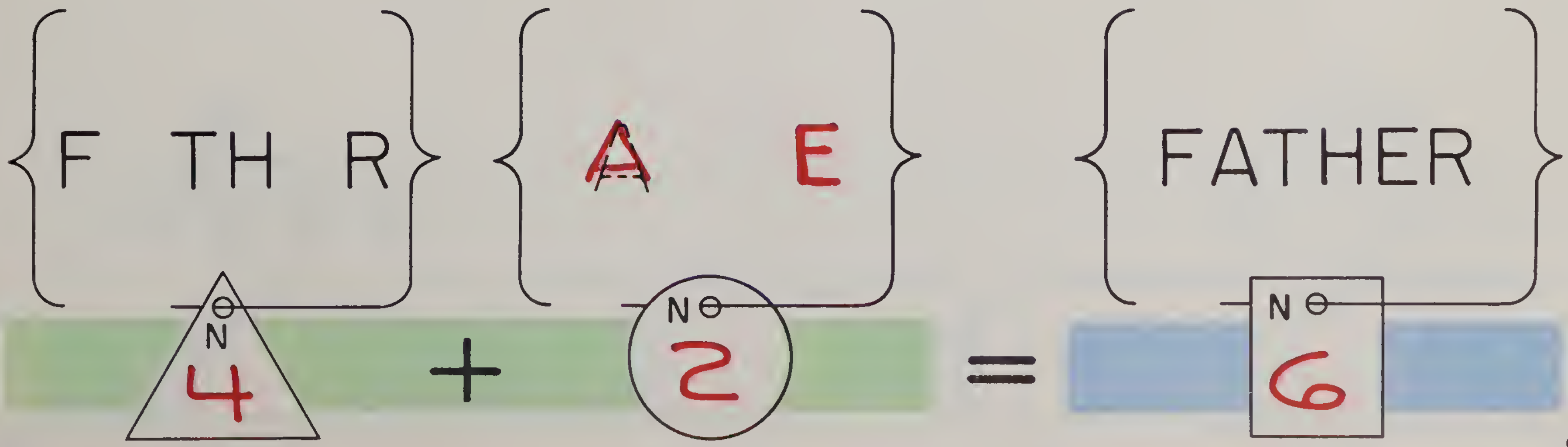
5 + 3 = 8
8 = 3 + 5

Headline Story: Bill spent 2 cents. If he had ___¢, he has ___¢ left. (Make a chart.)


N 3
+
N 5
=
N 8


N 1
+
N 3
=
N 4


N 2
+
N 4
=
N 6


{F TH R}
+
{A E}
=
{FATHER}
N 4
N 2
N 6

Notes on Pupil Page 40

Tasks: *Pupils examine each situation carefully and then complete each number sentence by writing in the tags the number or numbers which make each sentence true.*

Purpose: To further extend the pupils' understanding of addition and subtraction as operations performed on the cardinal numbers of sets.

Unifying Ideas: Sets; addition and subtraction.

Materials for Teacher: Demonstration size alphabet letters, square cutouts, triangular and diamond-shaped cutouts.

Materials for Pupils: Alphabet letters, desk number line, desk counters, green and blue square shapes, number tags.

The Lesson:

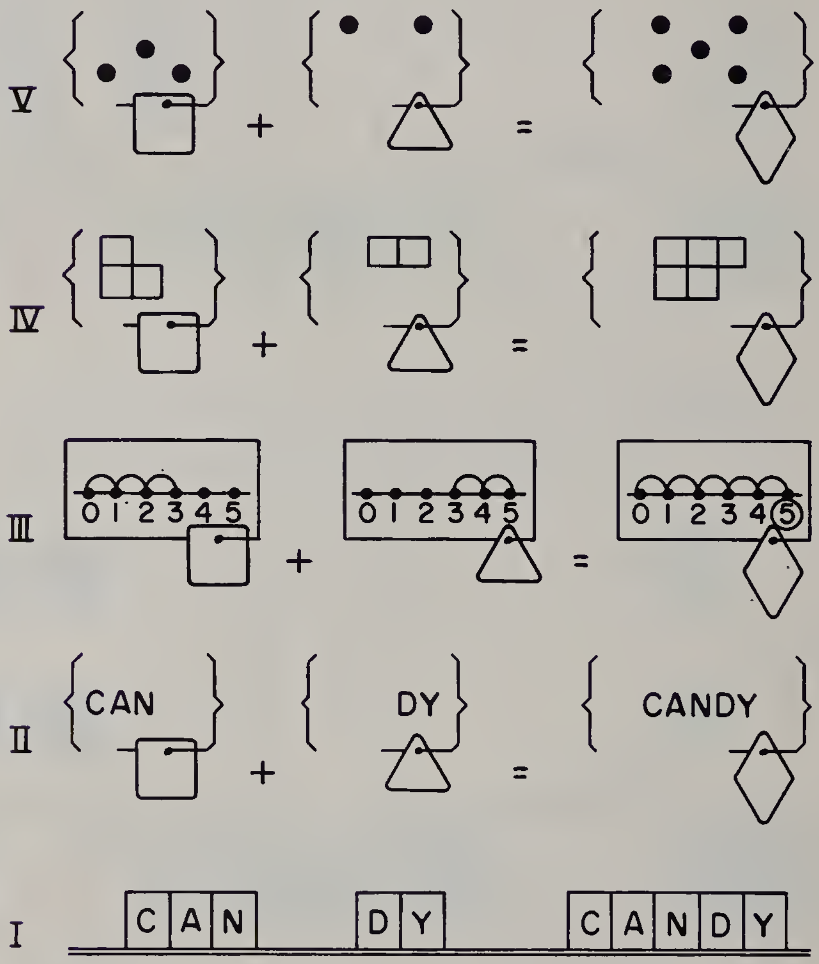
This lesson covers several days of classroom activities. Consider displaying the diagram shown in the next column on the chalkboard and chalkrail. Invite the class to examine each row carefully. Suggest that they start from the row of letters on the chalkrail and then read up the chalkboard row by row. Encourage volunteers to tell the class what is different about the two bottom rows. The same? Ask pupils to tell you what needs to be written in the second row of tags to make a true number sentence.

After the class completes the first number sentence, proceed to the next three rows with the pupils working at their desks using their number lines, square shapes, and counters.

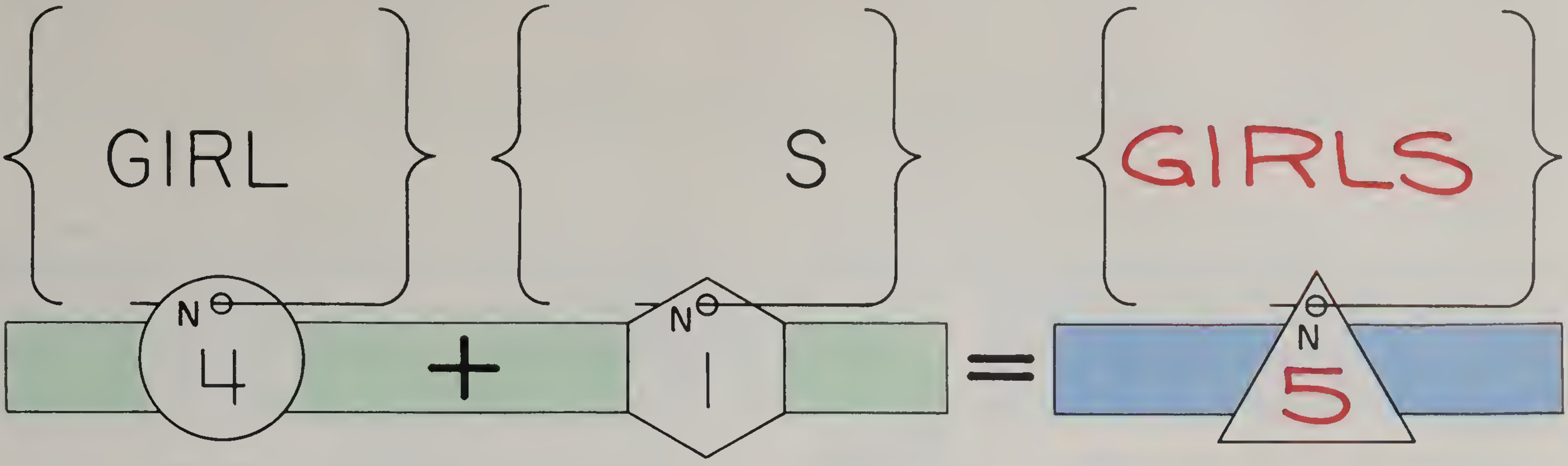
Now play the "missing sum game." Take a blank oak tag sheet (diamond-shaped) and cover at random the sum in any row. Have pupils read each complete number sentence. Follow this game with the "missing addend game." Have a square and triangular-shaped blank oak tag sheet in each hand. At random, cover the second addend in any row and ask volunteers to read the complete sentence to the class. Now randomly cover any first addend and ask for complete sentences.

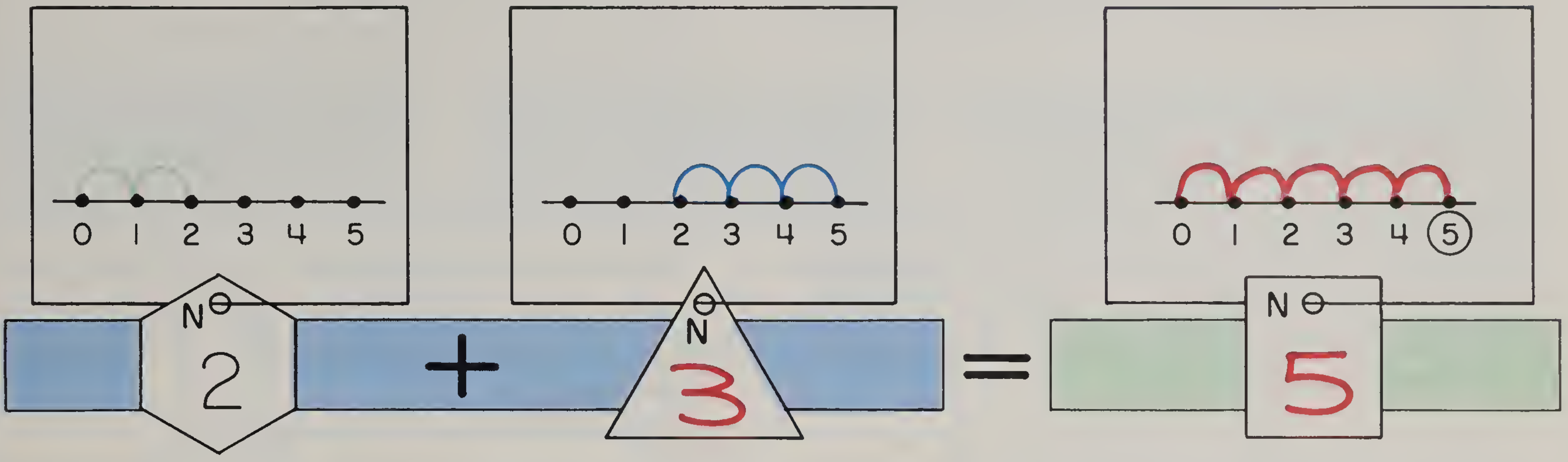
Finally, you might consider covering any sum or any addend at random and asking for a complete sentence.

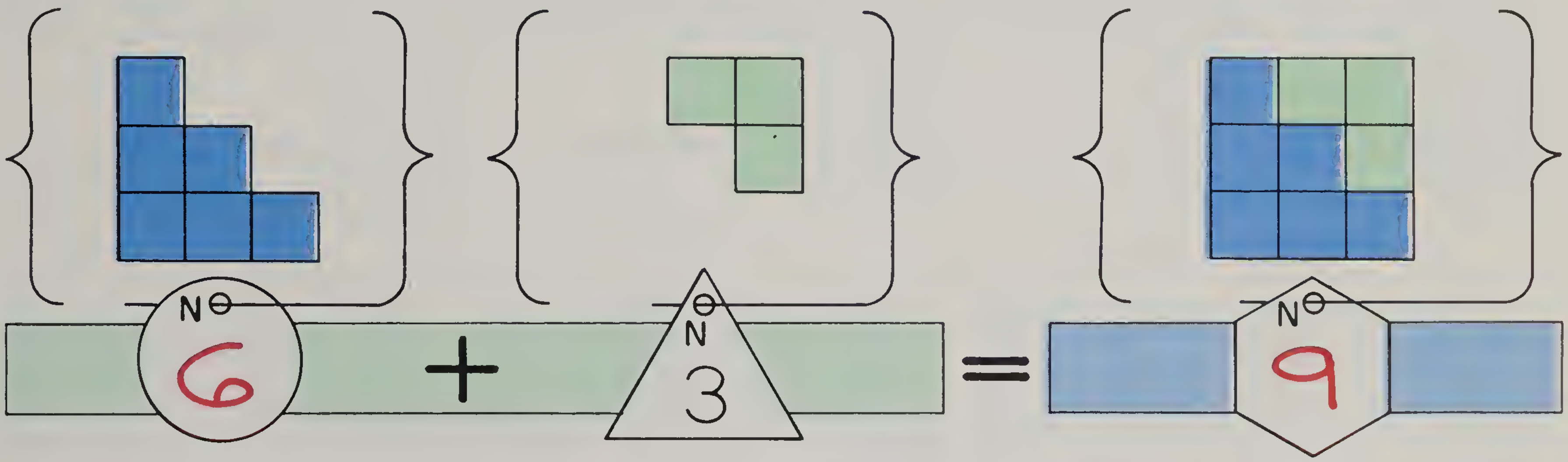
The class may now complete page 40 in their worktext.

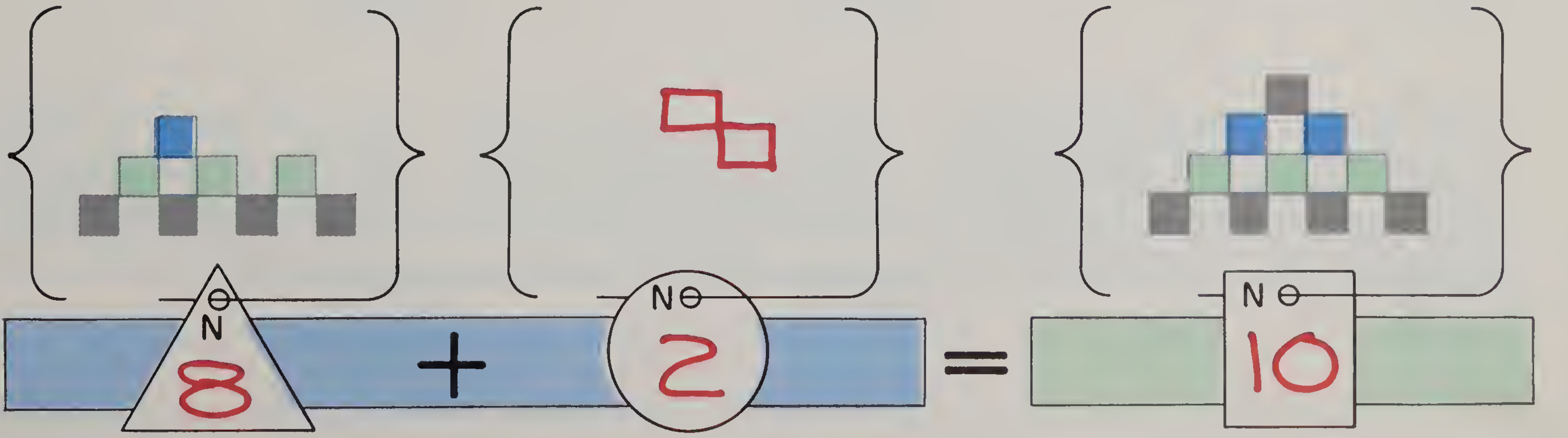


Headline Story: Tell a story to fit: 3 + 1 = □ 4 + △ = 5


GIRL
S
GIRLS
N⊖ 4
+
N⊖ 1
=
N⊖ 5


0 1 2 3 4 5
0 1 2 3 4 5
0 1 2 3 4 ⑤
N⊖ 2
+
N⊖ 3
=
N⊖ 5


N⊖ 6
+
N⊖ 3
=
N⊖ 9


N⊖ 8
+
N⊖ 2
=
N⊖ 10

Notes on Pupil Page 41

Tasks: *Pupils fill in labels for the number line at the top of the page and supply needed numbers and pictures.*

Purpose: To teach a standard written form of the addition facts with sum of 4.

Unifying Ideas: Sets; addition and subtraction; structure.

Materials for Teacher: A flannel board with objects and numbers to use on it for pupils who need help.

Materials for Pupils: Four counters for those who may need them.

The Lesson:

Discuss with pupils such questions as these:

1) If Jim and Dick were looking for four lost marbles and together found all four, how many did each find? A record will help:

Jim		Dick		
___	+	___	=	4
1	+	3	=	4
2	+	2	=	4

etc.

2) If I have four cents in my two pockets, how many do I have in each?

Left		Right		
___	+	___	=	4
3	+	1	=	4
0	+	4	=	4

etc.

Have the pupils turn to page 41 and complete labeling the number line. Let them discuss the goings-on in the first picture or row. Ask what they think they should do with the next row. Let those who are sure complete the page by themselves while you help the rest. For the third and last rows, the pictures the pupils draw do not necessarily have to match those in your answer copy. *Having the correct number of pictures in each section is what is required for a correct response.*

When all have finished, discuss the patterns on this page. Ask the pupils how many ways they used to get the sum 4. Are there other ways? (There are 5 ways. The answer to all such questions is 1 more than the sum if the addends are restricted to the cardinal numbers.)

Have the pupils look at the last two rows. How are they alike? How are they different? Have the pupils study the two rows above the last two rows in the same way.

Write on the board such examples as:

6 + 3 = 9 3 + 6 = ___

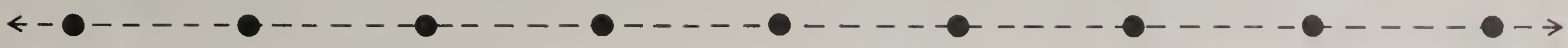
21 + 17 = 38 17 + 21 = ___

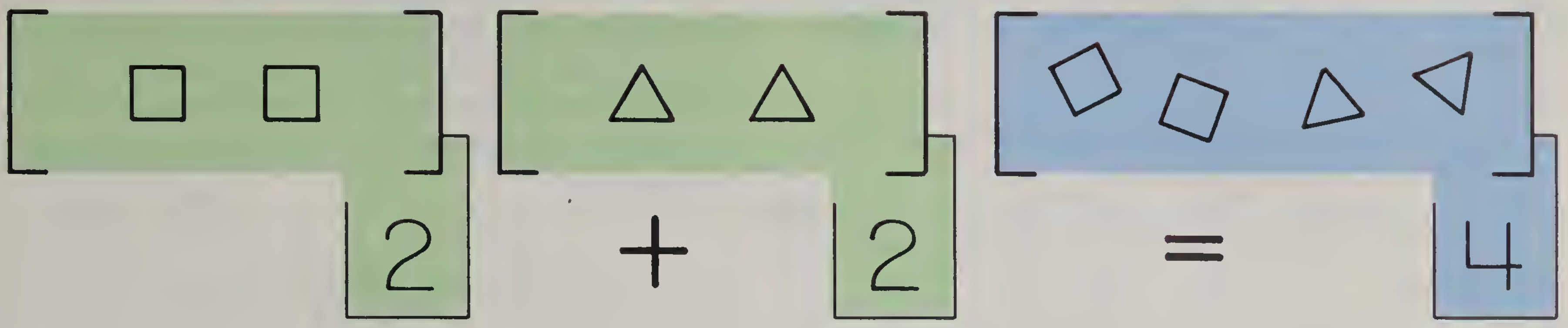
Can everyone fill in the blanks? By discovering that the order in which two numbers are added does not affect the sum, a pupil reduces by nearly one-half the number of addition facts he has to learn.

Be sure to review the accompanying booklet, *Games and Enrichment Activities*, for suggestions you may wish to incorporate in your daily lesson plan.

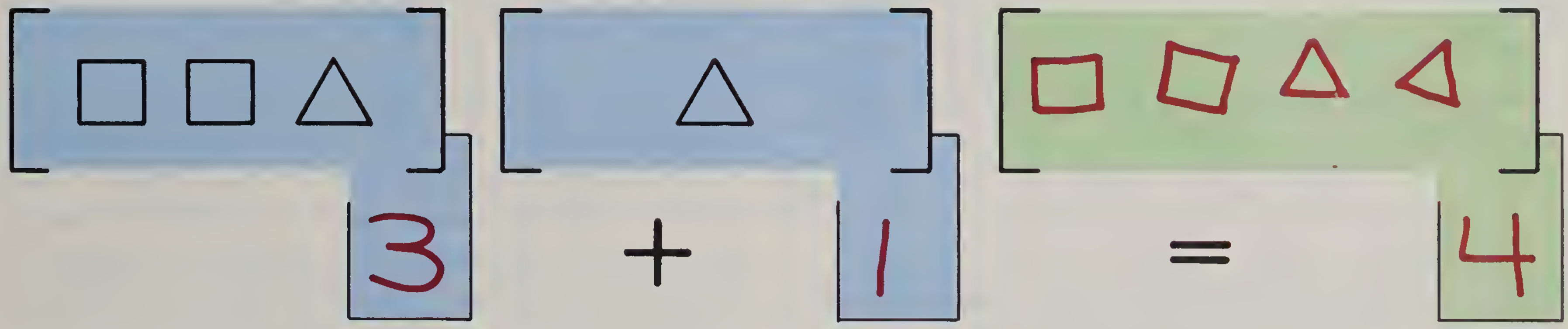
Headline Story: Together Sarah and Mary had 5 books. Mary had one more than Sarah.
Sarah had ______ books.

0 1 2 3 4 5 6 7 8

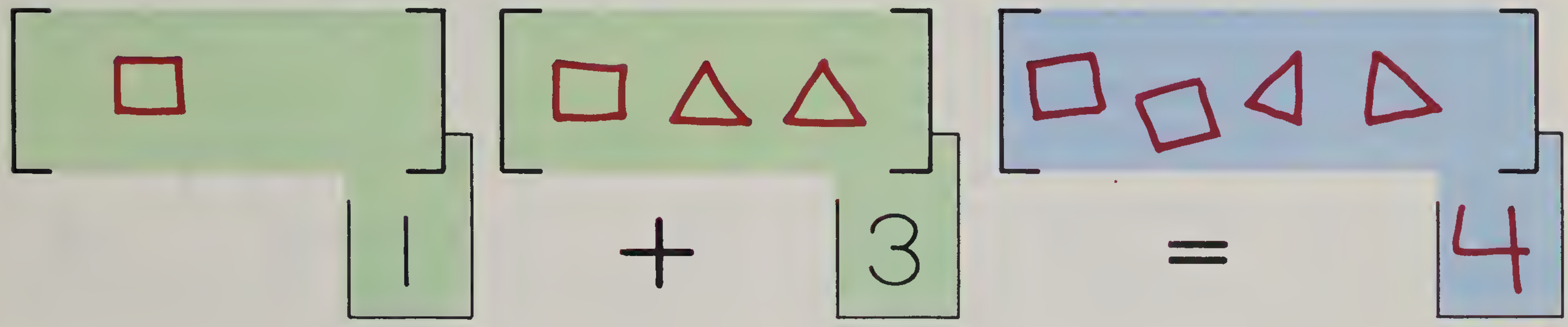




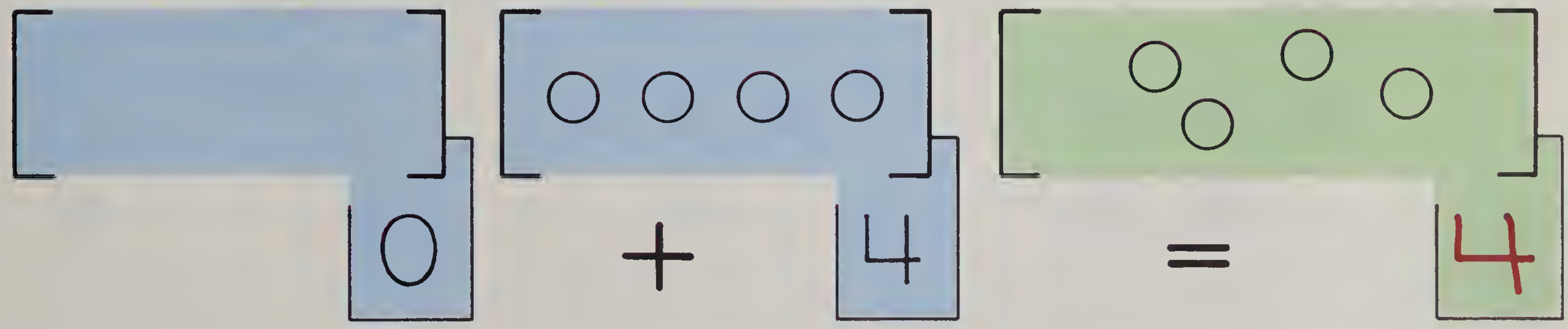
2 + 2 = 4



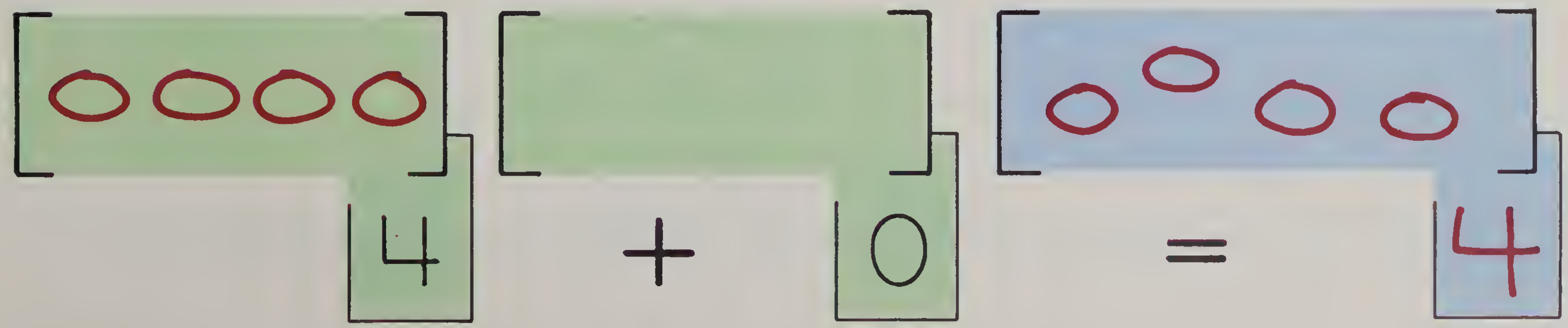
3 + 1 = 4



1 + 3 = 4



0 + 4 = 4



4 + 0 = 4

Notes on Pupil Page 42

Tasks: *Pupils fill in the number tags and draw pictures to represent the sums, which happen to be 5 in each case.*

Purpose: To teach a standard written form of the addition facts with sums of 5.

Unifying Ideas: Sets; addition and subtraction; structure.

Materials for Teacher: A number line on the chalkboard.

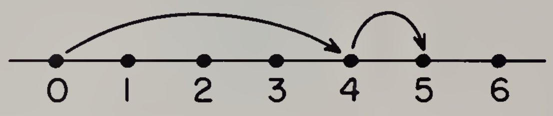
Materials for Pupils: A paper containing six lines with equally-spaced dots, to be used as number lines (see suggestion below).

The Lesson:

If the pupils have understood the lesson preceding this one, they should be able to do this page without help.

When all have finished the work on page 42, pass out copies of number lines. You can prepare number lines on a Ditto master. Draw six lines across a Ditto master, spaced about equally, including the top and bottom margins, down the length of the page. On each of these lines, you can put nine dots one inch apart or 17 dots one-half inch apart. For present purposes, the one-inch spaces may be better. Let the pupils do the numbering. They need practice in thinking about numbers in sequence and in writing numbers.

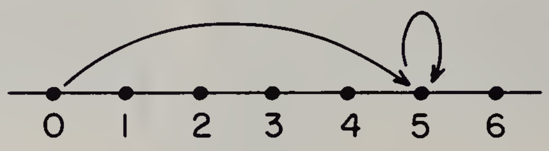
Instruct the pupils to label the first dot on each line 0, the next dot 1, and the rest in order. If your dots are one inch apart, the last number they write will be 8. Ask them to copy this picture of number line jumps on their papers, on the second number line.



4 and 1 → 5

$4 + 1 = 5$

Now, have your pupils look at page 42 in their books and see if they can find a row of pictures which tells the same story. (You have given them a hint; it is the second row.) Ask them to write the number sentence which goes with that row. Then see if anyone can suggest a picture on a number line to go with the next row ($3 + 2 = 5$). Have this done in the same way. Then tackle the top row. The picture for that should look like this:



5 and 0 → 5

$5 + 0 = 5$

Most of the pupils should now be able to do the other three rows without further help.

Headline Story: If ___ fingers on one hand are up, then ___ fingers are down. (Chart possible results.)

● ● ● ● ● 5 + 0 = ● ● ● ● ● 5

■ ■ ■ ■ 4 + ■ 1 = ■ ■ ■ ■ ■ 5

▲ ▲ ▲ 3 + ▲ ▲ 2 = ▲ ▲ ▲ ▲ ▲ 5

● ● 2 + ● ● ● 3 = ● ● ● ● ● 5

■ 1 + ■ ■ ■ ■ 4 = ■ ■ ■ ■ ■ 5

0 + ▲ ▲ ▲ ▲ ▲ 5 = ▲ ▲ ▲ ▲ ▲ 5

Notes on Pupil Page 43

Tasks: *Pupils fill in the Cross Number Puzzles to conform with the domino pictures above them; they fill in the missing numbers in the other exercises.*

Purpose: To provide pupils with more experiences in counting groups together (or adding) and to reinforce the pupils' concept of the "any order—any grouping" principle for addition.

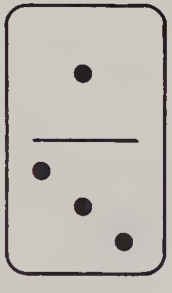
Unifying Ideas: Number and counting; structure.

Materials for Teacher: Demonstration domino cards.

Materials for Pupils: At least 19 counters, or a box of dominoes; a piece of paper folded into nine sections.

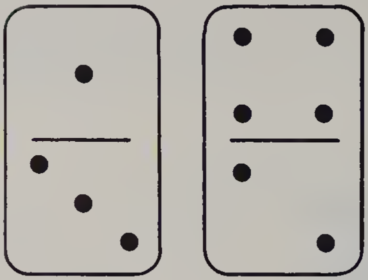
The Lesson:

Hold up one of your domino cards and ask the pupils to find a domino which matches it, or to arrange counters on their desk to match the dots in the two parts of your card. Have them fill in the left-hand column of boxes on their paper with a record of the number of dots in the two parts of the domino and the number of dots on the whole domino.



1		
3		
4		

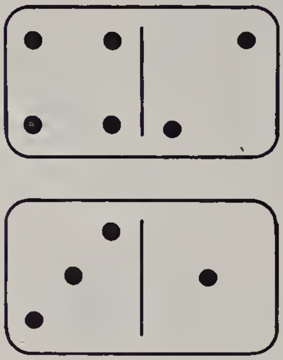
Hold up another domino card beside the first one and repeat the activity, filling in the second column of boxes in the record.



1	4	
3	2	
4	6	

Have pupils complete the Cross Number Puzzle.

The pupils may use the other side of their papers to keep a record of another activity of this kind, using the same dominoes but changing their order.

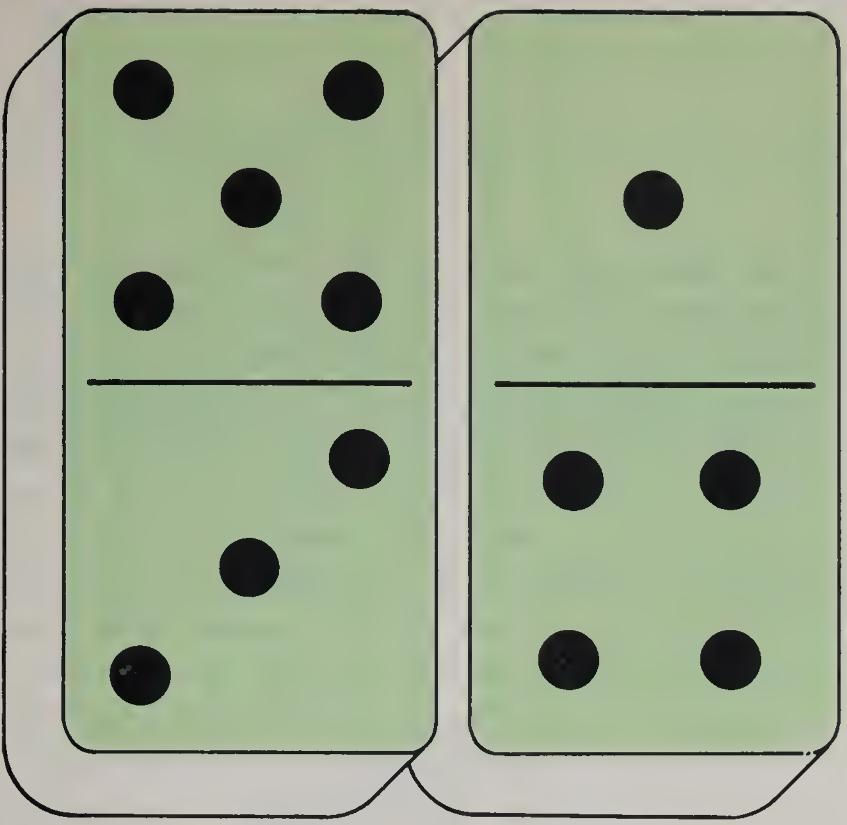


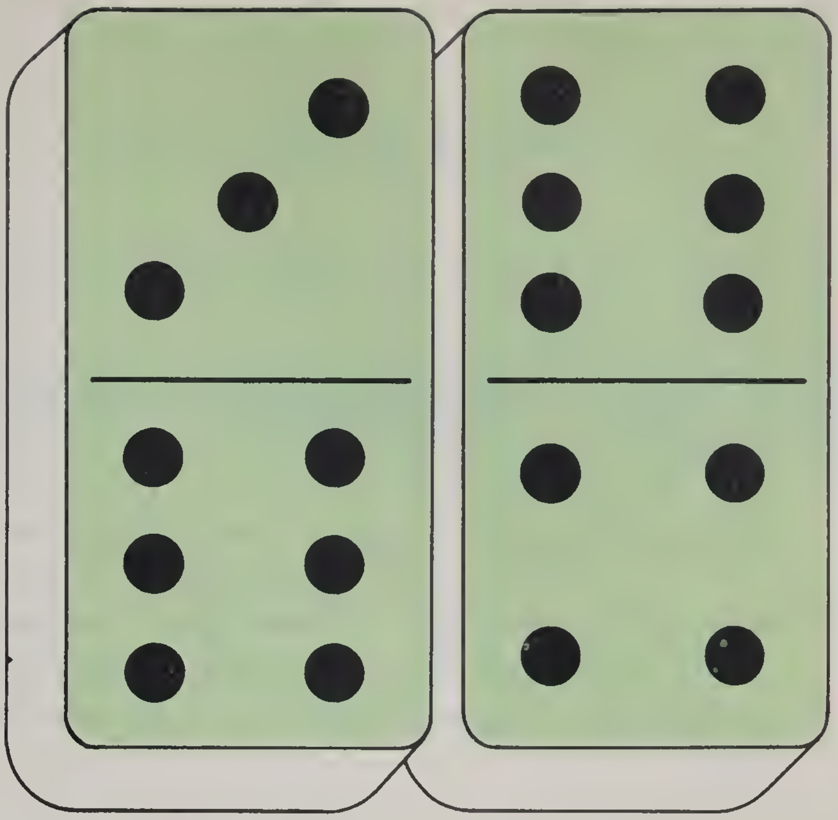
Assign page 43 for individual study. Allow any pupil who needs them to use counters or number lines to help him find answers to the exercises on the bottom half of the page.

Focus the class's attention on the breakout picture at the bottom right of the page. Discuss with them the difference between talking about a number of coins and the value of these coins.

Headline Story: Complete headlines and tell stories that fit:

$3 + 3 = \square$ $\triangle + 5 = 7$





5	1	6
3	4	7
8	5	13

3	6	9
6	4	10
9	10	19

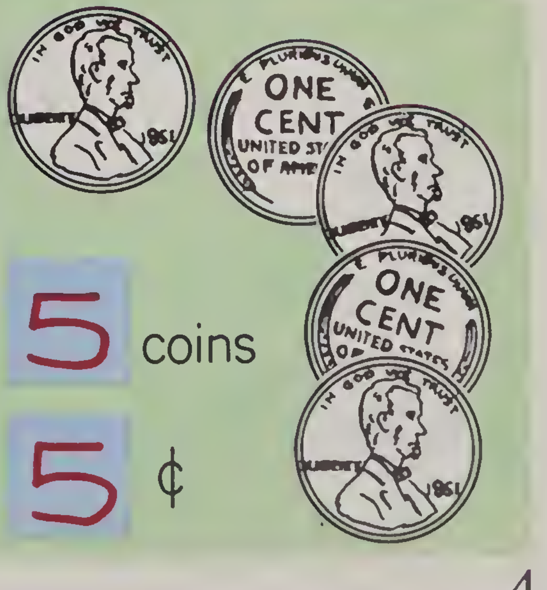
1	2	2	3	3	4	4	5
+ 2	+ 1	+ 3	+ 2	+ 4	+ 3	+ 5	+ 4
3	3	5	5	7	7	9	9

5 + 2 = $\boxed{7}$

2 + 5 = 7

6 + 3 = 9

3 + 6 = 9



5 coins

5 ¢

Notes on Pupil Page 44

Tasks: *Pupils fill in the number tags to describe the action in the picture.*

Purpose: To introduce the "take-away," or decomposition, idea of subtraction.

Unifying Ideas: Sets; addition and subtraction.

Materials for Teacher: Cards with compound words your pupils can read (Sunday, playhouse, sometimes); demonstration rods or counters; scissors.

Materials for Pupils: Objects which can be moved easily.

The Lesson:

Hold up one of your cards and ask a pupil to read the word. Ask who sees two little words in this big word. Cut the card between the syllables (or cover the second syllable) and say clearly what you have done when you have removed the second part; for instance: "*Cupcake*, take away *cake* leaves *cup*." Put the card back together and say, "Now, let's count the letters. How many letters in the big word?" (Write a 7 on the board.) "How many letters did we take away?" (Write —4 beside the 7.) "How many letters were left?" (Write = 3.) Call attention to the new kind of number sentence on the board. Tell the pupils we say "minus" for the new kind of sign. We read the number sentence, "7 minus 4 equals 3." Repeat this activity a few times.

Have several pupils come to the front of the room. List their names on the board. Ask some of these pupils to take their seats. Cross out the names of those who sat down. Now, say what happened: (Peter, Albert, Mary, Ed, Pat), take away (Mary, Pat) leaves (Peter, Albert, Ed). Write the number sentence associated with the same situation: 5—2 = 3. Read this sentence, "5 minus 2 equals 3." Go over what is written on the board so that everyone sees how the list of names with some crossed out and some left in is one way of writing the story of the pupils who came to the front of the room, while the number sentence is another way of keeping a record of the same story. Then repeat this activity using different numbers of pupils each time.

Think of other ways to dramatize a "take-away" subtraction situation. Pupils may hold up 4 fingers, turn down 1 finger, and note how many are left. They should say the number sentence: 4 minus 1 equals 3. They may put a stated number of counters on their desks, take a given number of them away, and see how many are left. In all such activities, they should say the appropriate number sentence.

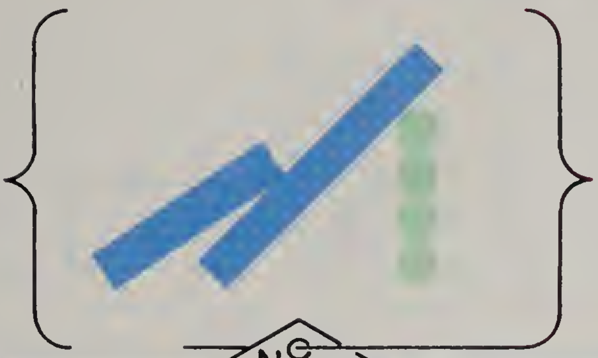
Assign page 44 for individual study. Any pupil who has difficulty should be encouraged to cross out in the first picture in each row those objects which the second picture tells him to take away. He should count how many there were at first and write that number in the first tag. He should count how many he took away and write that number in the second tag. He should count how many are left and write that number in the last tag. He should see how what he has done goes with the material on the page.

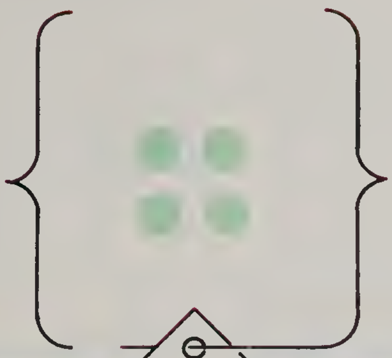
Headline Story: Martha had 5 dresses. Two were dirty. How many clean dresses?

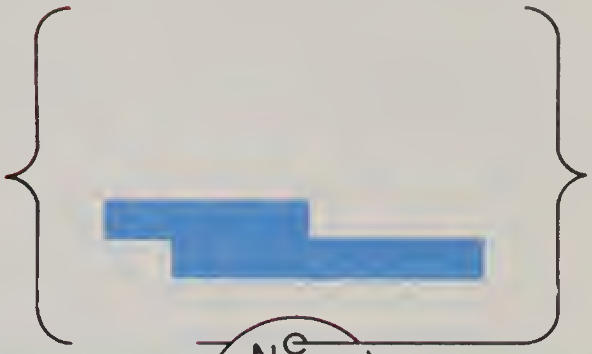
{ SUNDAY } { DAY } { SUN }

N 6 − N 3 = N 3

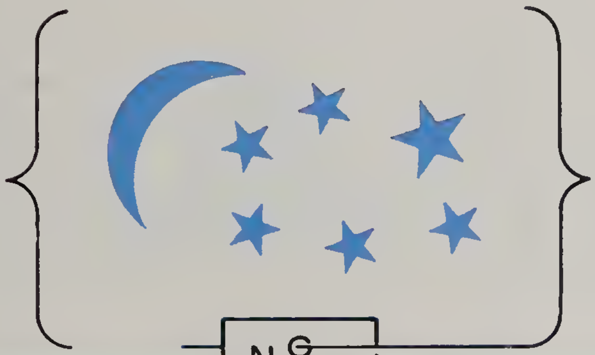
"6 minus 3 equals 3"

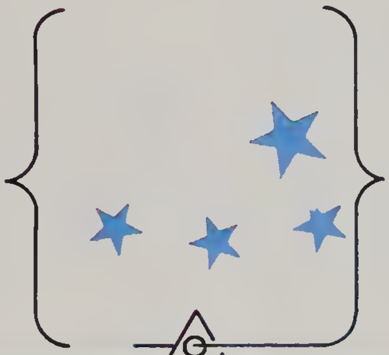


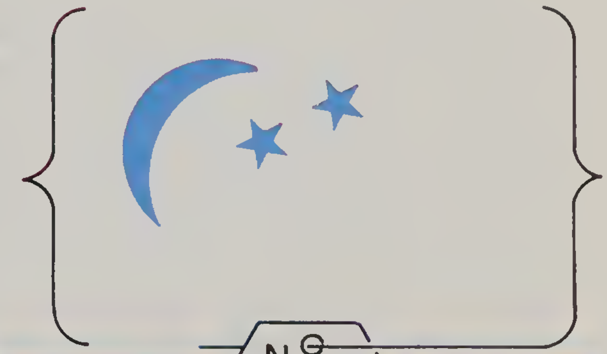




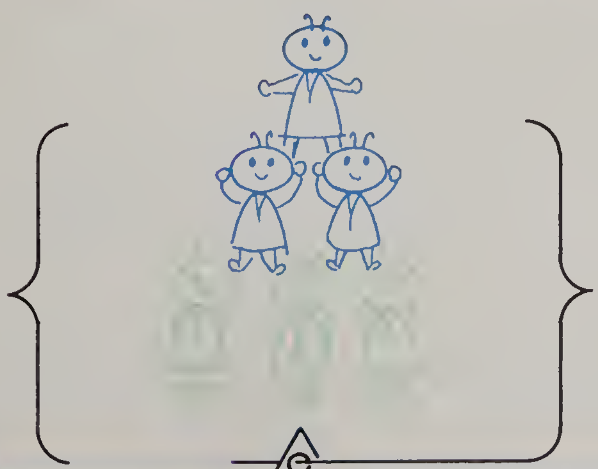
N 6 − N 4 = N 2

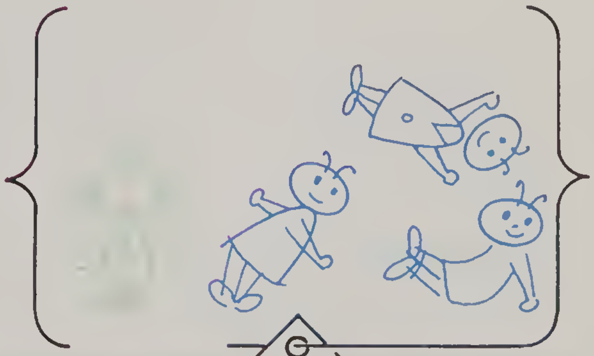






N 7 − N 4 = N 3





N 6 − N 2 = N 4

Notes on Pupil Page 45

Tasks: *Pupils fill in number tags and missing pictures to describe the actions in each row.*

Purpose: To continue the development of the decomposition idea of subtraction.

Unifying Ideas: Sets; addition and subtraction.

Materials for Teacher: Colored chalk.

Materials for Pupils: A worksheet, as described below.

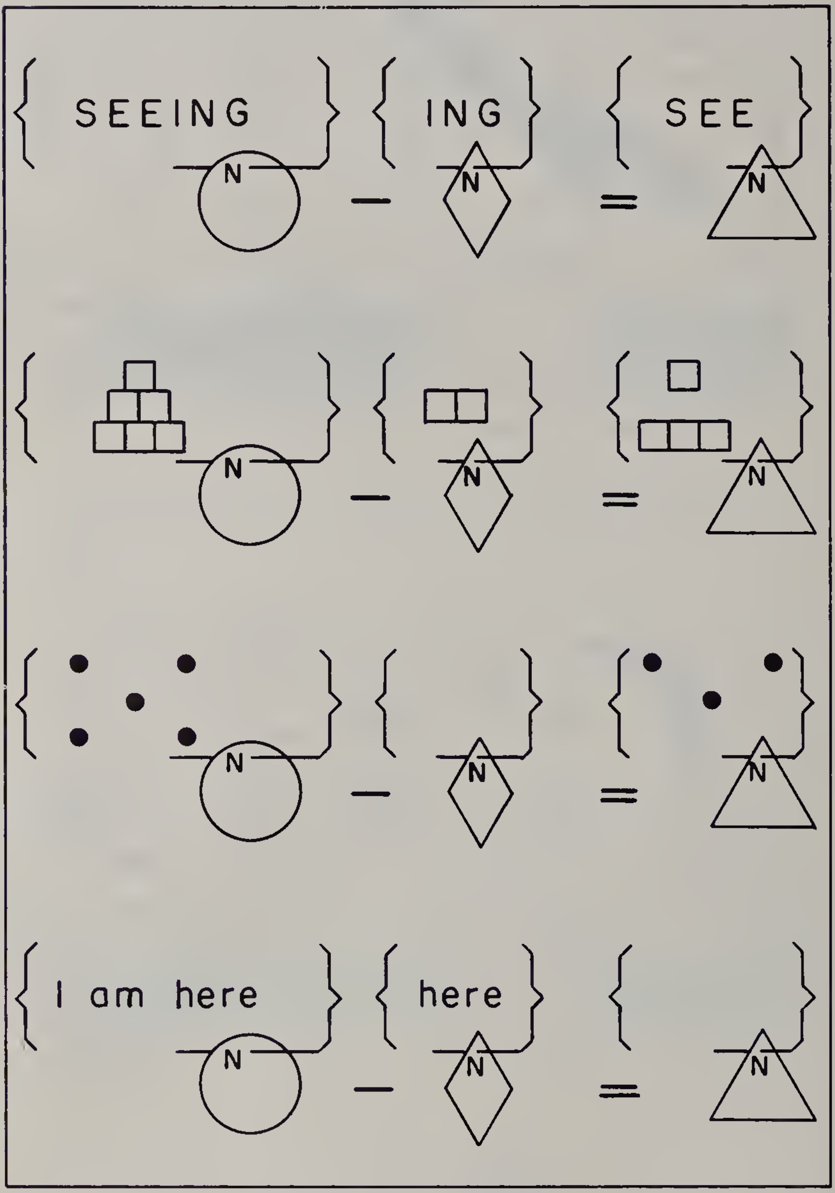
The Lesson:

Provide each pupil with a worksheet which looks somewhat like the one in the next column. Copy one row of this worksheet at a time on the board and work with the pupils as you discuss it together. You might merely ask, "What is going on here?" Most pupils have the resourcefulness to make every important observation with no further prompting.

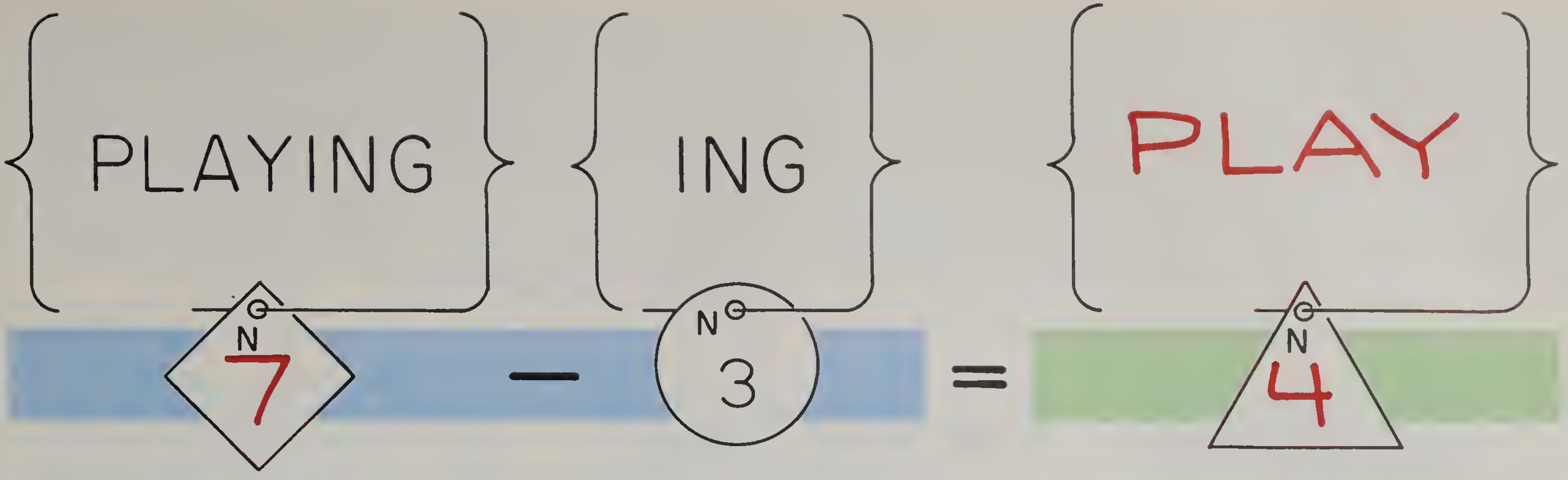
A few may need more specific help. In discussing the second row, for example, say, "How many squares are in the first picture? Fill in the tag to tell that number. How many squares are to be taken away? Fill in that tag. In the first picture, cross out the squares that are taken away. In the space at the right, draw a picture of what is left. Write the number that tells how many are left. Now read the number sentence: 6 minus 2 equals 4."

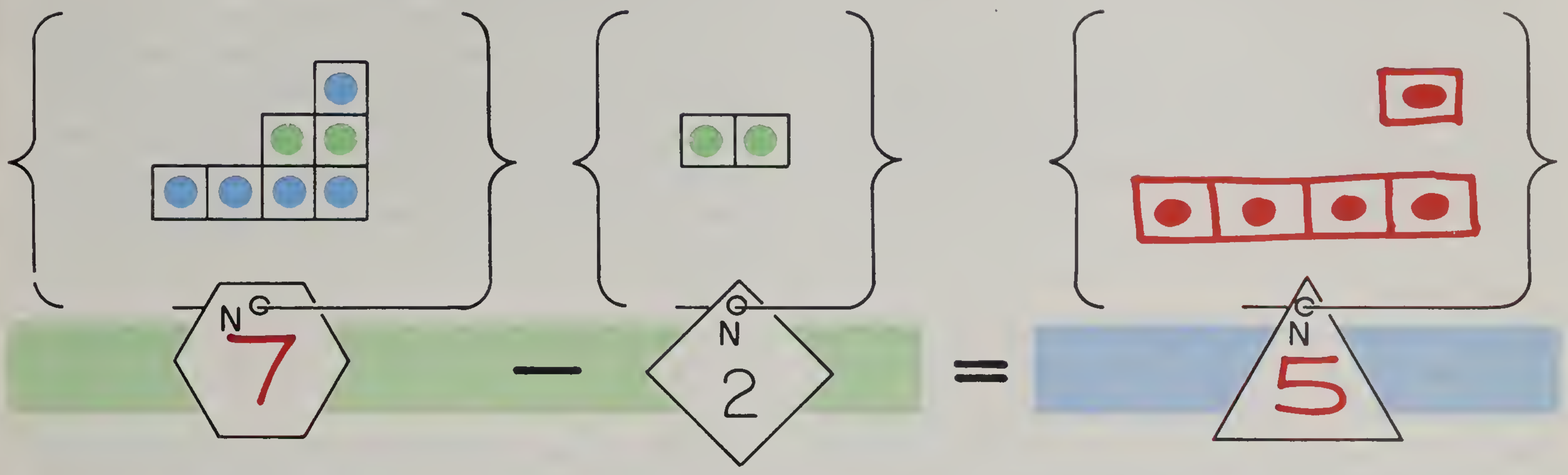
Crossing out the objects taken away is better than erasing or covering them, because the pupils need to see all three of the numbers involved. Crossing out some objects allows the pupils to see all that were there at first, those which were "taken away" (crossed out), and those which are left. In the third row, the pupils should cross out parts of the first picture which are not left in the third, then draw a picture of what they crossed out.

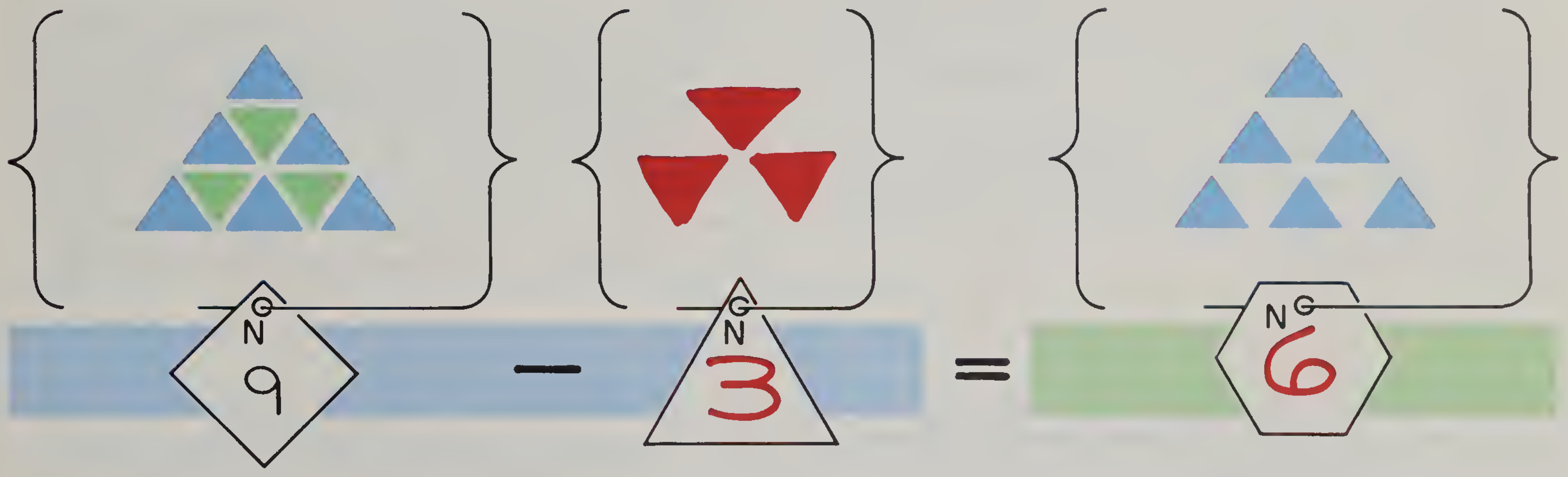
Assign page 45 for individual study. Pupils who can perform satisfactorily without crossing out the objects taken away should not be required to use this aid. When all have finished, read the number sentences together.

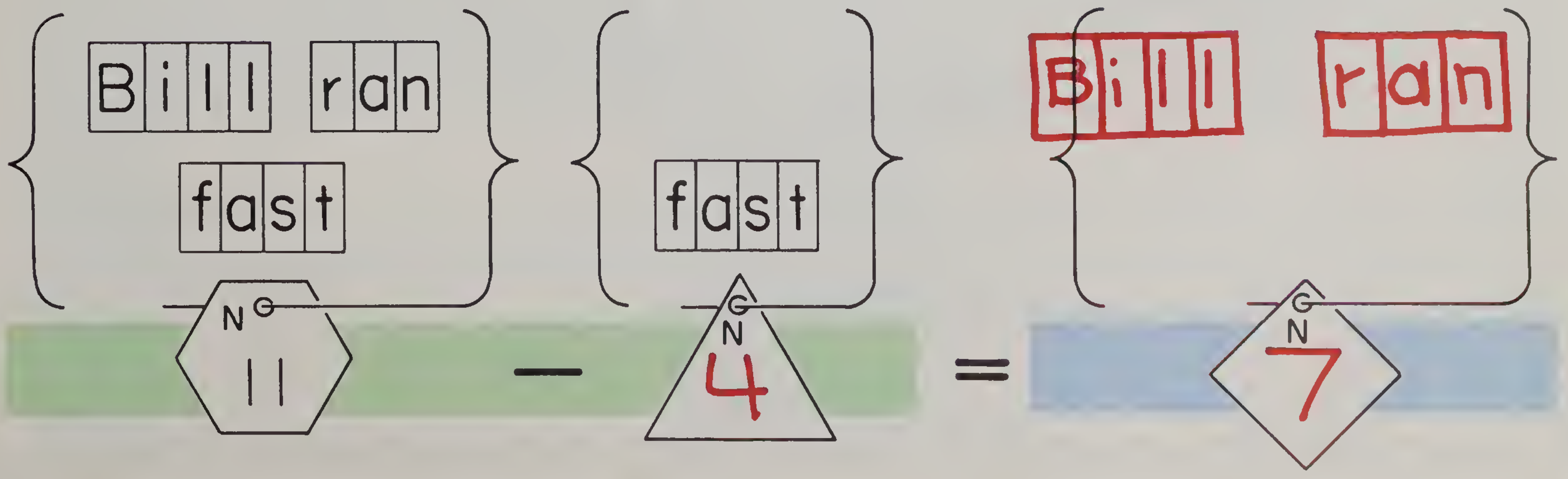


Headline Story: Pat lived 6 blocks from school. After he walked 4 blocks, how far has he to go?


PLAYING
ING
PLAY
N 7
−
N 3
=
N 4


N 7
−
N 2
=
N 5


N 9
−
N 3
=
N 6


Bill ran
fast
fast
Bill ran
N 11
−
N 4
=
N 7

Notes on Pupil Page 46

Tasks: *Pupils draw arrows between appropriate points of the number line, circle the numbers each arrow points to, and fill in the missing numbers in the boxes.*

Purpose: To show subtraction in terms of motion to the left along the number line.

Unifying Ideas: Functions and relations; addition and subtraction; structure.

Materials for Teacher: Demonstration number lines on the floor, chalkrail, table, or chalkboard.

Materials for Pupils: Number line worksheets as described for page 42.

The Lesson:

In previous lessons, pupils saw addition associated with jumps forward on the number line. This lesson should strengthen the feeling that there is something "opposite" about addition and subtraction. Exercises O and P on page 46 reinforce this idea.

Begin by having several pupils actually perform the appropriate actions on your demonstration number lines. Three kinds of questions are involved:

First, do a number of exercises which involve finding the answer to a question like, "If you start at 5 and jump back 3, where do you land?" Record this action as follows:

$$5 \;\overset{\curvearrowleft}{3} \longrightarrow \boxed{2}$$

After several examples like this, introduce this way to keep the same record:

$$5 - 3 = \boxed{2}$$

Second, proceed to questions like, "If you start at 8 and want to land at 7, in which direction will you have to jump? How big a jump will you take?" Record the action at first as:

$$8 \;\overset{\curvearrowleft}{\triangle 1} \longrightarrow 7$$

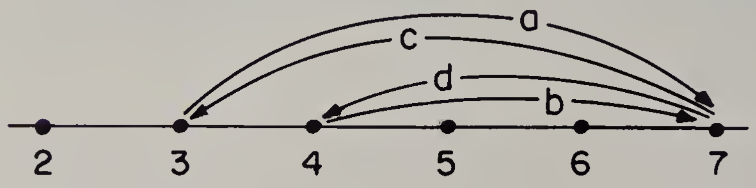
Later, introduce $8 - \boxed{1} = 7$ as another way to keep the same record.

The third kind of question is by far the hardest for the pupils to handle. It may be wise to work for a day or two on the first two questions before tackling this third one. An instance of this is, "If you want to jump back 2 and land at 7, where do you have to start?" Again, two kinds of records should be kept.

$$\textcircled{9} \;\overset{\curvearrowleft}{2} \longrightarrow 7 \quad \text{and} \quad \boxed{} - 2 = 7$$

After you are satisfied with the work your pupils are doing on the demonstration number lines, have them do page 46 and, if needed, exercises on worksheets.

This is a good time to ask pupils to tell you what is going on in the following pattern and record. (It illustrates the inverse relationship between addition and subtraction, and the commutative principle of addition. Of course, we are not interested in these words. The important thing is the exploration, discussion, and investigation of similar patterns.)

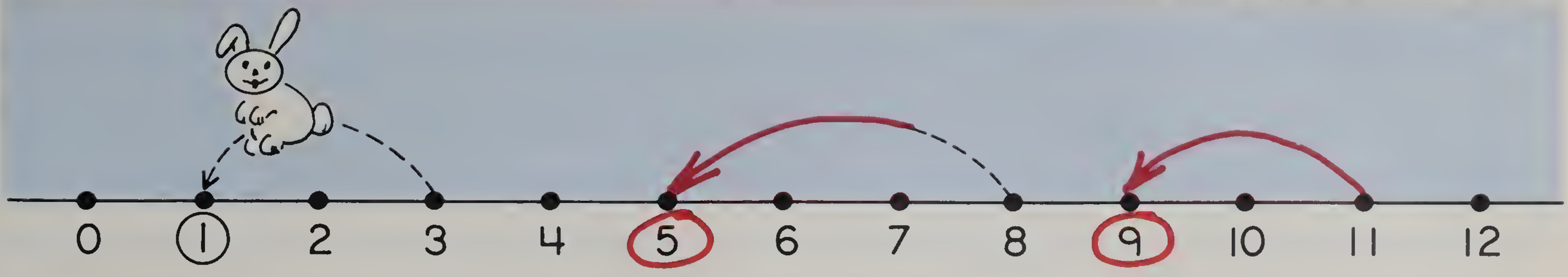


(a) $3 + 4 = 7$ (c) $7 - 4 = 3$

(b) $4 + 3 = 7$ (d) $7 - 3 = 4$

Headline Story: Complete headline and tell a story that fits:

$$20 - 10 = \boxed{}$$

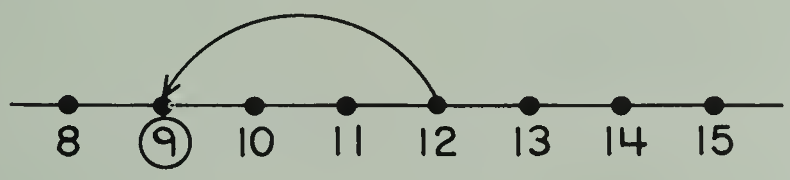

0
1
2
3
4
5
6
7
8
9
10
11
12

$3 \overset{2}{\curvearrowleft} \rightarrow 1$

$8 \overset{3}{\curvearrowleft} \rightarrow \boxed{5}$

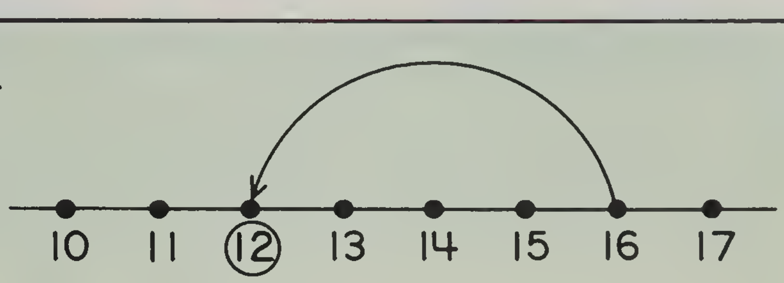
$11 \overset{2}{\curvearrowleft} \rightarrow 9$

K


8
9
10
11
12
13
14
15

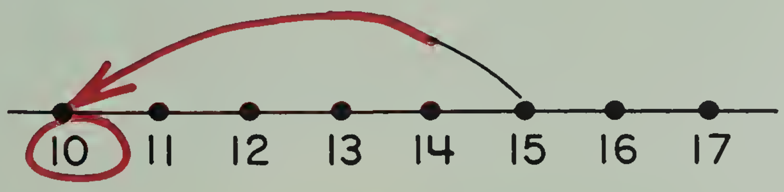
$12 - 3 = 9$

L


10
11
12
13
14
15
16
17

$16 - 4 = 12$

M


10
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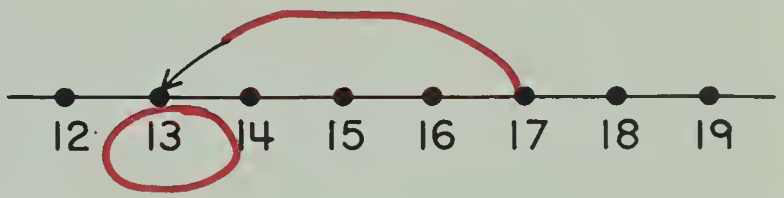
16
17

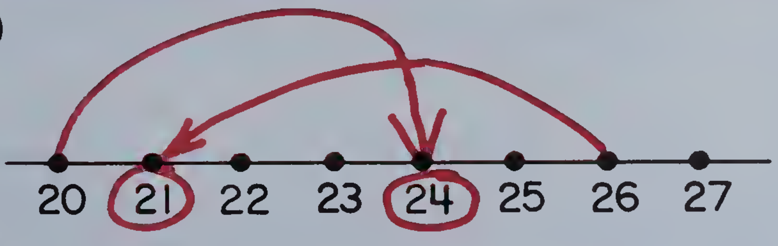
$15 - \boxed{5} = 10$

N


12
13
14
15
16
17
18
19

$\boxed{17} - 4 = 13$

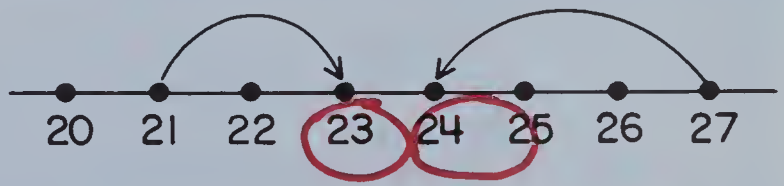
O


20
21
22
23
24
25
26
27

$26 - 5 = \boxed{21}$

$20 + 4 = 24$

P


20
21
22
23
24
25
26
27

$\boxed{21} + 2 = 23$

$27 - 3 = 24$

Notes on Pupil Page 47

Tasks: *Pupils list the various ways of writing about the number 3 in one circle and similarly for the number 5 in the other circle.*

Purpose: To review the idea that numbers have many names and to recall some addition facts in this context.

Unifying Ideas: Numeration; addition and subtraction; functions and relations.

Materials for Teacher: Chalkboard or flannel board, number tags, three boxes—red, white, and blue.

Materials for Pupils: Counters; Cuisenaire® Rods or teacher-made facsimilies.

The Lesson:

Write the words zero, one, two, three, four, and five, along the top of the board. Now have pupils come to the board and write the ordinary (Arabic) number symbols in the appropriate places. Call attention to the fact that you have a record of three ways to write the names of some of the numbers. Some will be interested in the fact that there is no Roman numeral for the number 0.

Now, let's explore a number like 4. "Who can show us a picture suggesting four?" A pupil might put four objects on the flannel board. Another pupil might pile four counters on top of one another. Another might show four with his Cuisenaire® Rods. All these are representations of four. Then ask the pupils to regroup the rods or counters which represent four. Record the results on the board. If the pupil with rods used 4 white ones, write 1 + 1 + 1 + 1. If he used 2 red rods, write 2 + 2. If he used a light green rod and a white rod, write 3 + 1. We might agree that a purple rod can be recorded as 4, as 4 + 0, and as 0 + 4. Continue regrouping with four counters or with objects on the flannel board. Continue until you have listed all the possible combinations in all the possible orders. Then pick another number, say five, and repeat the activity.

Another interesting exploration gets under way if we have three boxes—red, blue, and white. In how many ways can we put one object in the boxes? . . . two objects? . . . three objects?

ONE OBJECT

	red	blue	white	
Barbara	1	0	0	1 + 0 + 0 = 1
Jim	0	0	1	0 + 0 + 1 = 1
Harry	0	1	0	0 + 1 + 0 = 1

TWO OBJECTS

	red	blue	white	
Barbara	2	0	0	2 + 0 + 0 = 2
Jim	1	0	1	1 + 0 + 1 = 2
Harry	0	0	2	0 + 0 + 2 = 2
Etc.				

THREE OBJECTS

	red	blue	white	
Barbara	3	0	0	3 + 0 + 0 = 3
Jim	0	3	0	0 + 3 + 0 = 3
Harry	0	0	3	0 + 0 + 3 = 3
Etc.				

Be sure to review the accompanying booklet, *Games and Enrichment Activities*, for suggestions you may wish to incorporate in your daily lesson plan.

Headline Story: There are 5 children in a family. If there are __ boys, there are __ girls.

1 + 0

2 + 0

[✓ 3 + 0]

4 + 0

two

II

3 + 1
[✓ 3 + 2]
3 + 3

I

1 + 2 + 1
[✓ 1 + 2 + 2]
1 + 2 + 3

3

3 + 0

three

III

2 + 1

0 + 3

1 + 1 + 1

1 + 2

six

[✓ 4 + 1]

[✓ three]

[✓ III]

[✓ five]

[✓ 2 + 1]

0 + 1
0 + 2
[✓ 0 + 3]
0 + 4

eight

IV

seven

[✓ 1 + 4]

four

nine

5

3 + 2

1 + 2 + 2

4 + 1

five

1 + 4

V

2 + 3

0 + 1 + 1
[✓ 1 + 1 + 1]
2 + 1 + 1

[✓ V]

[✓ 1 + 2]

ten

1 + 3
[✓ 2 + 3]
3 + 3

one

VI

Notes on Pupil Page 48

Tasks: *Pupils fill in the blanks to show the number of dots in each triangle, and, where the triangles are connected, to show how many in all. They can count to find any answer they do not see at a glance.*

Purpose: (a) To show the interesting configurations associated with what are called the "triangular numbers"; (b) to review some previously learned addition facts; and (c) to anticipate the development of facts not yet learned.

Unifying Ideas: Geometry; addition and subtraction.

Materials for Teacher: Demonstration models of the triangular numbers shown on this page, and one more—15.

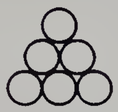
Materials for Pupils: About twenty counters, all the same size; Cuisenaire® Rods.

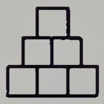
The Lesson:

The triangular numbers are those which are 1, the sum 1 + 2, the sum 1 + 2 + 3, the sum 1 + 2 + 3 + 4, and so on.

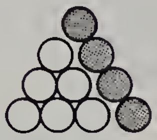
Ask your pupils if they have ever seen acrobats stand on one another's backs, always building up to one person at the top. Have the pupils describe how this is done. They should mention the fact that the more people there are in the bottom row, the higher the "pyramid" can be. Draw stick men on the board to show this so that everyone understands. Draw a triangular "fence" around these pictures and ask the pupils what they call the shape you have just drawn. Count the men inside the triangles and record the number underneath each picture.

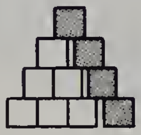
Ask the pupils to use their counters to build triangular structures.





Now ask, "How many in the top row?" (Write a 1 on the board.) "How many in the next row?" (Write: +2.) "How many in the bottom row?" (Write: +3.) "How many in all?" (Write: =6.) Ask how many counters, or how long a rod, should be needed in building this figure one row higher. Have the pupils do this. They can check with your demonstration model. Do any of them see that, if they are using separate round or square counters, they can build up one of the sides as well as the bottom in making the triangle one row higher?



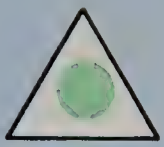


Repeat this activity with five in the bottom row. Then have the pupils build two triangular numbers side by side. Have them find how many counters they needed for both of these together. Write the appropriate addition fact on the board.

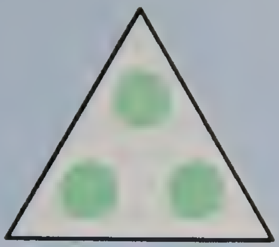
Then assign page 48 for individual study.

Headline Story: Complete headlines and tell stories that fit:

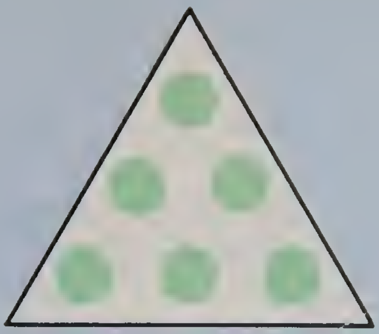
$5 + \square = 10$ $10 - \square = 5$



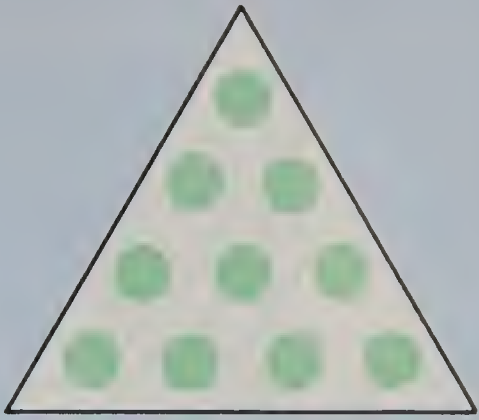
A. 1



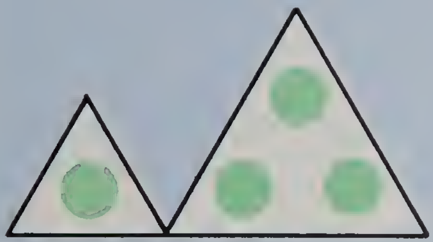
B. 3



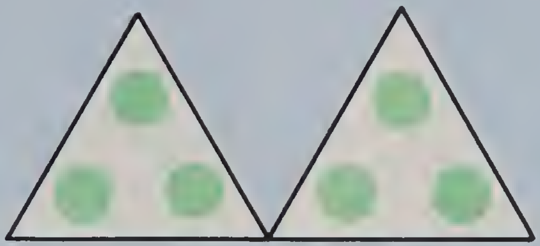
C. 6



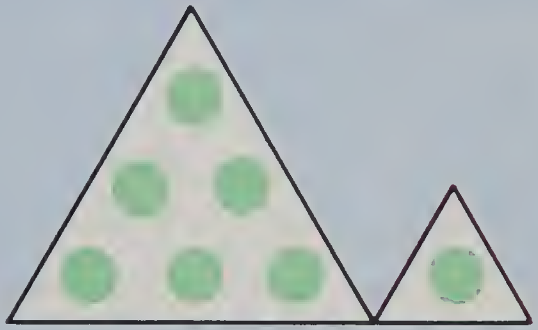
D. 10



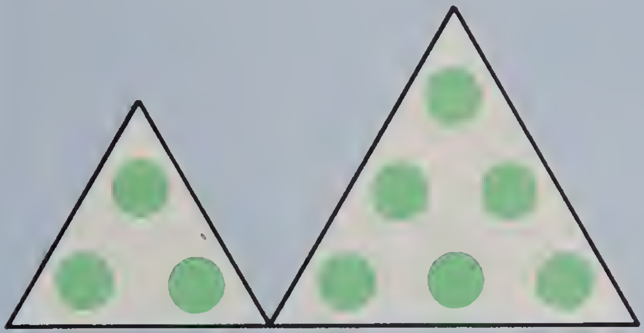
E. 1 + 3 = 4



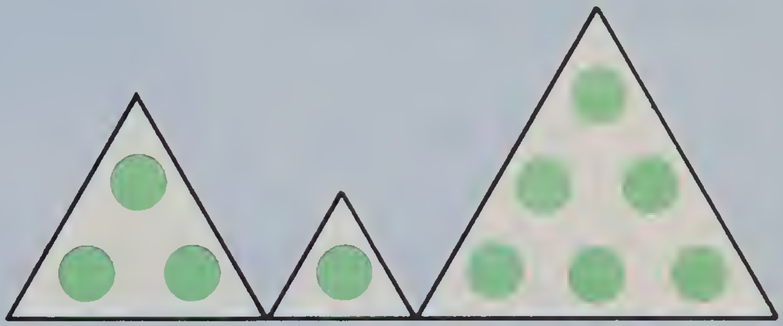
F. 3 + 3 = 6



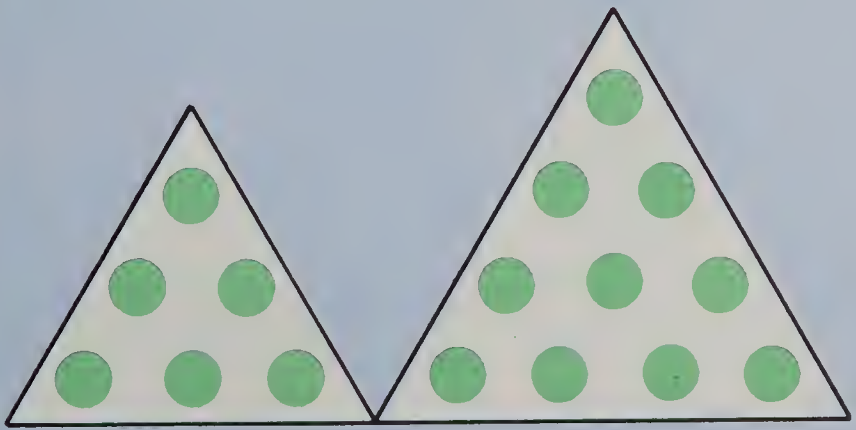
G. 6 + 1 = 7



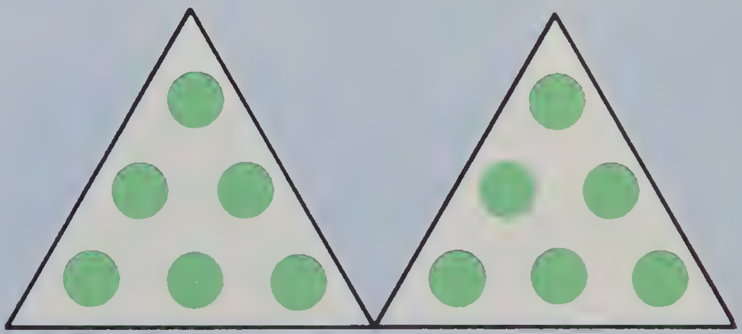
H. 3 + 6 = 9



I. 3 + 1 + 6 = 10



J. 6 + 10 = 16



K. 6 + 6 = 12

Notes on Pupil Page 49

Tasks: *Pupils fill in the boxes to tell how many in each part and how many in all.*

Purpose; To reinforce the pupils' understanding of the "teen" numbers as "10 and so many more."

Unifying Ideas: Numeration; number and counting.

Materials for Teacher: At least fourteen objects to use on the flannel board, or colored chalk.

Materials for Pupils: Counters, not necessarily all alike.

The Lesson:

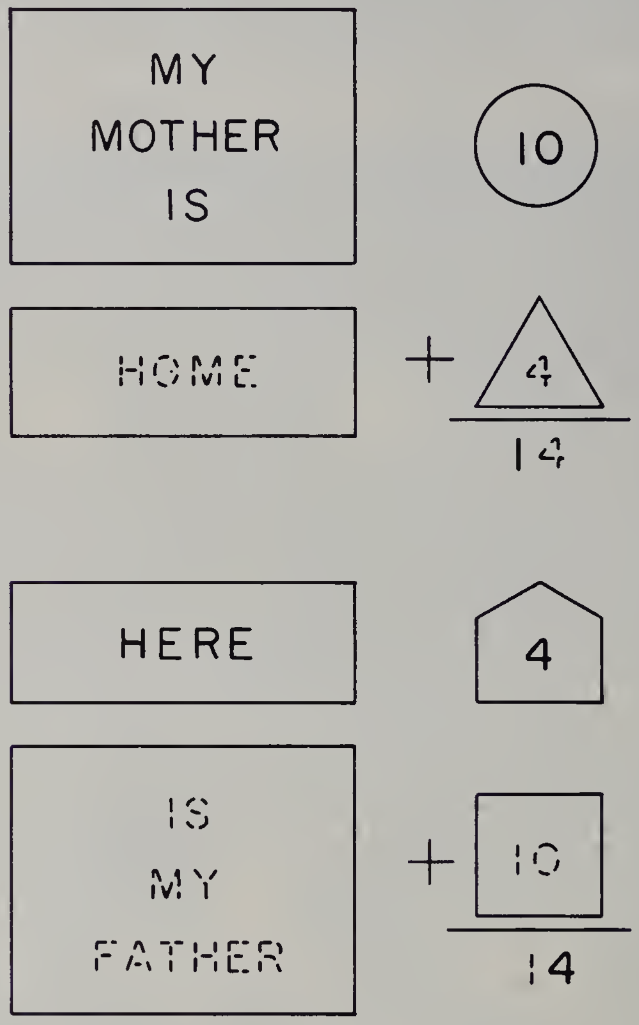
Have the pupils use their counters to show "10 and so many more" while you draw pictures on the board or place objects on the flannel board to help them, if necessary. Proceed somewhat as follows:

Ask pupils to put 10 counters in one group on their desks and 2 counters in another group. On the board, write both of the following forms and have someone complete the examples.

$$10 + 2 = \qquad \begin{array}{r} 10 \\ +\ 2 \\ \hline \end{array}$$

Repeat this activity a number of times, sometimes having the 10 first and sometimes having the other number first. Do not stop until everyone can tell without counting from the first object to the last that 10 and 3, or 3 and 10, *are* 13.

When you are satisfied that the pupils will think "14" whenever they see either 10 + 4 or 4 + 10, sketch the following example on the board, using two colors of chalk. The numbers are to be filled in after the pupils have done the appropriate counting and adding.



Assign page 49 for individual study. Pupils who do not finish this quickly need special help with these teen numbers.

Headline Story: Complete headline and tell a story to fit:

$$10 + \square = 13$$

I

DOGS
CAN
RUN

14

FAST

10

4

+

14

II

IT

12

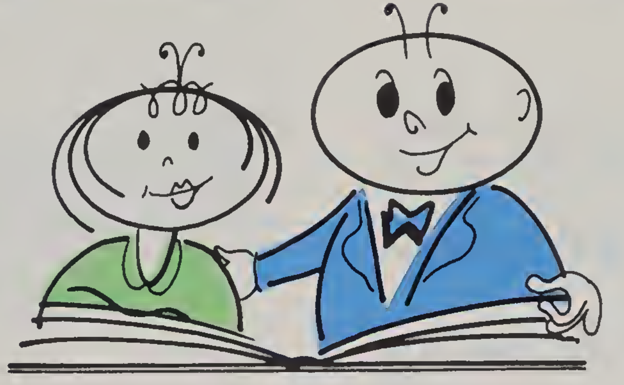
IS A
NICE
DAY

2

10

+

12



III

SHE

IS MY
MOTHER

3

10

+

13

IV

I

PLAY
WITH
ED

1

10

+

11

Notes on Pupil Page 50

Tasks: *Pupils fill in the boxes to complete the pattern in the number sequences or in the graphic designs.*

Purpose: (a) To test achievement; (b) to provide further experiences with patterns of number sequences and with graphic representations of numbers.

Unifying Ideas: Number and counting; numeration.

Materials for Teacher: Number lines on the floor, the chalkrail, or the chalkboard; five containers; at least 50 small objects to use as counters; numbers 0-20 written on small pieces of oak tag or paper.

Materials for Pupils: Containers; objects; and numbers as described above, if you want each pupil to do some of the activities individually.

The Lesson:

Pages 50 and 51 might be used as test pages. Tell the pupils that something is missing in each picture or pattern on pages 50 and 51. Do the first example together. No further instructions should be given.

Play a "Something is Missing" game, using any or all of the following activities after the pupils have completed the test pages.

Draw a section of a number line on the chalkboard. Label appropriate points 1, 2, 4, and 5. What is missing?

On a number line on the floor, have one pupil stand on 0, another on 2, another on 4, and another on 8. What is missing?

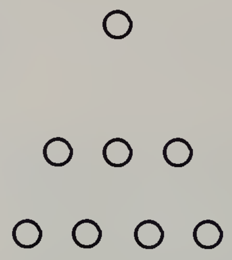
Above your chalkrail number line, label points for 0, 5, 10, and 20. Have a pupil fill in the "something missing."

In your five containers, put 3 counters in the first, 4 counters in the second, nothing in the third, 6 counters in the fourth, and 7 counters in the fifth. What is missing?

Try putting counters in the containers to indicate a descending sequence; for instance 5, 4, 3, ___, 1. What is missing?

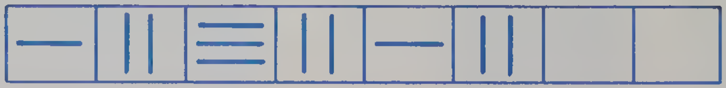
Use number cards instead of counters in your containers if you do not want to use this lesson particularly for practicing counting. Put a slip of paper showing a number in all but one of the containers, to show a broken sequence such as: 8, 9, ___, 11, 12. Repeat this activity using other number sequences, ascending and descending. Your pupils will need more help than you might expect, perhaps, in dealing with sequences which do not start at 1 or 0. (Try saying five letters of the alphabet starting with *j* or *s* or *u*. Do you have to think twice?)

Draw a picture similar to the one below. Have a pupil supply the missing row.



Ask pupils individually and in teams to supply a pattern in which "something is missing."

Headline Story: Extend this design:



0	1	2	3	4

2	3	4	5	6

1	2	3	4	5

5	4	3	2	1

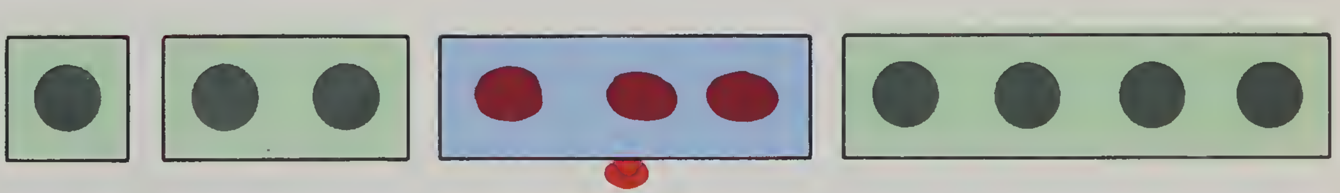
0	2	4	6	8

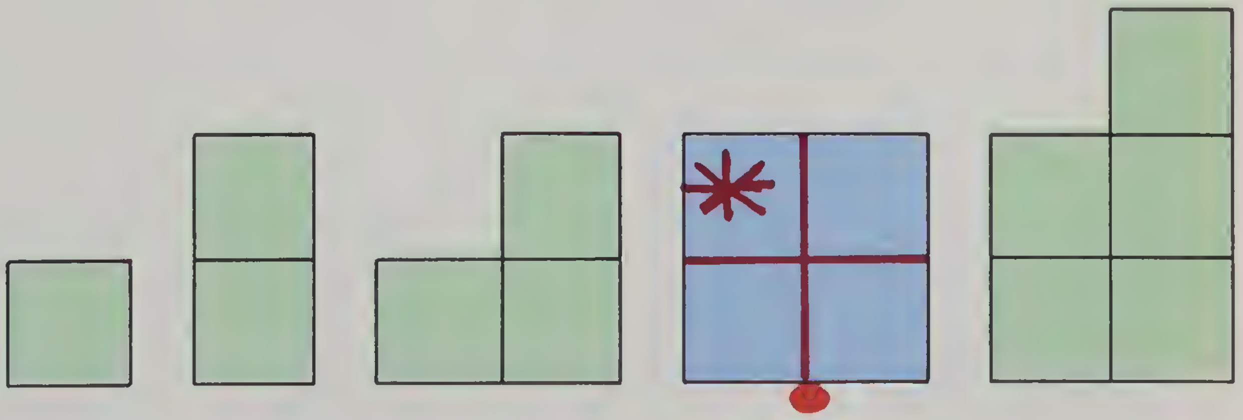
1	3	5	7	9

0	3	6	9	12

6	7	8	9	10

16
17
18
19
20





*Other logical answers for "4" acceptable.

Notes on Pupil Page 51

Tasks: *Pupils write a number in each blue space. In the upper part of the page, this number tells how many objects are pictured in both green boxes together. At the bottom of the page, this number tells how many "bricks" are missing.*

Purpose: (a) To test each pupil's ability to count or add to find how many in all, and (b) to assess his competence in dealing with simple spatial relations.

Unifying Ideas: Number and counting; measurement.

Materials for Teacher: Blocks or tiles or cardboard squares, if necessary.

Materials for Pupils: No special materials needed.

The Lesson:

Have the pupils turn directly to page 51 in their books. Discuss each part of the page just enough for each pupil to understand what to do. Have everyone look at the first item at the top of the page. Ask why a 4 is written in the blue space. If everyone knows, direct the pupils to complete the other examples which look like that. When all have finished, direct their attention to the "brick wall" which is pictured at the bottom of the page. Ask why a 1 is written in the first blue space in the top row. Find the 3 in a blue space in the bottom row. What does the 3 tell? If a large number of pupils do not know, use your blocks (or tiles or cardboard squares) to help them see that the 3 refers to the number of green "bricks" it would take to fill that blue space. Then have the pupils complete the lesson.

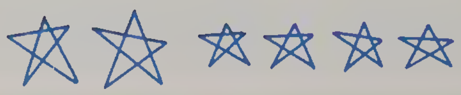
Evaluation

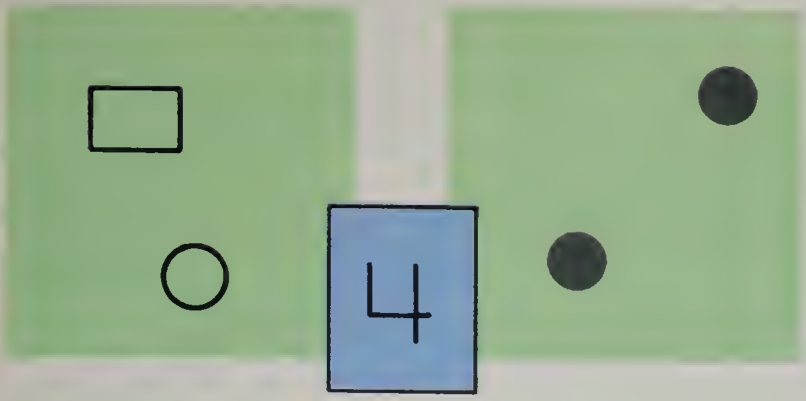
Every test can serve a variety of purposes. The material on page 50 can be used to assess certain aspects of a pupil's number sense. It can be used to help determine his readiness for addition, subtraction, or multiplication. It can be used to appraise his ability to tell what is appropriate, and what is not, in a reasonably well-defined pattern. It can be used to test his mastery of counting forward and backward by 1's and of counting forward by groups. It can be used to identify the pupils who need help with any or all of these things.

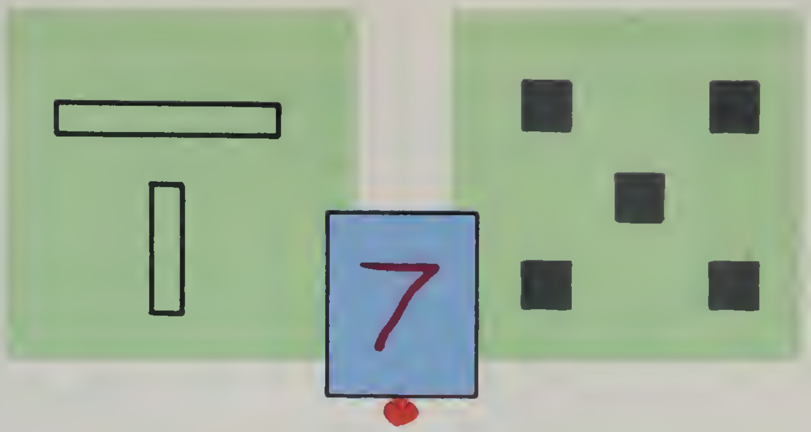
The material on the top of page 51 can be used as a test of the pupil's mastery of counting to find how many in all. A pupil who had correct responses for all these items may have done some adding, or he may have gotten all his answers by counting. Pupils who had more than one mistake on the top part of this page require immediate individual attention.

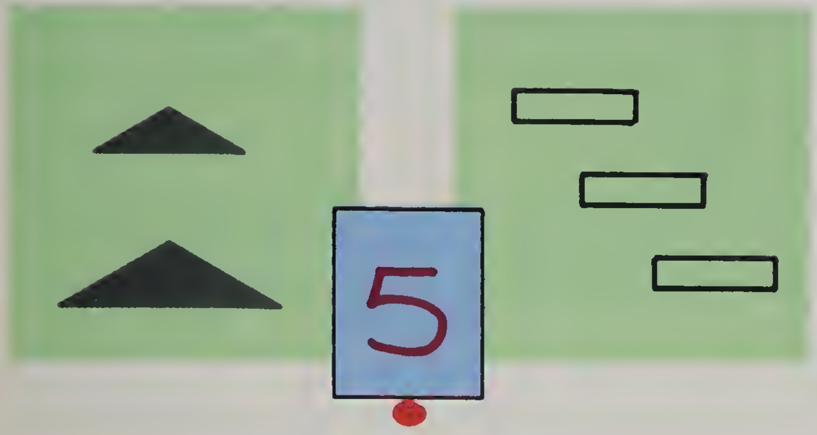
The material on the bottom of page 51 may be regarded as a power test, diagnostic or prognostic in character. Pupils who did well with this will have no trouble with the lesson which follows this, and will need a minimum of help with the two pages after that. Note the pupils who did poorly with the bottom of page 51. They are the ones who will need the most help with the next three lessons.

Headline Story: Tell a story suggested by the sketch:




4


7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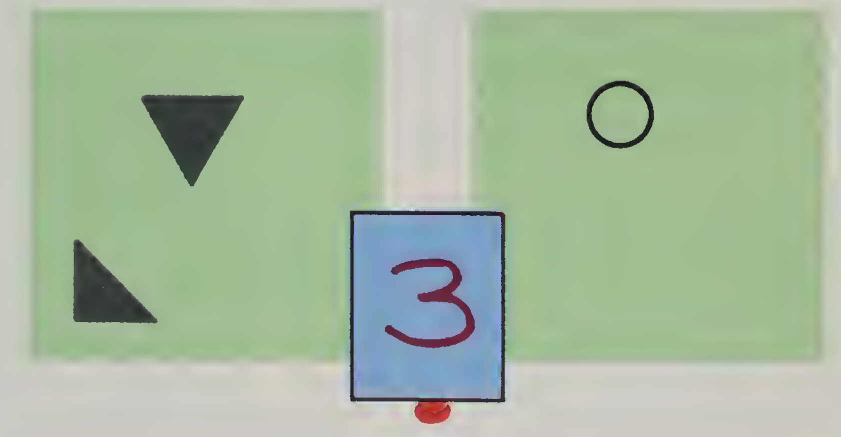
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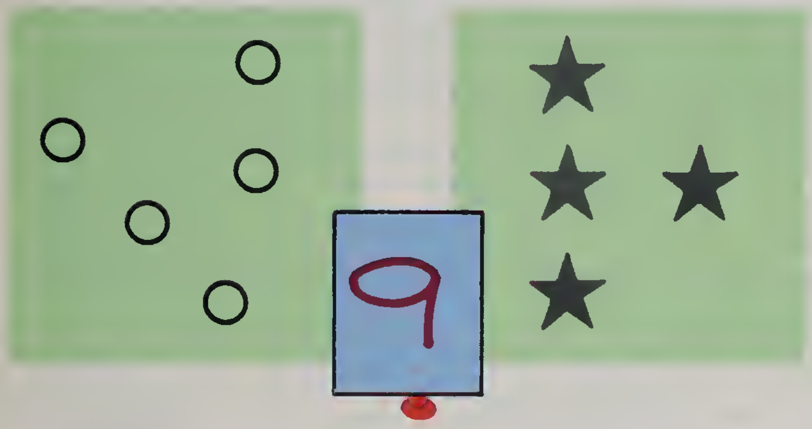
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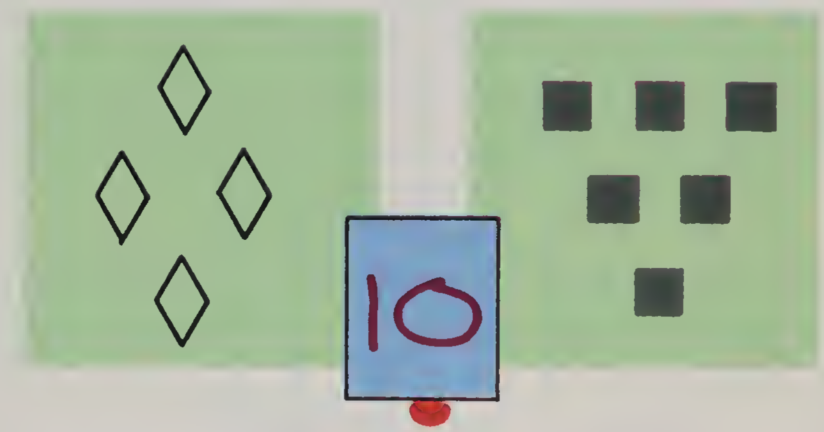
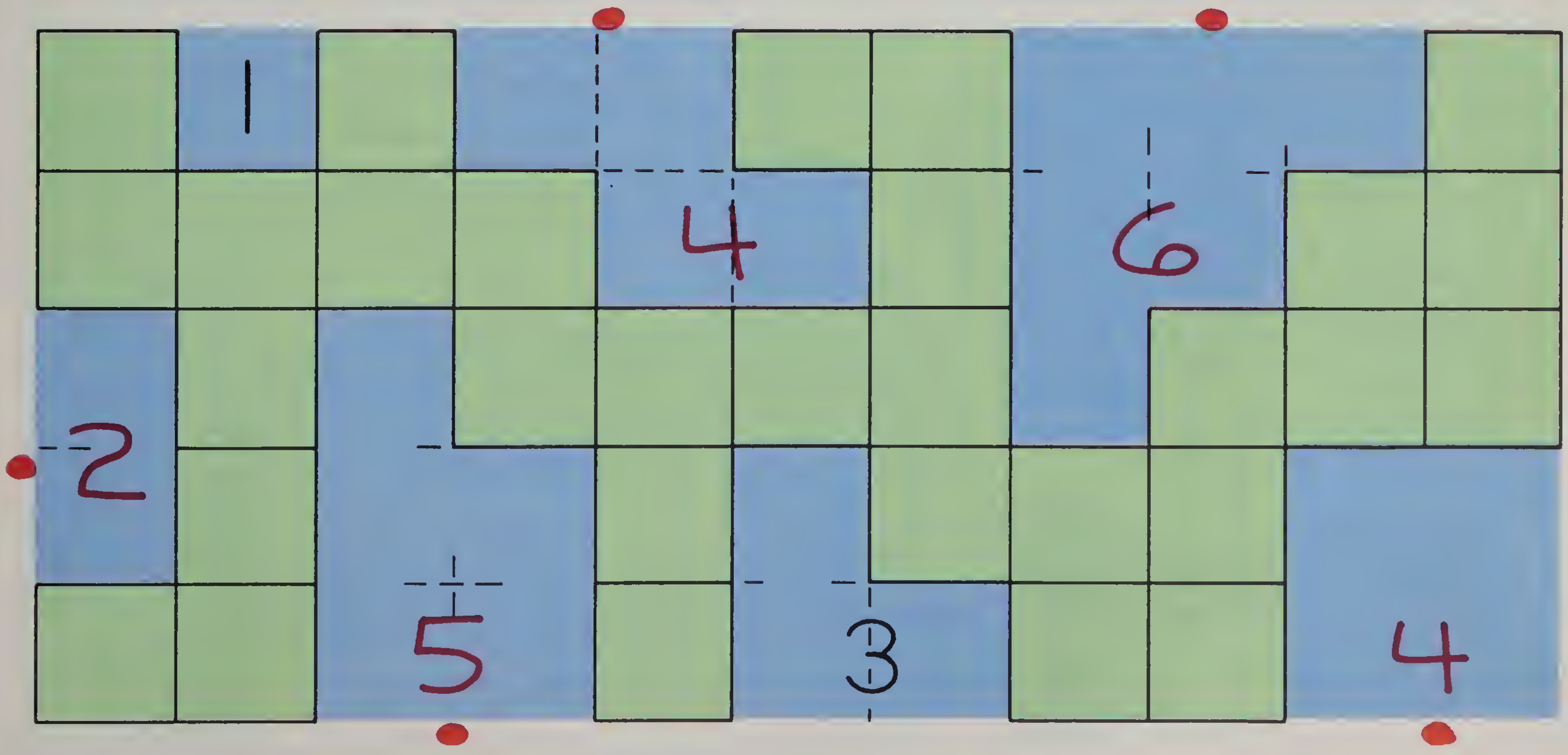

10


1
4
6
2
5
3
4

Notes on Pupil Page 52

Tasks: *In parts* m *and* n, *pupils fill in the blank in front of each strip with a number which tells the size of the strip in terms of the indicated unit; then they color strips of the same size with the same color. In parts* o *through* s, *they write appropriate numbers in all the empty boxes and blanks. In the lower right-hand corner, they write how many coins and how many cents.*

Purpose: To introduce the idea of "how much" associated with "how many."

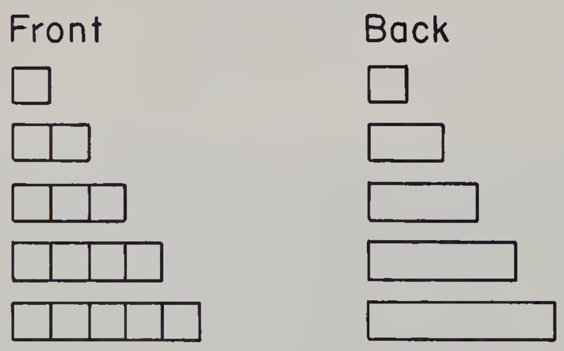
Unifying Ideas: Number and counting; measurement.

Materials for Teacher: Strips of cardboard or tag prepared as described below, or the equivalent; real or toy pennies.

Materials for Pupils: Individual strips or Cuisenaire® Rods or square counters; crayons.

The Lesson:

For the next three lessons, cardboard or tag strips prepared as shown below will prove to be effective learning aids. Unit squares in the teacher's set should be about 2 inches on a side. Make five of the unit squares and one or two of each of the others.



Provide each pupil with individual models of these strips or with Cuisenaire® Rods, or have them use square counters laid side by side to produce the same effect. If you make the pupils' unit strip a half-inch square, their strips will be the same size as those pictured on this and the next two pupil pages.

Hold up your unit square. Say, "When I look at this, I think of 1. Hold up the thing you have on your desk which makes you think of 1." Pick up your strip which is 2 units long, with the front of the strip facing the pupils. Ask, "What number should I think of when I look at this?" Have the pupils represent 2 with their materials. Repeat this activity for all the numbers 1-5, first in consecutive order and then in random order.

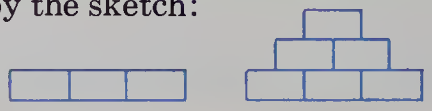
Conduct a similar activity showing the pupils the back of your strips, the side which does not have the units marked on it. This will be more of a challenge for most of the pupils. In each case, have the answers verified by finding out how many unit squares it takes to cover the unmarked strip.

Have the pupils put their materials aside, get out their books, and look at page 52. Discuss the picture of coins. If possible, use real money in helping the pupils recognize pictures of pennies.

Discuss with the pupils what they are to do on this page. Tell them that if they wish to they may draw lines between appropriate dots to make squares, or use their representational materials to help them carry out their tasks.

Be sure to review the accompanying booklet, *Games and Enrichment Activities*, for suggestions you may wish to incorporate in your daily lesson plan.

Headline Story: Tell a story suggested by the sketch:



m.

1 A

2 B

3 C

4 D

5 E

n.

3 C

4 D

1 A

5 E

2 B

Same size- same color (above)

Label boxes with no's — "□" suggests "1".

o.

2 1 1

2 2

p.

4 2 2

4 4

q.

3 2 1

3 3

r.

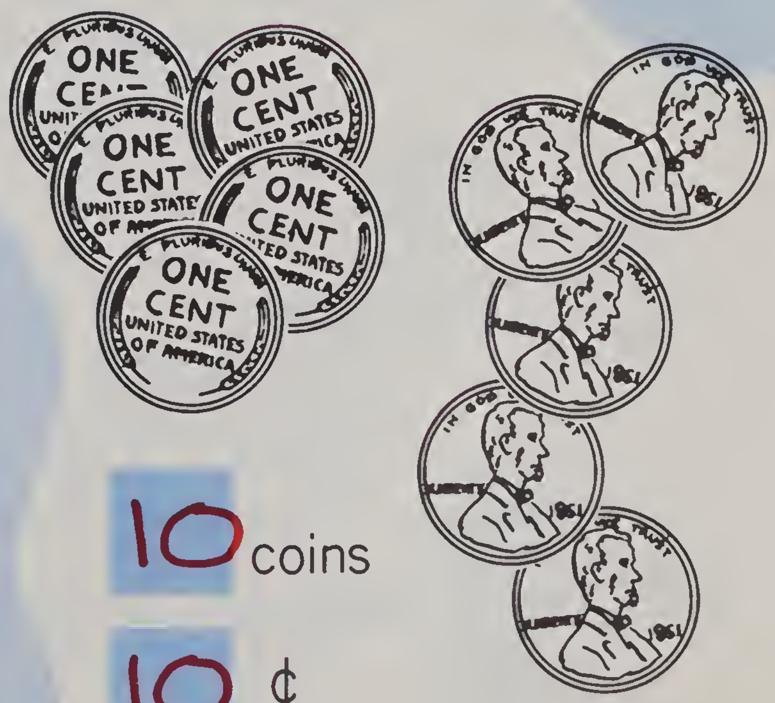
4 3 1

4 4

s.

5 2 1 2

5 5


ONE CENT
UNITED STATES OF AMERICA
IN GOD WE TRUST
LIBERTY
1961

10 coins

10 ¢

Notes on Pupil Page 53

Tasks: *In parts* a *and* b, *pupils fill in the blank in front of each strip with a number which tells the size of the strip in terms of the indicated unit; then they color strips of the same size with the same color. In parts* c *through* g, *they write appropriate numbers in all the empty boxes and blanks. In the lower right-hand corner, they write how many coins and how many cents.*

Purpose: To extend the idea of "how much" associated with counting by 2's.

Unifying Ideas: Number and counting; measurement.

Materials for Teacher: The same strips of cardboard or tag prepared for the preceding lesson, or the equivalent; real or toy pennies and nickels.

Materials for Pupils: Individual strips or Cuisenaire® Rods; crayons.

The Lesson:

This lesson is like the one on page 52, except that a unit square represents 2 instead of 1 this time and a nickel has replaced five of the pennies in the money section. Your procedure should be altered slightly to take care of these added elements of difficulty.

Hold up one of your unit squares. Say, "Suppose that when I look at this, I think of 2. Hold up the thing you have on your desk which would make you think of 2 in the same way." Check to see that each pupil is holding up a unit square or rod. Then hold two of your unit squares in the same hand. Say, "If I think of 2 when I look at one of these, what number should I think of when I see two of them together?" Have the pupils represent 4 in similar fashion with their materials. Then hold up your single strip which is 2 units long. Ask the pupils what number it represents. Repeat this activity for all the even numbers 2-10, first in consecutive order and then in random order.

Conduct a similar activity showing the pupils the side of your strip which does not have the units marked on it. In each case, have the answers verified by finding out how many unit squares it takes to cover the unmarked strips and then counting these squares as 2's. Extend this activity by using a combination of strips to represent 6 or 8 or 10 instead of a single strip.

Using real coins, if possible, discuss how to identify a nickel and a penny (color, size, details of design), the amount each of these coins is worth, and the fact that a nickel may be exchanged for five pennies, and vice versa. Emphasize the two-way aspect of this exchange. Then have the pupils find the value of 1 nickel and 1 penny, 1 nickel and 2 pennies, and so on, hopefully coming out with the conclusion that, using only nickels or pennies, or both, there are three ways you can have 10 cents.

Assign page 53 for individual study. Pupils who wish to may connect dots to make squares available for counting as 2's or may use their representational materials.

Headline Story: Complete the headline and tell a story to fit:

$$6 + 2 = \square$$

a.

2 A

4 B

6 C

8 D

10 E

b.

6 C

8 D

2 A

10 E

4 B

Same size- same color (above)

Label boxes with no's — "□" suggests "2".

c.

4 2 2

4 4

d.

8 4 4

8 8

e.

6 4 2

6 6

f.

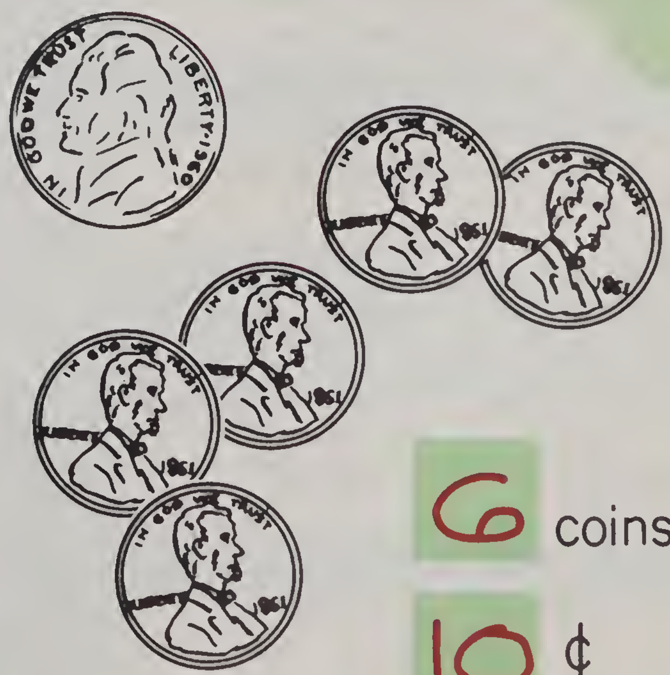
8 6 2

8 8

g.

10 4 2 4

10 10



6 coins

10 ¢

Tasks: *In parts* a *and* b, *pupils fill in the blank in front of each strip with the number which tells the size of the strip in terms of the indicated unit; then they color strips of the same size with the same color. In parts* c *through* g, *they write appropriate numbers in all the empty boxes and blanks. In the lower left-hand corner, they write how many coins and how many cents.*

Purpose: To extend the idea of "how much," using 3 as a measuring unit.

Unifying Ideas: Number and counting; measurement.

Materials for Teacher: Five small containers, all the same size and shape; rectangular strips cut to just cover 1 container, 2 containers . . . 5 containers; 15 counters; real or toy pennies and nickels; or the same materials used in the two preceding lessons.

Materials for Pupils: Strips of 1-5 units for which one unit is ½ inch; crayons.

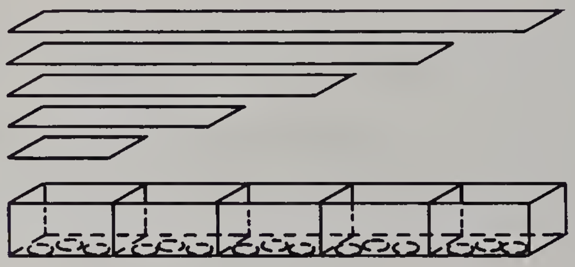
The Lesson:

This lesson looks very much like the two preceding lessons, but it has two significant elements of increased difficulty: (1) a unit square represents 3 on this page, and for some pupils it is much more of a challenge to count by 3's than by 1's or 2's; (2) on this page there are no dots or lines to show unit divisions on the longer strips.

One way to treat this lesson is to let those pupils who can do so complete the page with no specific introduction while you work with the other pupils on concepts they need help with.

Another way to treat this lesson is to conduct the same activities you used in the preceding lesson, but to specify that a unit square refers to 3 this time. If you do this, expect that almost all of your pupils will be able to follow the discussion involving the use of concrete materials, but not all of them will be able to complete the page in the book correctly without further help.

A third way to treat this lesson involves the materials listed above. While the pupils watch you, put three counters in each of your five containers. Tell the pupils you are going to leave the counters just like that all through the lesson. Count the counters by 3's. Then you can play a sort of game. Your rectangular strips are lids for 1-5 containers, all lined up touching one another as shown below:



Set out one container, cover it with your unit strip, and ask, "How many counters (use the name of whatever you used for counters) did I hide?" Ask the pupils to find the strips they have which look like a smaller version of the lid you just used. Repeat this activity using 2 containers, 3 containers, and so on up to 5—first in consecutive order and then in random order. Have the pupils turn to their books and show them that their strips just match those pictured on page 54. Let them use their strips in completing that part of the lesson.

Use real money, if possible, in assisting any pupils who may need help with the part of the lesson which deals with coins.

Headline Story: Complete headlines and tell stories to fit:

$3 + \triangle = 6$ $\square + 6 = 12$

a

3 A

6 B

9 C

12 D

15 E

b

9 C

12 D

3 A

15 E

6 B

Same size - same color (above)

Label boxes with no's — "□" suggests "3".

c

6 3 3

6 6

d

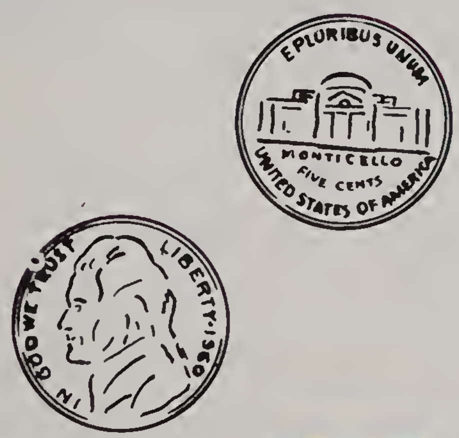
12 6 6

12 12

e

9 6 3

9 9



2 coins

10 ¢

f

15 15

15 12 3

g

15 9 6

15 15

Tasks: *Pupils finish drawing the lines to connect* a *and* b *with* x *and* y *in the lower left-hand box; they supply missing numbers in the frames and fill in missing* = *signs in the gray boxes.*

Purpose: To show the application of one-to-one correspondence in establishing the equivalence of sets and in counting.

Unifying Ideas: Sets; number and counting; functions and relations.

Materials for Teacher: Objects and pieces of string for use on the chalkboard or flannel board as described below.

Materials for Pupils: A random number of two different kinds of objects.

The Lesson:

However simple this lesson may appear, it is among the more important lessons your pupils will have. The concepts involved here are very significant.

One concept, exemplified by the material in the first box in each row, is that of two sets having the same number. *Note that it is not necessary to know what that number is.* How does one tell whether or not two sets have the same number of members? He matches the members (pairs them) and if this matching (or pairing) "comes out even," he knows that the number of the two sets is the same.

Pick up a handful of pencils, enough so that the pupils cannot see at a glance how many you are holding. Have an appropriate number of pupils come to the front of the room and ask, "How can we tell without counting whether I have enough pencils in my hand for each of these pupils to have a pencil?" Someone will surely say, "Pass them out and see." Do this, and emphasize the fact that if you have just enough to go around, each pupil with a pencil and each pencil with a pupil, there is *the same number* of pencils as pupils. Repeat this activity using a variety of other objects to answer questions such as, "Is there the same number of crayons in this box as erasers in this other box?"

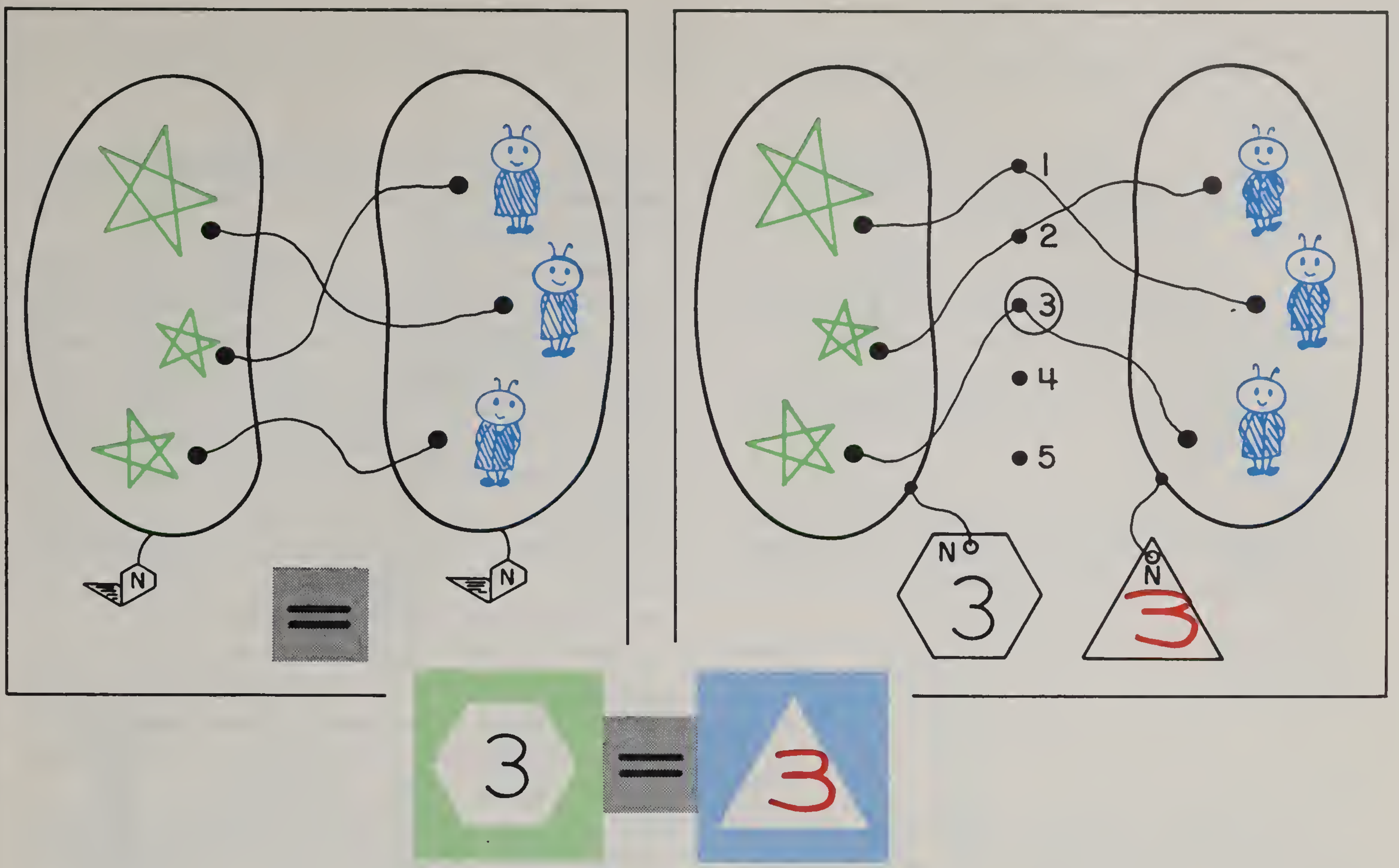
Have the pupils put their counters in two groups on their desks with the counters which are alike together. Ask them to show you, without counting, whether or not they have the same number of each of the two kinds of counters. Elicit that they do have the same number if each counter of one kind has a partner of the other kind.

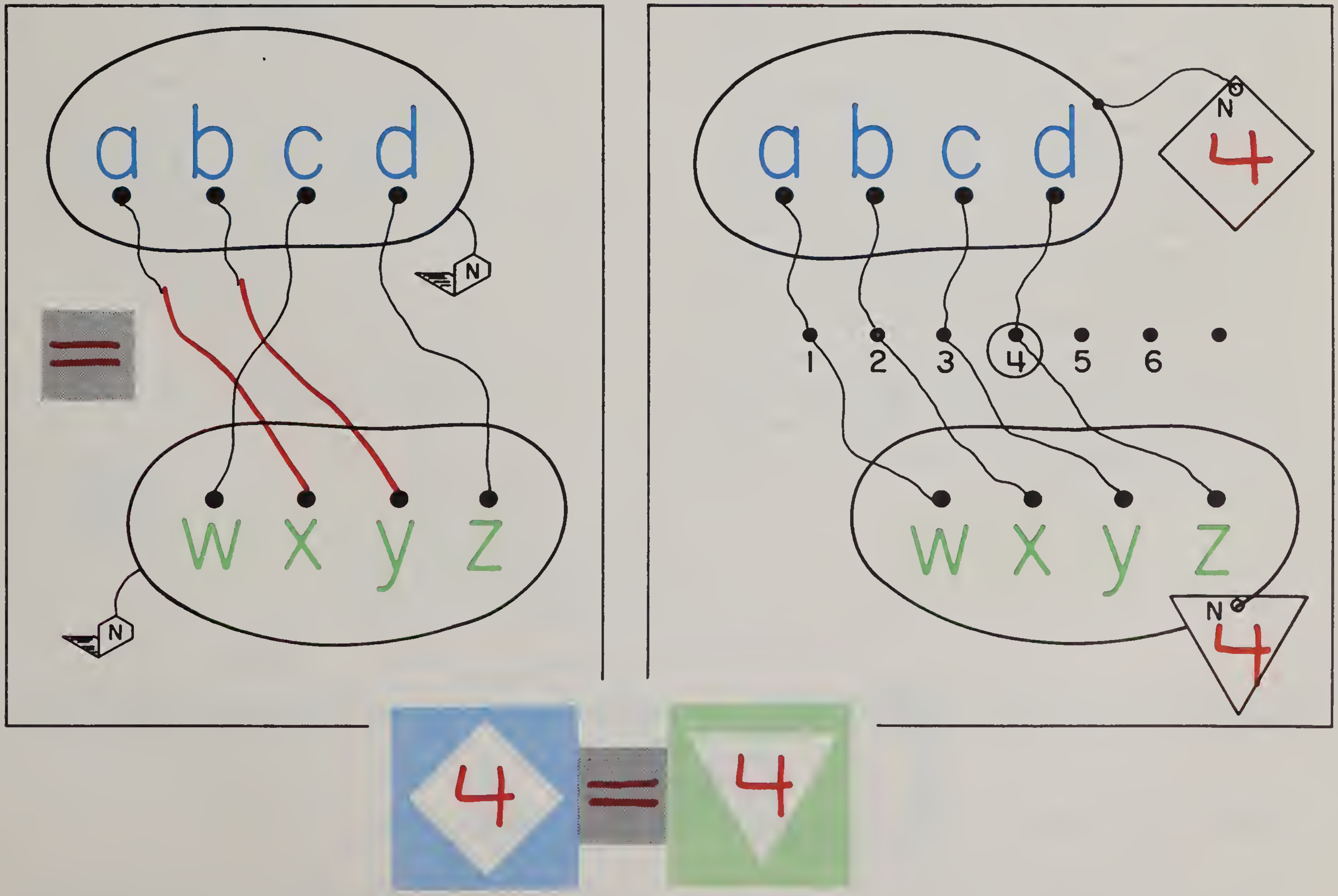
The other important concept involved in this lesson is that of pairing members of sets with subsets of the counting numbers. This is the rationale of counting.

Affix objects, with strings attached, to your chalkboard or flannel board and have the pupils connect each string to an appropriate point on a number line on the chalkrail or some other convenient place. Note emphatically that the *objects may be considered in any order* as long as each object is used exactly one time, but the *numbers must be used in consecutive order* beginning with 1.

Ask the pupils how page 55 in their books is like what they have just been doing. Elicit that the = sign, as used here, tells that two sets have the same number of members.

Headline Story: Mary and Bill each had the same number of books. Draw a sketch to fit.


N
N
=
N
N
=


a b c d
N
=
w x y z
N
N
4
a b c d
1 2 3 4 5 6
w x y z
N
4
4
=
4

***Tasks:** Pupils supply the missing numbers and fill in the one missing > sign.*

Purpose: To introduce the relation *is greater than* and the symbol used to express this relation.

Unifying Ideas: Sets; number and counting; functions and relations.

Materials for Teacher: Ten or more objects, with strings attached, that may be affixed to the chalkboard or flannel board; number tags.

Materials for Pupils: Number tags, 0-5; a piece of plain paper.

The Lesson:

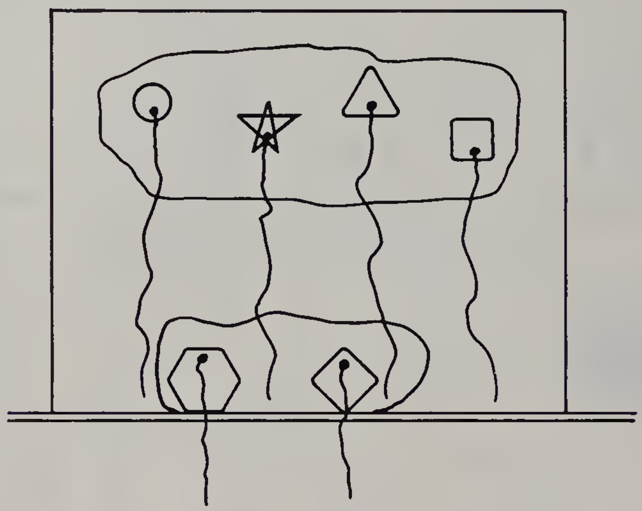
In this lesson, your pupils will learn a part of the rationale of establishing order relations among the cardinal numbers. If the phrase "is more than" makes more sense to them than the standard terminology associated with the > sign, allow them to use the nonstandard phrase for the time being. Note that the relation "is greater than" pertains to numbers, not sets, in this context.

On your chalkboard or flannel board, set up a display somewhat like the one in the next column. Ask the pupils how they could tie the strings so that one object which is "up in the air" is tied to one object which is "on the ground." They should see that they can fasten any one of the top objects to the first object in the bottom group, any one of the remaining objects in the top group to the other object at the bottom, and there will be a couple of strings still dangling. Ask them what the situation would be if the two sets had the same number of members. Ask how the dangling strings show which set has more members.

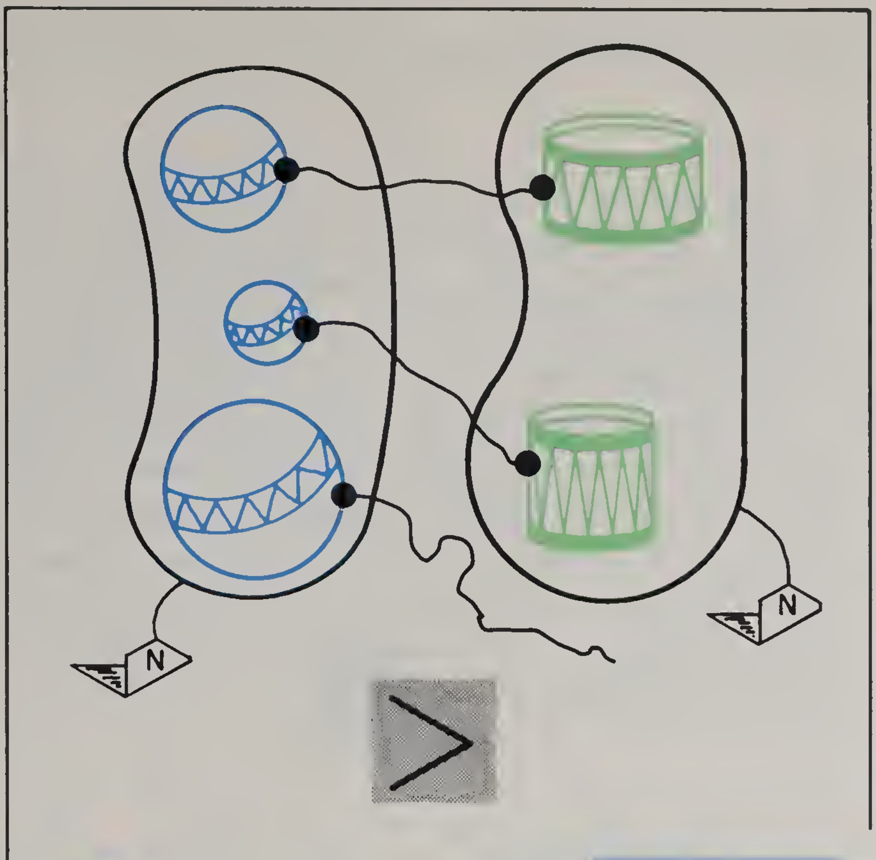
Have the pupils lay on their desks the number tags that go with the sets you have displayed. Tell them that the number of the set with more members is greater than the number of the other set. Show them our "shorthand" way of writing this: 4 > 2. Have several pupils read what you have written. Have them arrange their number tags properly on their piece of paper and write the sign between them. Repeat this activity using different numbers of objects. Be careful to arrange that the set you consider first has the larger number.

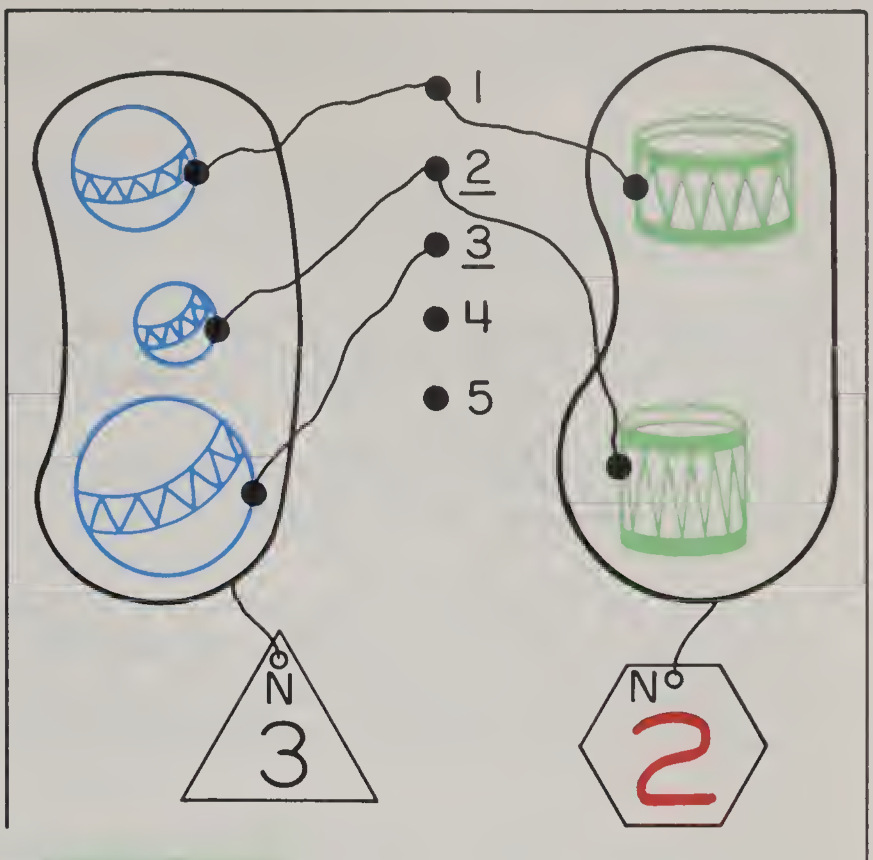
Mention other examples, such as: "Imagine that the ceiling is covered with balloons that float, each balloon with a string hanging down. How can we tell whether the number of balloons is greater than the number of pupils in the room?"

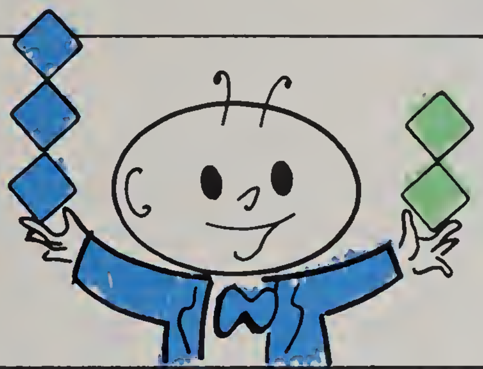
Have the pupils do page 56 in their books, then discuss it with them. Be sure each pupil learns how to write the > sign correctly and knows what it means.



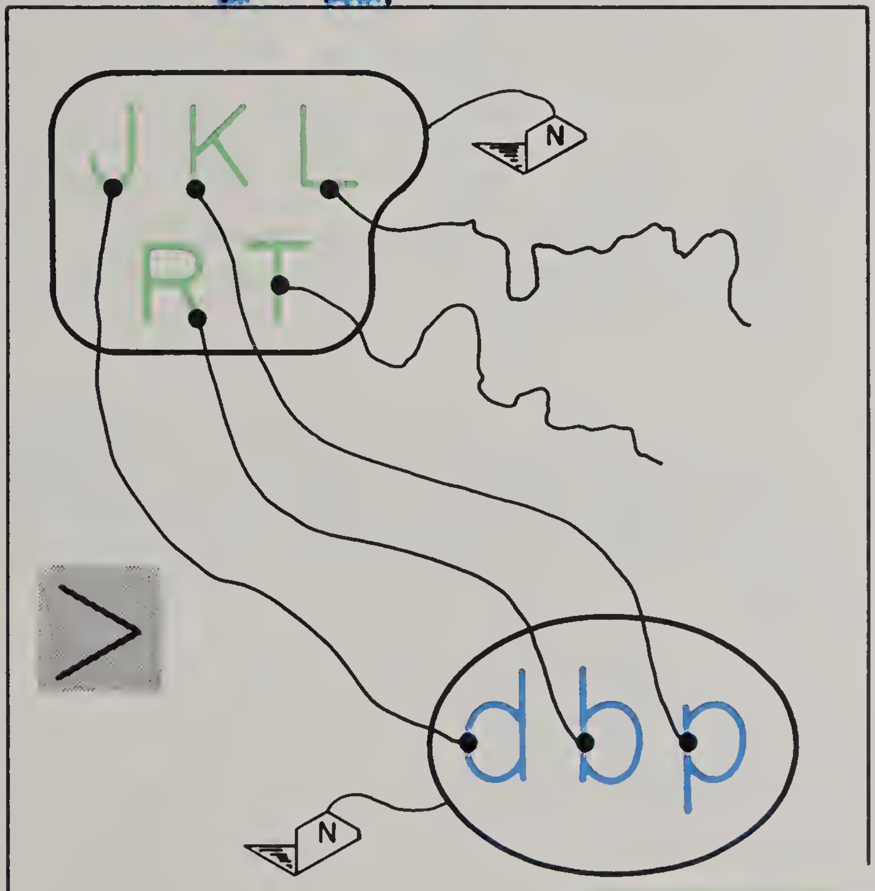
Headline Story: Harry had more blocks than John. Draw a sketch to fit.

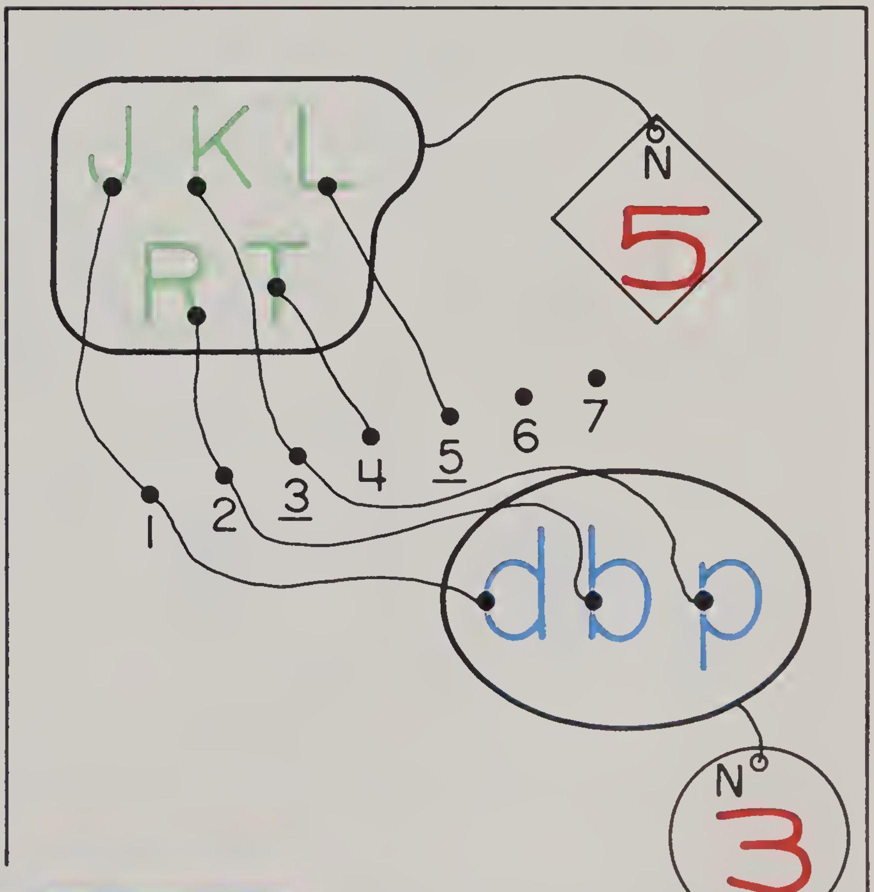






3 > 2


J K L
R T
N
d b p
N
>


J K L
R T
N
5
1
2
3
4
5
6
7
d b p
N
3

5 > 3

Notes on Pupil Page 57

Tasks: *Pupils supply the missing numbers and fill in the missing $<$ signs.*

Purpose: To introduce the relation *is less than* and the symbol used to express this relation.

Unifying Ideas: Sets; number and counting; functions and relations; measurement.

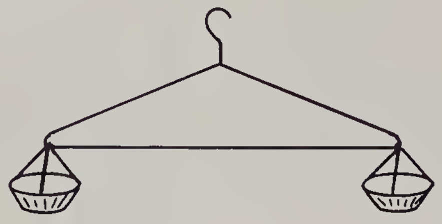
Materials for Teacher: A wire coat hanger, some string or wire, and two containers, all made into a simple balance, as described below; some uniform "weights," such as pennies or Cuisenaire® Rods.

Materials for Pupils: A piece of paper to write on.

The Lesson:

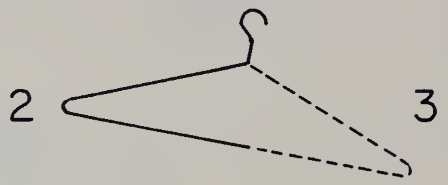
Some of your pupils may see at once that each "is greater than" sentence implies an "is less than" sentence. For instance, if $3 > 2$ then $2 < 3$. If the phrase "is smaller than" makes more sense to your pupils than the standard terminology associated with the $<$ sign, allow them to use the nonstandard phrase for the time being.

You can make a simple balance (like the scales of justice) by attaching two containers, such as the aluminum dishes frozen meat pies are sold in, by wires or strings to each end of a wire coat hanger as shown below. You need six pieces of string or wire (three for each side), each about 6 inches long.



If you rest the hook of the hanger on your index finger, the balance will stay level, or more appropriately, as you add equal weights to the dishes. Conduct all of your discussions on the basis of what the pupils see as they look from left to right.

Put two weights in the dish on the pupils' left and three in the dish on their right. Observe what happens and discuss it. Then show the pupils how to write in shorthand, "two is less than three," and have them write this on their papers. Notice that the contour of the hanger in the position it now has suggests the $<$ sign:

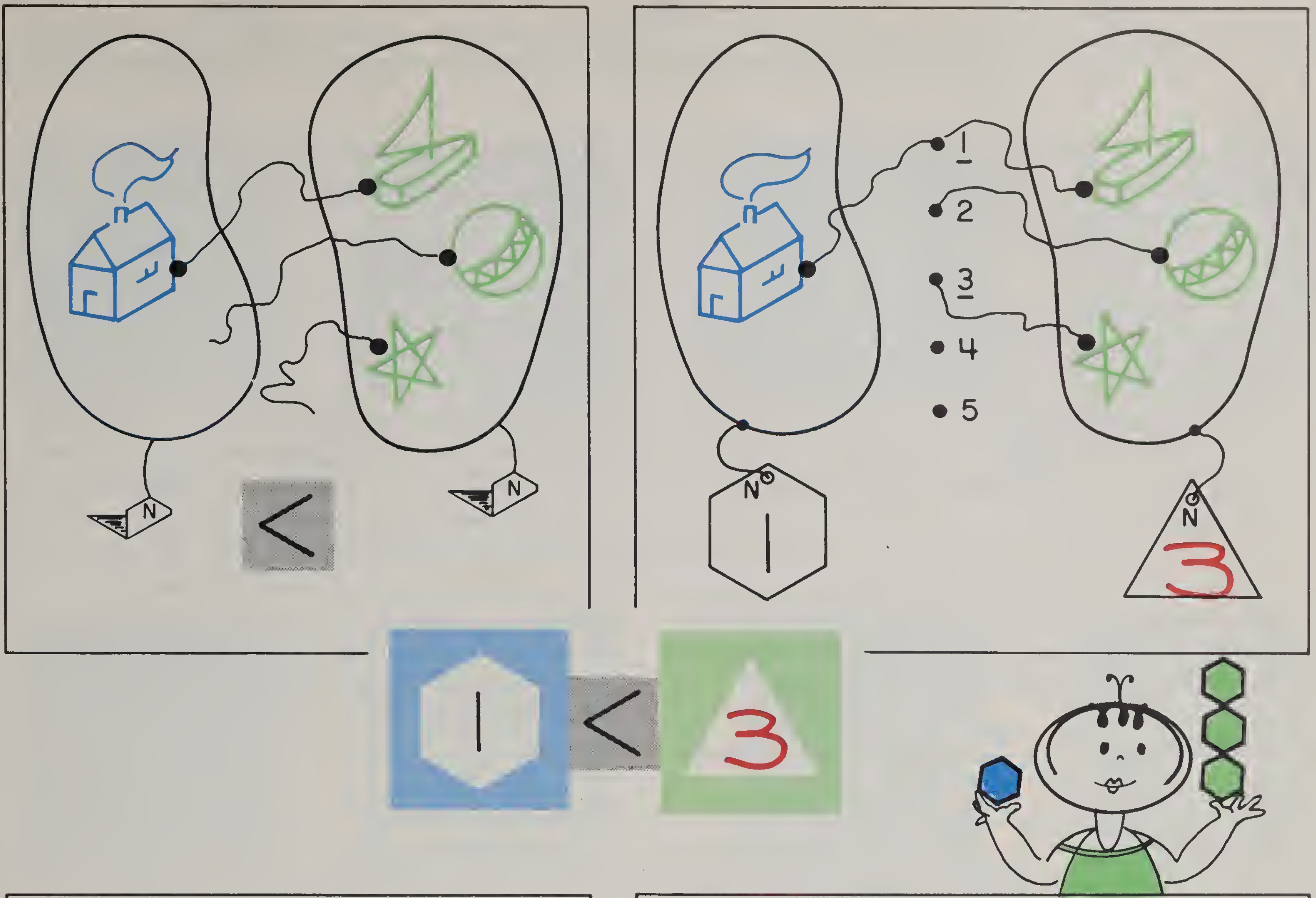


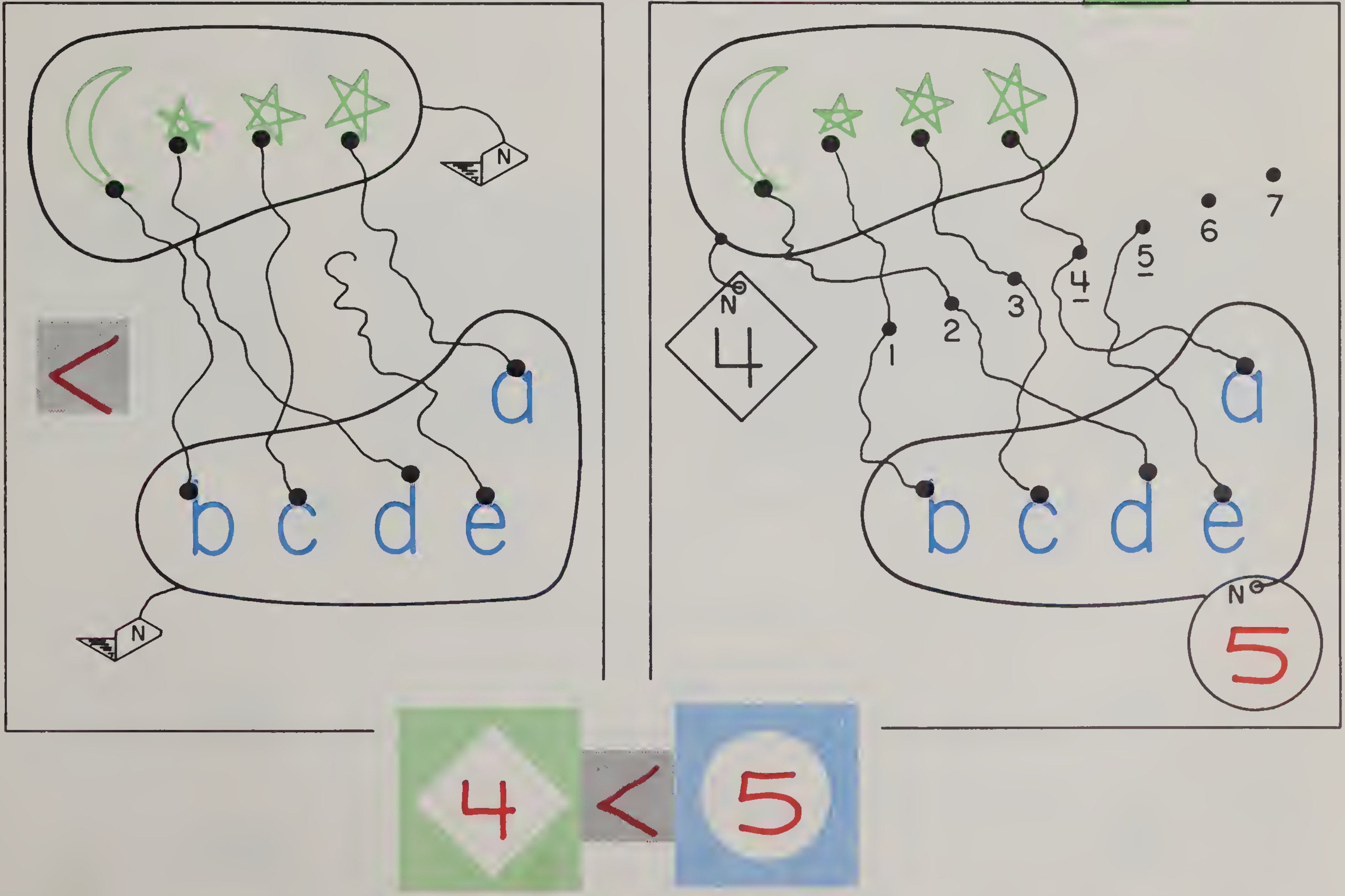
Repeat this activity several times, always putting the lesser weight in the dish on the pupils' left. Each time, have the pupils write the corresponding number sentence. When you are sure the pupils have the "is less than" relation clear in this situation, ask them what they think would happen if you added more weights to the heavier side. Then ask them what they think would happen if you added more weights to the lighter side, one weight at a time. Whatever they say, have them explain it, and demonstrate, if necessary.

Have the pupils do page 57 in their books, then discuss it with them.

Be sure to review the accompanying booklet, *Games and Enrichment Activities*, for suggestions you may wish to incorporate in your daily lesson plan.

Headline Story: There were fewer cups than saucers. Draw a sketch to fit.


N
<
N
1
2
3
4
5
N
1
N
3
1 < 3


N
<
a
b c d e
N
1
2
3
4
5
6
7
N
4
a
b c d e
N
5
4 < 5

Notes on Pupil Page 58

Tasks: *Pupils supply missing numbers and fill in the missing =, >, and < signs. In one section, they supply pictures of the members of a set whose number (8) is given.*

Purpose: To bring together as parts of one large concept the three relations, "is equal to," "is greater than," and "is less than" as these relations apply to cardinal numbers.

Unifying Ideas: Sets; number and counting; functions and relations.

Materials for Teacher: A seesaw balance, as described below; a corkboard or large piece of cardboard to which this seesaw balance may be fastened with a thumb tack; number cards.

Materials for Pupils: A piece of paper to write on.

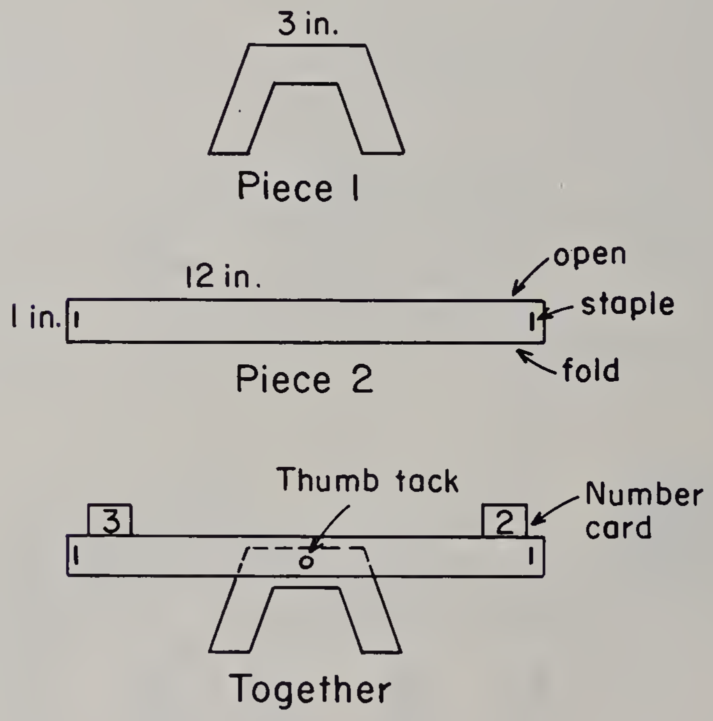
The Lesson:

Use construction paper to make the seesaw balance, following diagram in the next column. The suggested measurements are approximate, of course. The "plank" of the seesaw is made double and open at the top to hold the number cards.

This learning aid is another version of the same idea involved in the coat hanger balance, but this one requires more understanding on the part of the pupils because they have to tell which way the balance should move under given conditions without actually seeing it happen. Explain this to your pupils. Then put a number card in each side of the balance and ask who can fix the position of the seesaw to show which number is greater, or to show that they are the same number, if it happens that this is the case. Have each pupil write the appropriate number sentence on his paper. Check to see that he has used the proper sign and has written it correctly.

The observation that the slant of the plank on the seesaw suggests the way the bottom half of the applicable sign goes may help some of the pupils. Repeat this activity several times. You can make the examples as difficult or as easy as you please by choosing large or small numbers, or by using number cards with combinations such as 3 + 2 and 7 − 3 on them on one or both sides of the seesaw.

Assign page 58 for individual study. In the green box in the lower right-hand side of the page, the pupils should draw 8 pictures of some kind. One might expect them to write 8 letters, but 8 objects of any other kind constitute a correct response. Note that in the frame in the middle of the bottom row, 8 or any number larger than 8 makes a true sentence.



Headline Story: Tell a story that fits the headline:

7 > 4

1 < 2 3 > 2 2 = 2

A B C

D E F G

3 < 4

H M
I
J
K
N L

O P
Q R

7 > 4

S V
T W
U

X
Y
Z

5 > 3

MOTHER

FATHER

6 = 6

THE
BOY

RAN
HOME

6 < 7

A B C
D E F
G H

8 > 0

---

20 = 20

9 < 19

1 > 0

0 < 1

7 < 8
or more

21 > 12

Notes on Pupil Page 59

Tasks: *Pupils supply missing numbers on the number lines and in the frames; they fill in missing relation signs; in the picture which is not colored, they color each length a different color.*

Purpose: To show the "is greater than" and "is less than" relations in terms of numbers associated with lengths.

Unifying Ideas: Number and counting; measurement.

Materials for Teacher: An opaque projector, or *copies* on the chalkboard of the first two drawings on this page.

Materials for Pupils: Crayons.

The Lesson:

Tear page 59 from a pupil book and cover the bottom part with blank paper so that only the top drawing can be projected on the chalkboard (or copy this drawing on the chalkboard if a projector is not available).

Ask the pupils what they think this drawing shows. In mature language, the drawing shows the measure of each of the strips in terms of the indicated unit. Your pupils cannot be expected to speak in those terms. Let them express their ideas in their own way as long as they see that as the length of a strip increases, the number associated with that length also increases.

Have selected pupils write the missing numbers associated with points on the number line, and be sure that everyone sees that each of those numbers is also associated with the length of a particular strip.

Then develop the idea that, of two numbers on the number line, the one on the left is less than the one on the right. You might proceed somewhat as follows: "Which of the two green strips is longer? What number goes with the shorter green strip? (Write a 1.) What number goes with the longer green strip? (Leave a space and write a 4 to the right of the 1). Is 1 less than 4? (Insert the relation sign.) Where is 1 on the number line? Where is 4? Is each of the numbers which comes before 4 on the number line also less than 4? (Test them one at a time, if necessary.) Which of the gray strips is the shorter? Which of the blue strips is the shorter?

Having done this, choose pairs of numbers the larger of which is not shown in your drawing, such as 4 and 9, write them on the board with a space between them, and select pupils to fill in the missing relation signs.

Conduct an analogous discussion of the left-hand drawing near the middle of the page, but this time develop the idea that, of two numbers on the number line, the one on the right is the greater.

Have the pupils turn to page 59 in their books. Explain to them what they are to do. Notice that any one of the numbers 0, 1, 2, 3, and 4 constitutes a correct response to the exercise in the middle of the bottom row.

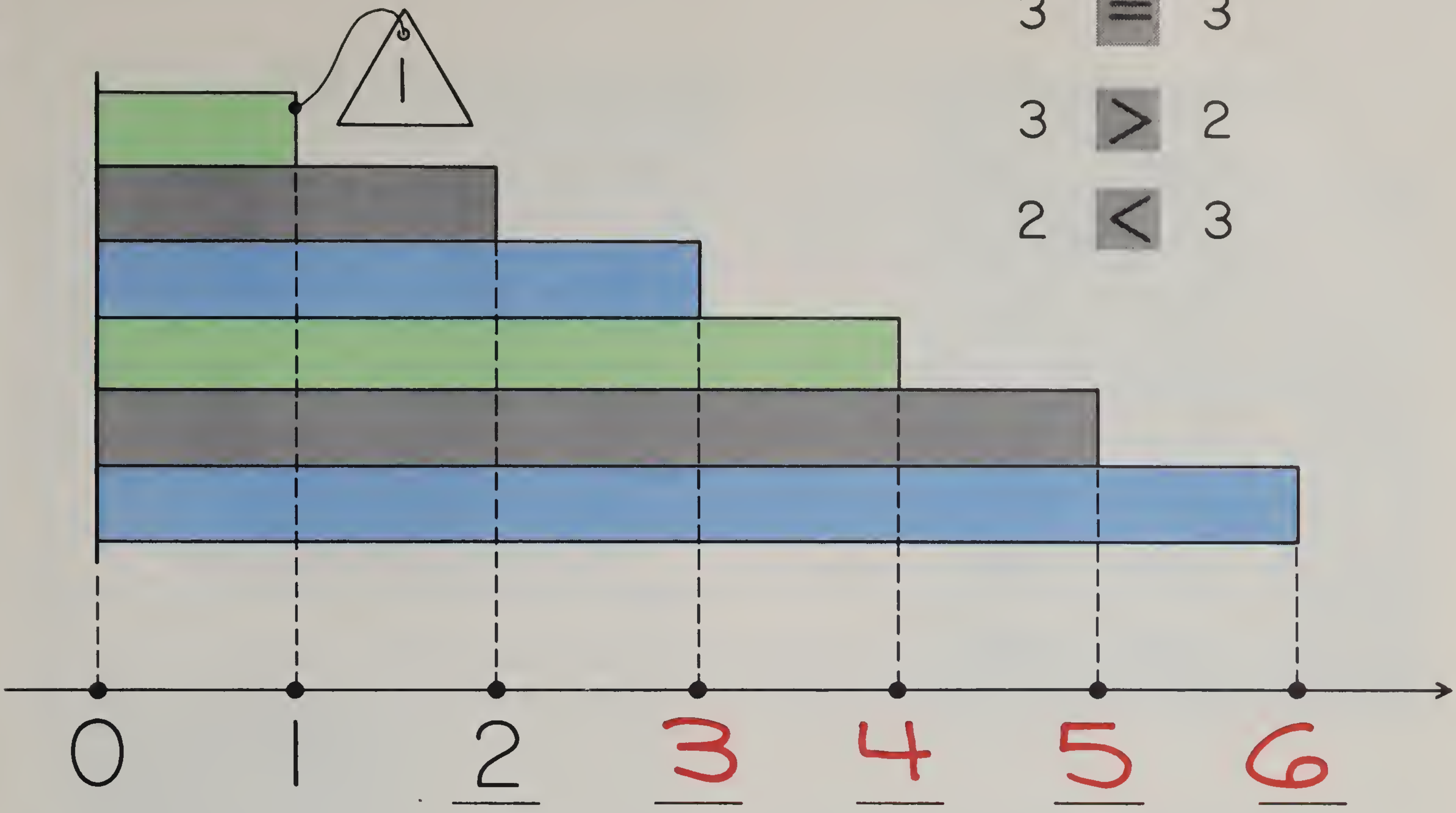
Headline Story: Tell a story that fits the headline:

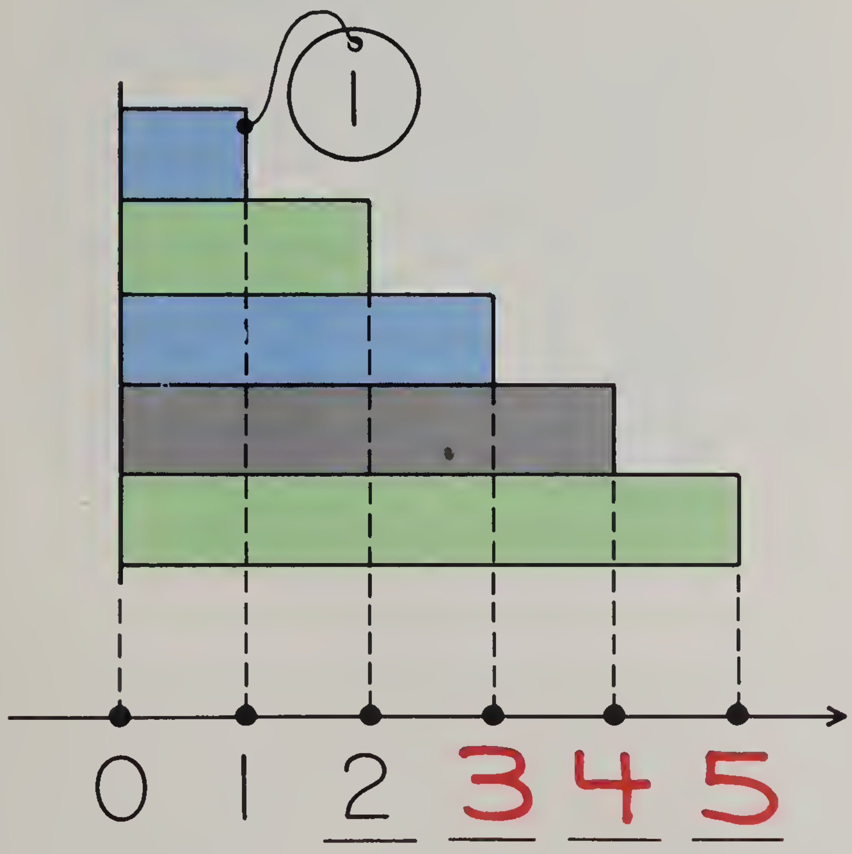
$6 < \square$

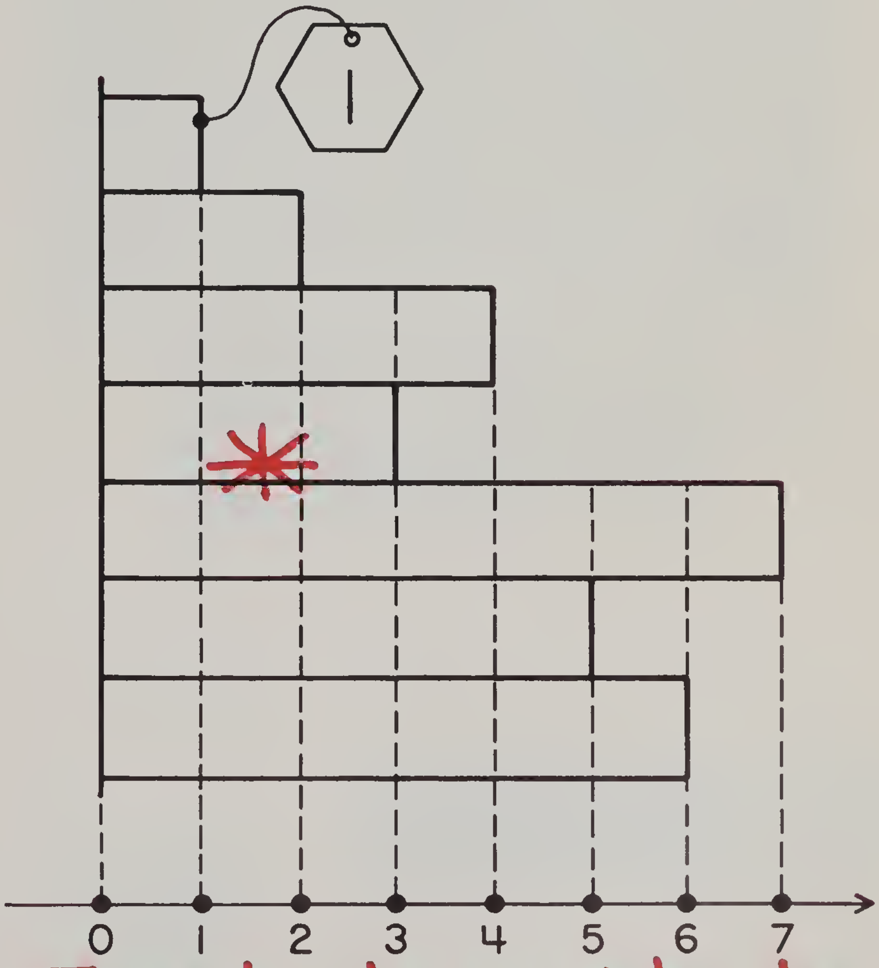
3 = 3

3 > 2

2 < 3

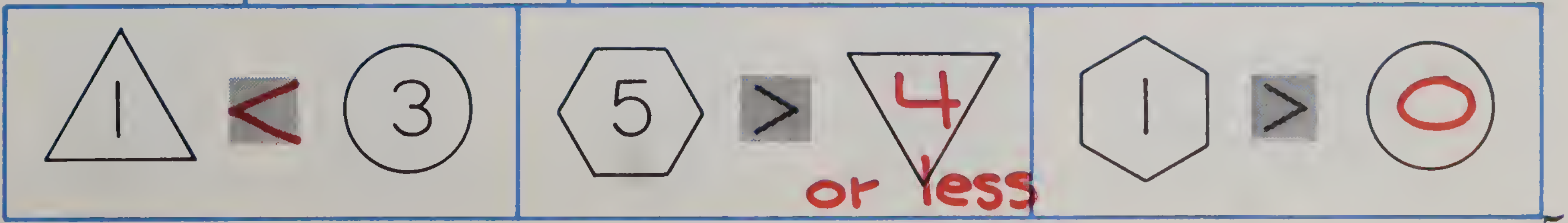






* Each length to be a different color.

4 > 2


1
<
3
5
>
4
or less
1
>
0

Notes on Pupil Page 60

Tasks: *Pupils supply numbers to complete the addition sentences in the boxes and draw appropriate pictures in the blank domino to show 4 objects in each part.*

Purpose: (a) To check up on each pupil's ability to associate a number with a set and a set with a number; (b) to assess mastery of addition facts.

Unifying Ideas: Sets; addition and subtraction.

Materials for Teacher: No special materials are needed.

Materials for Pupils: Counters or rods on hand for use if needed.

The Lesson:

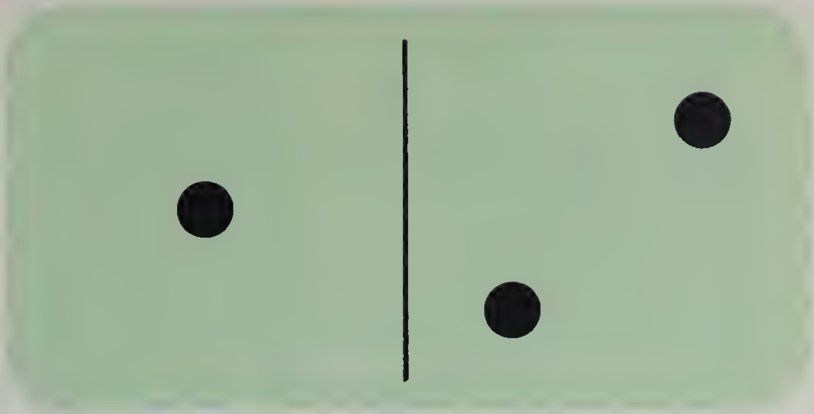
Give just enough introduction to this lesson so that pupils know what to do, then assign page 60 for individual study. Look for any pupils who appear to be having no success with the part of the page which furnishes no picture clues to the number facts. Make a note to yourself about which pupils are in this category. Allow them to use counters or rods to help them find the answers. Accept as correct for the blank domino any pattern of 4 dots or 4 objects of any kind on each half. Furthermore, the objects pictured do not necessarily have to be alike.

It is wise to keep individual records of pupil performances on lessons of this kind. Sometimes a brief interview will reveal the source of a difficulty. Note those who can find answers correctly when allowed to use concrete materials but who are insecure without them. Note those who appear to have mastered addition facts with sums of 6 or less, but who have trouble with larger sums. Note those who do better with facts written horizontally than those written vertically.

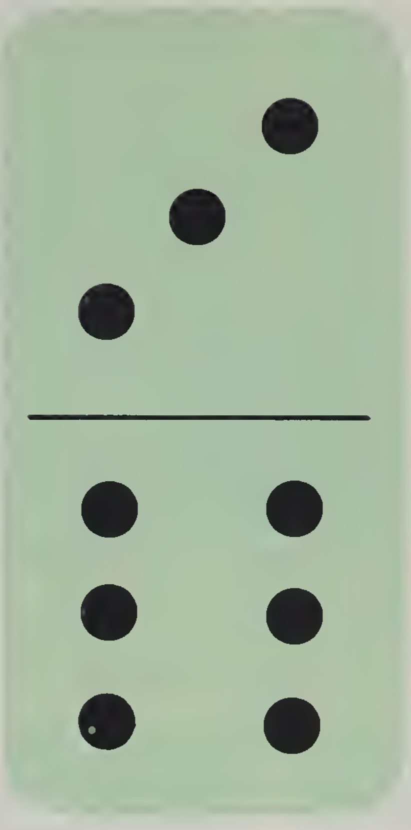
After checking the page, ask your pupils how one might find the answers for the top part of the page. Then have the pupils study the examples at the bottom of the page. What patterns do they notice? Do they see $2+0$, $2+1$, $2+2$, $2+3$ . . .? Do they see a series of examples in which the first number is 2 and the second number gets 1 larger each time? Does this give them an idea of how their answers should go? Does any pupil have answers which do not go like that? Can he find his mistake? Why is the answer to the very last example not 1 more than 7? Can anyone supply the two examples which have 2 as a first addend and 8 and 9, respectively, as sums? What example with 2 as a first addend has 10 as its sum? Does $2+8=5+5$? What other combinations of numbers have the same sum as $5+5$? Given that $2+2=4$, how much is $4+4$? Is $4+3$ one less than 8? Is $4+5$ one more than 8? Pupils who develop the ability to relate addition facts in this way have a useful tool at their disposal.

Headline Story: Tell stories that fit the headlines:

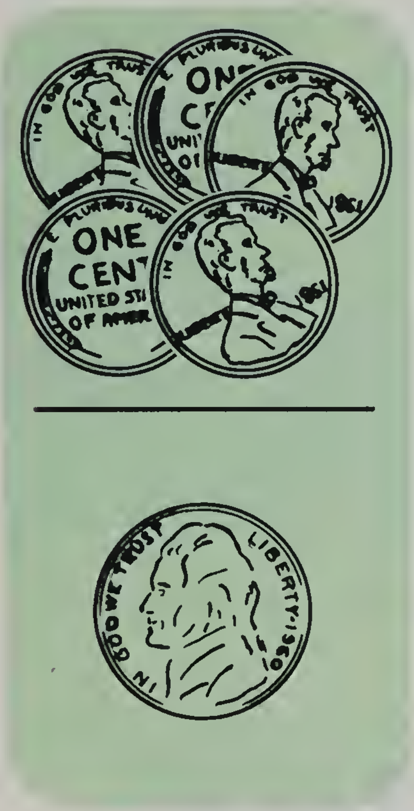
$\square + 5 = 5$ $\qquad$ $10 < 15$



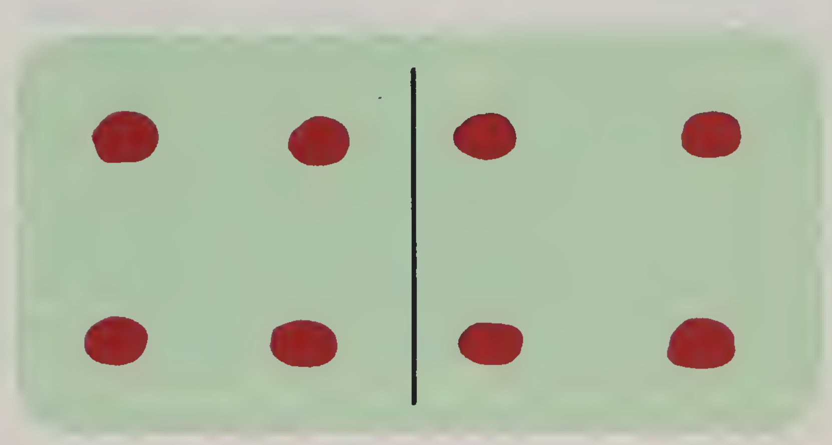
1 + 2 = 3



3
+ 6
9



5
+ 5
10



4 + 4 = 8

2
+ 0
2

2
+ 1
3

2
+ 2
4

2
+ 3
5

2 + 4 = 6

2 + 5 = 7

5 + 5 = 10

Notes on Pupil Page 61

Tasks: *In the upper half of the page, pupils write a number in each blue space to tell how many objects are pictured in both green boxes together. At the bottom of the page, they write how many green triangles, how many white triangles, and how many triangles there are in all.*

Purpose: To test each pupil's ability to count or add to find how many in all.

Unifying Ideas: Number and counting; addition and subtraction.

Materials for Teacher: No special materials are needed.

Materials for Pupils: No special materials needed.

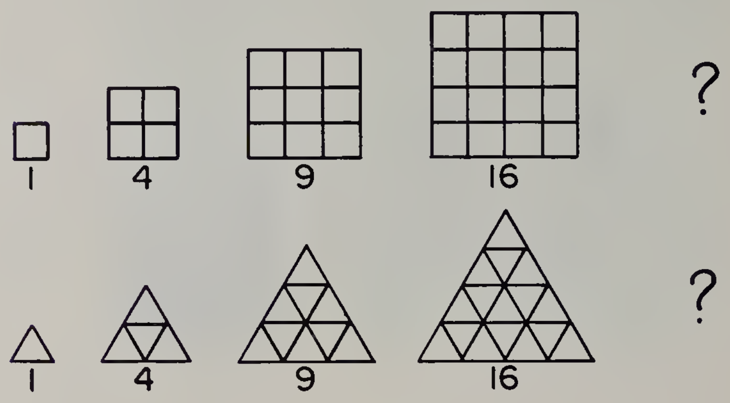
The Lesson:

Have the pupils turn to page 61 in their books. Have everyone look at the first item at the top of the page. Ask why a 3 is written in the blue space. As soon as everyone understands this, direct attention to the first example at the bottom of the page. Ask what the 1 in the blue space refers to. Ask what the 0 refers to. Ask what the 1 in the green space tells. The 1 and the 0 in the blue space tell you that there is 1 *large* green triangle and 0 *large* white triangles in the array pictured just above the triangle markers. The 1 in the green space tells you the total number of large triangles in the array. Have someone explain the 3 and the 1 in the next blue space. Then assign the page for individual study.

Observe the pupils as they work. See if you can tell who is counting laboriously, one object at a time, and getting correct answers, who is counting by groups, and who is adding to find how many in all.

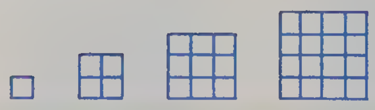
After you have checked the papers, work individually with any pupils who had more than one incorrect response. You may find some cases where mistakes were due largely to carelessness. Carelessness is a habit which is difficult to break, but it is less serious at this stage than conceptual deficiencies. You may find some pupils whose attention span is too short for them to be able to count accurately beyond a certain number. They will have to deal with smaller numbers than others in the class until they mature a little more. You may find some pupils who lack the visual discrimination to deal with drawings like those at the bottom of page 61. Work with concrete materials with those pupils, and if they can count up to 16 correctly with those materials, they are not having conceptual difficulties with the numbers.

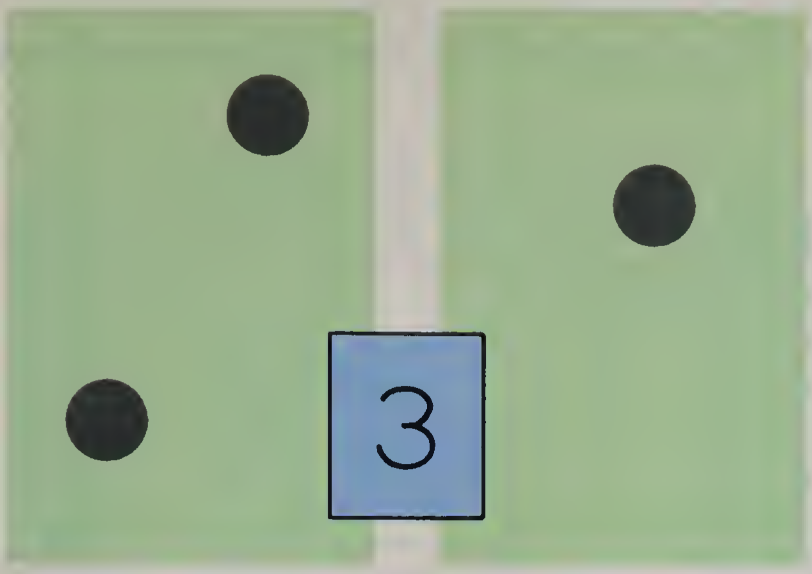
Give 25 square counters to those pupils who had perfect responses on this page. Ask them to use their counters to build "houses" which have the same shape as a single counter. They can do this by building a house out of 4 counters, 9 counters, 16 counters, and 25 counters.

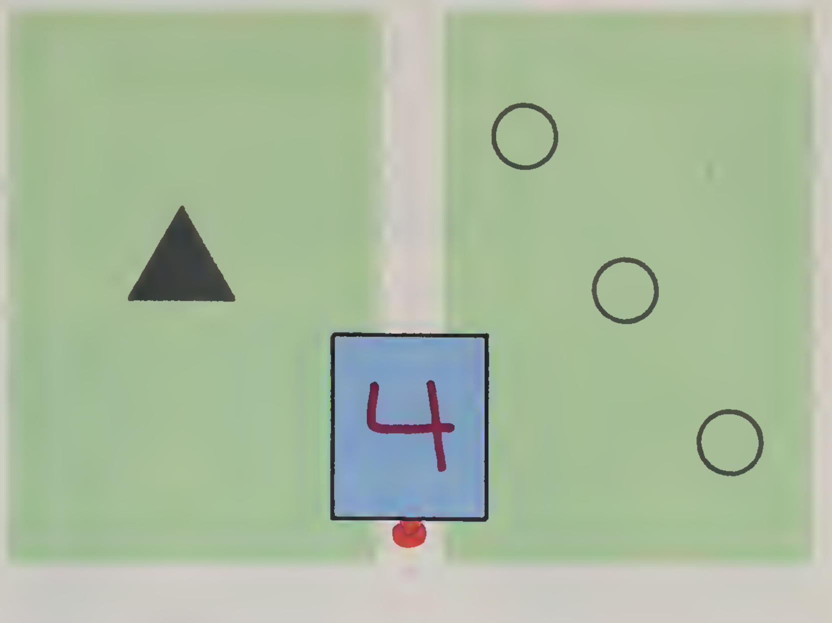


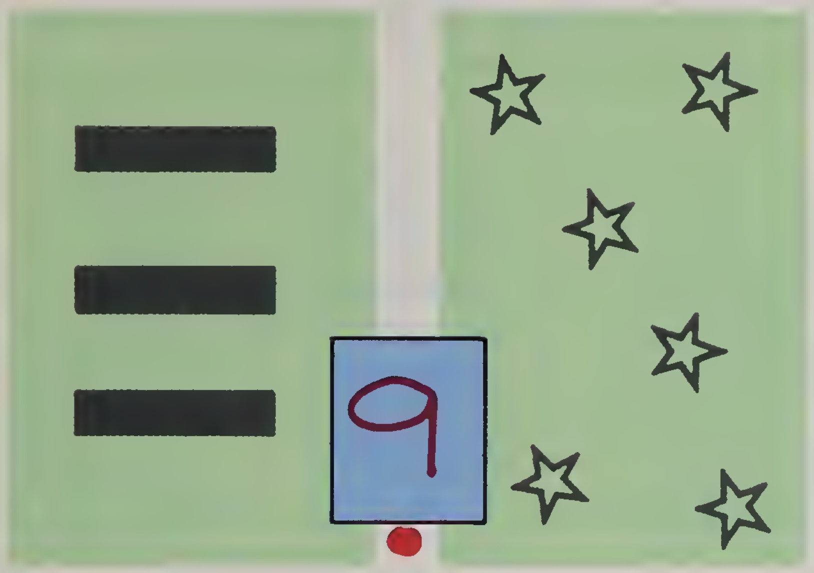
Perhaps some pupils will note the relationship between the square patterns obtained by using squares and the triangular patterns on the bottom of page 61. What would the next construction be?

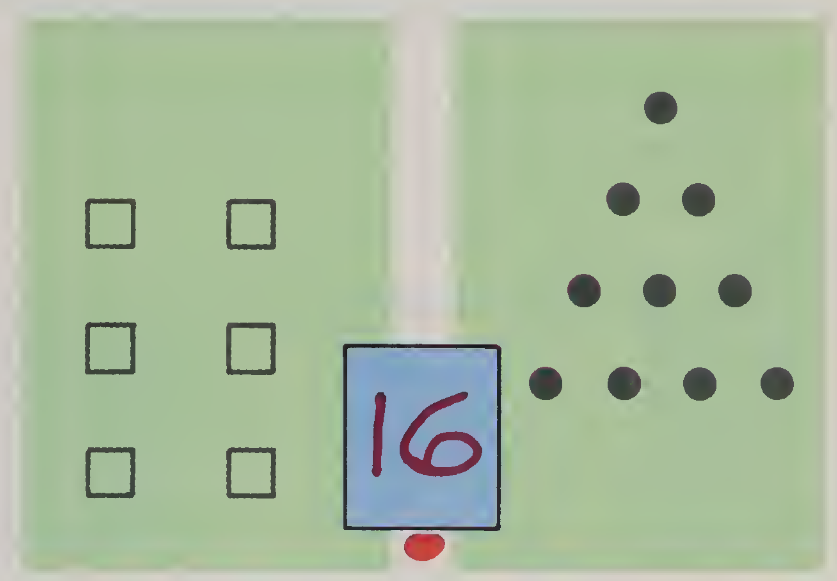
Headline Story: What sketch comes next in this series?

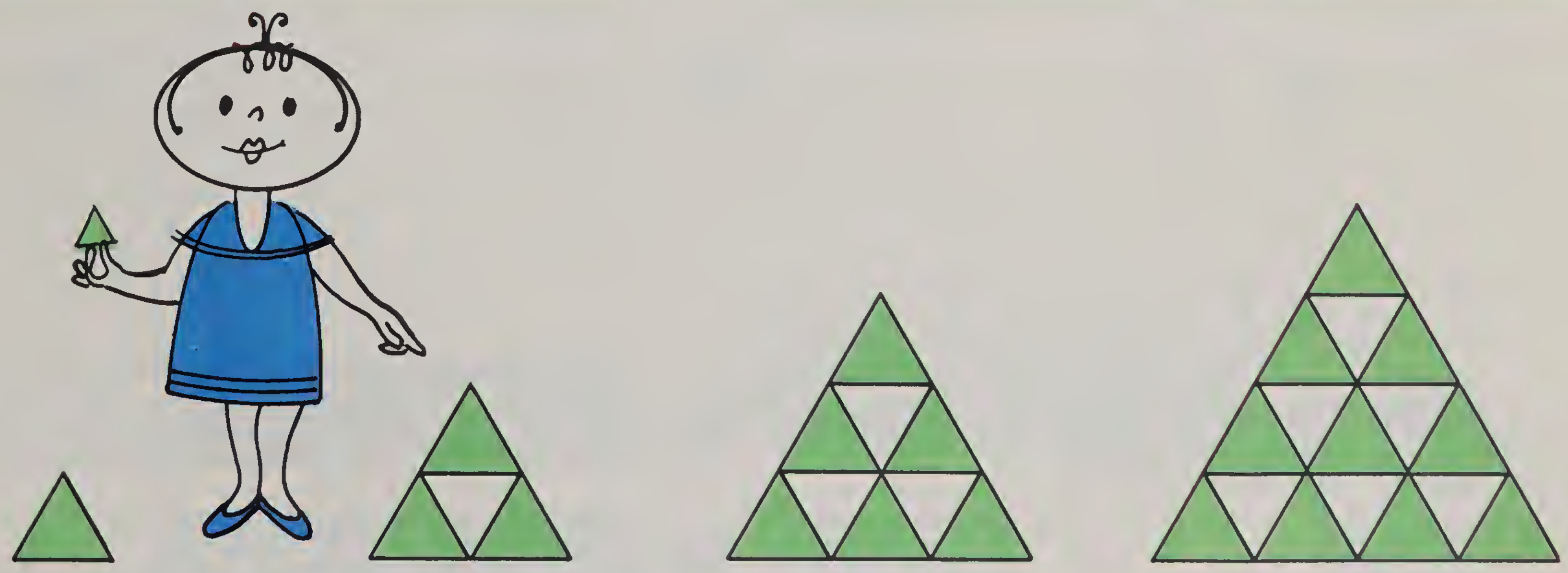












▲	▽
1	0
1	

▲	▽
3	1
4	

▲	▽
6	3
9	

▲	▽
10	6
16	

Notes on Pupil Page 62

Tasks: *Pupils fill in missing relation signs in the gray squares and supply the missing numbers in the frames.*

Purpose: To further develop pupil understanding of equality and inequality in terms of numbers associated with measurement.

Unifying Ideas: Measurement; functions and relations.

Materials for Teacher: Demonstration rods, or cardboard models of polygonal shapes marked off into unit areas like those pictured on page 62.

Materials for Pupils: Individual rods, or square counters.

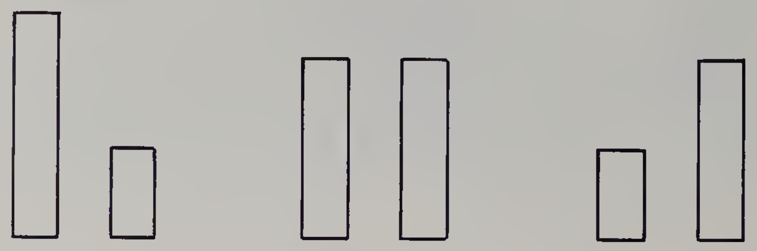
The Lesson:

Put on the board something like the following:

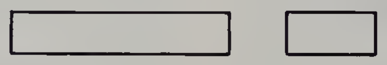
1 < 2	13 = 13	7 > 4
2 3	7 0	□ > △
5 1	2 4	◇ < ○
6 6	0 1	□ = □
3 8	9 9	△ < ⬡

Have the pupils fill in the missing relation signs in the first two columns. Have them write appropriate numbers in the frames in the last column. They should notice that, in this last column, there is an endless list of correct answers.

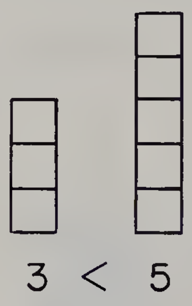
Use the flannel board, the chalkrail, or a table top (depending upon the kind of materials you are using for demonstration) to display pairs of lengths, one pair at a time.



Each time, ask the pupils what sign they would use to express the relation between the lengths of the two objects you used. Then conduct a similar activity displaying the lengths horizontally



Have each pupil use his counters as bricks to build two small "houses" side by side on his piece of paper. Ask him to count the number of bricks he used to build each house and to write the number beneath it. Then have him write the appropriate relation signs between the numbers.

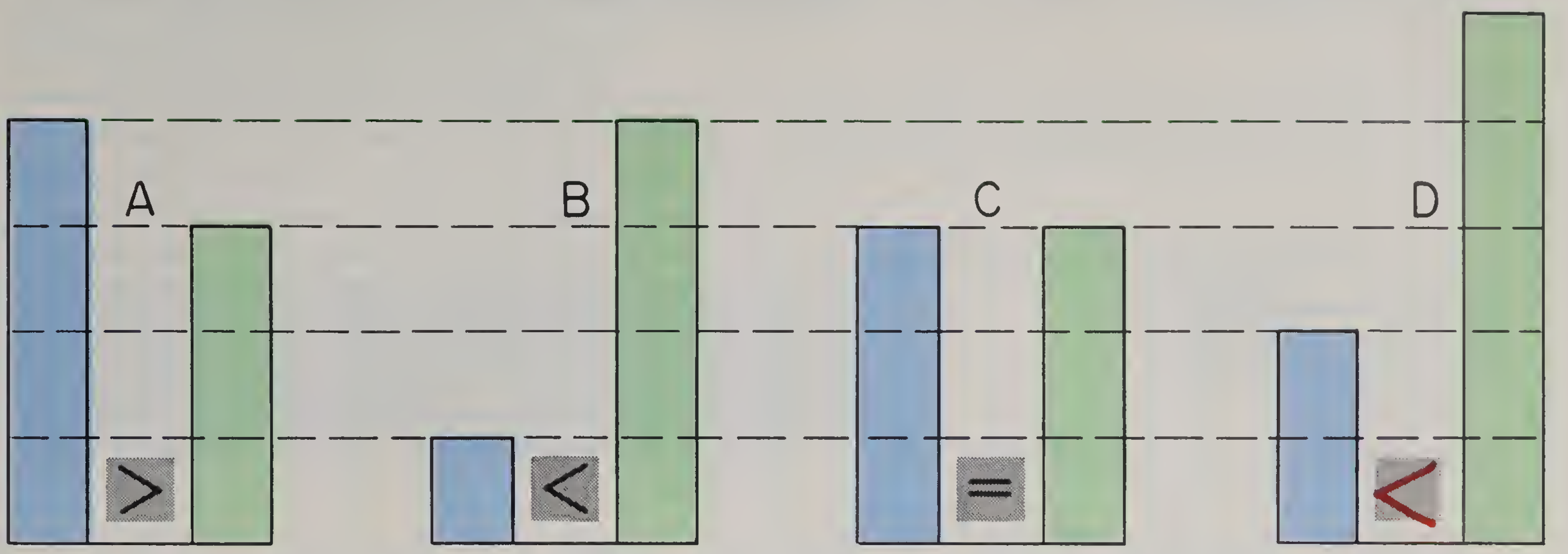


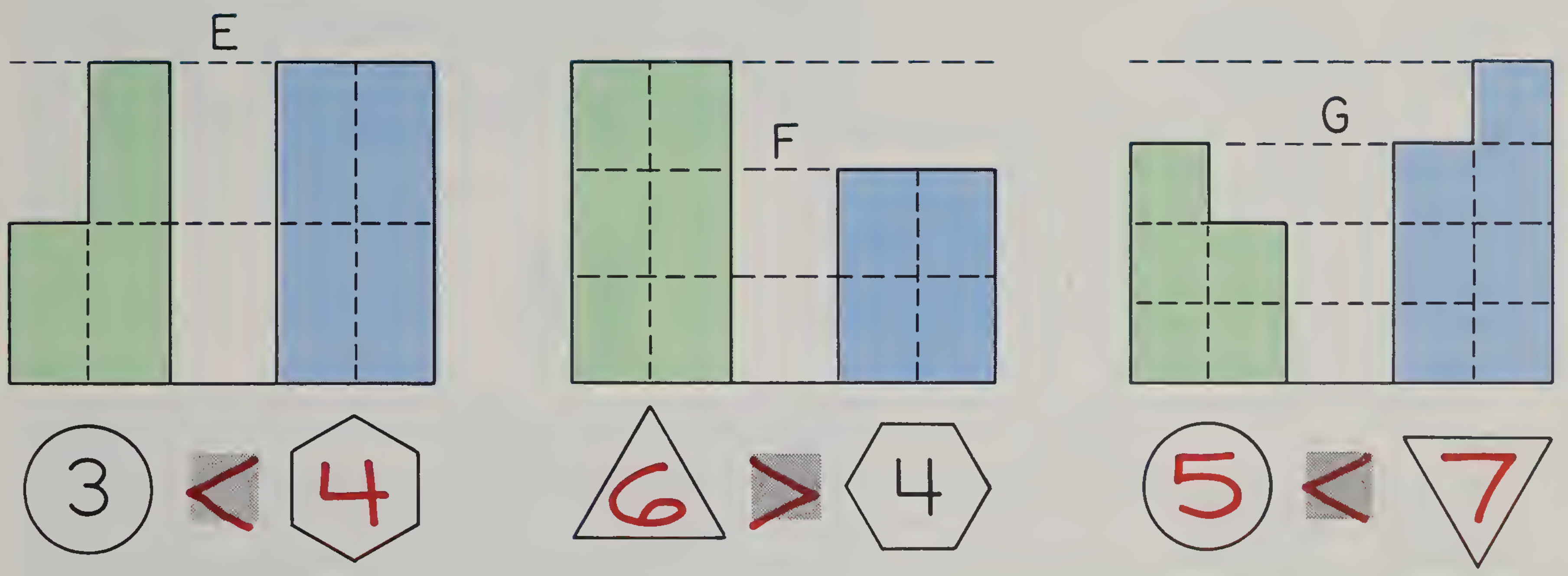
Vary this last activity by specifying one of the three relation signs and having the pupils build houses so that their brick counts will fit the sign.

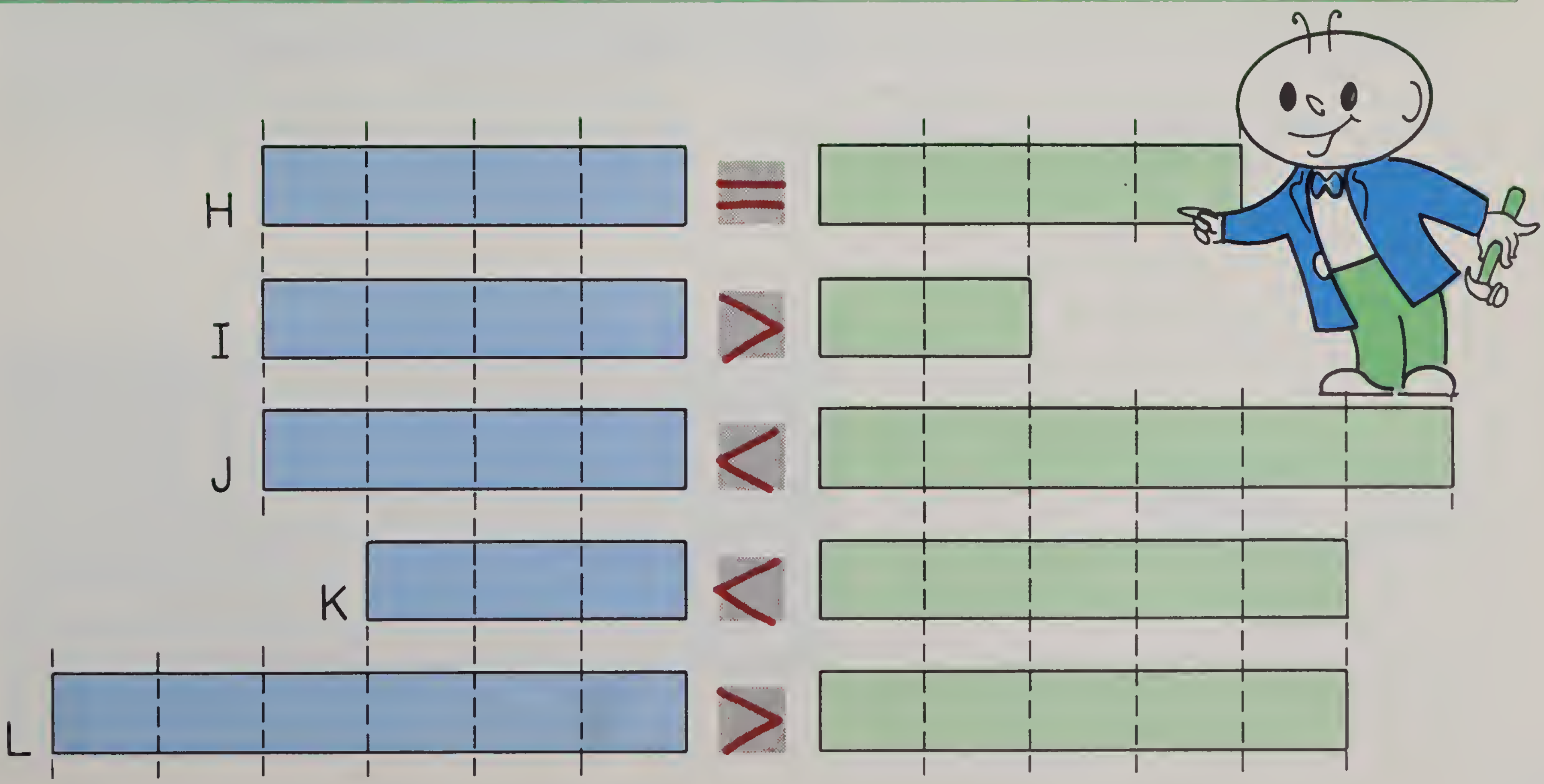
Headline Story: Tell stories to fit these headlines:

14 < 16   1 > □   △ > 9

4 = 4 4 < 5 5 > 4







Notes on Pupil Page 63

Tasks: *Pupils tell how many blue squares, how many green squares, and how many squares there are in all.*

Purpose: To strengthen pupil understanding of the decimal system of numeration.

Unifying Ideas: Numeration.

Materials for Teacher: Colored chalk.

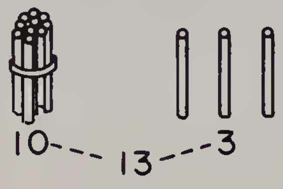
Materials for Pupils: At least 50 swab sticks, tickets, or other items easy to bundle into sets of 10 each; rubber bands.

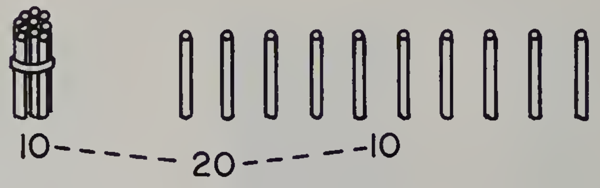
The Lesson:

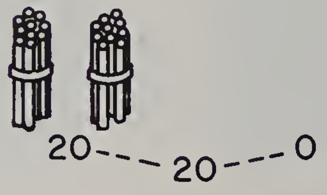
Young children need a great many experiences in seeing and handling relatively large numbers of objects so that they will get accurate ideas of the relative sizes of numbers and so that they will see the need for some kind of organization in counting. In activities such as those described below, your pupils may work in teams of two to four rather than individually, if you wish.

Choosing numbers less than 10, have your pupils represent a variety of numbers with their sticks (or tickets, or other materials). Check on the work the pupils are doing, and note which pupils are starting to count from 1 each time and which are making appropriate alterations in the representation they just had. Then ask them to take 10 sticks and bundle them together with a rubber band. Tell them to remember that they have 10 sticks in this bundle; they can think of it as one (bundle of) ten.

Ask the pupils to represent one of the teen numbers, say 14, with their sticks. Let them do this in their own way at first. Those who did not use their bundle of 10 and 4 more should be encouraged to find a shortcut. Keep a record, as suggested below, using different colors of chalk for the tens and ones shown by the top row of figures. Repeat this activity with other numbers between 10 and 20. Then ask for a representation of 20. Do the pupils see that they could use two bundles of 10 for this?



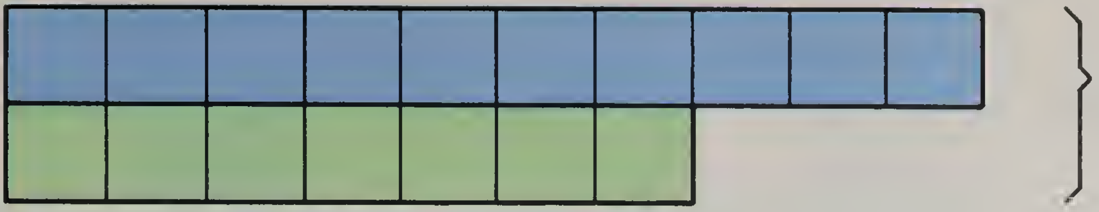




Extend this activity to larger numbers. Remind the pupils that "40" means 4 tens, "30" means 3 tens, and so on. Have them count by tens, saying, "One ten, ten; two tens, twenty; three tens, thirty; . . . ten tens, one hundred."

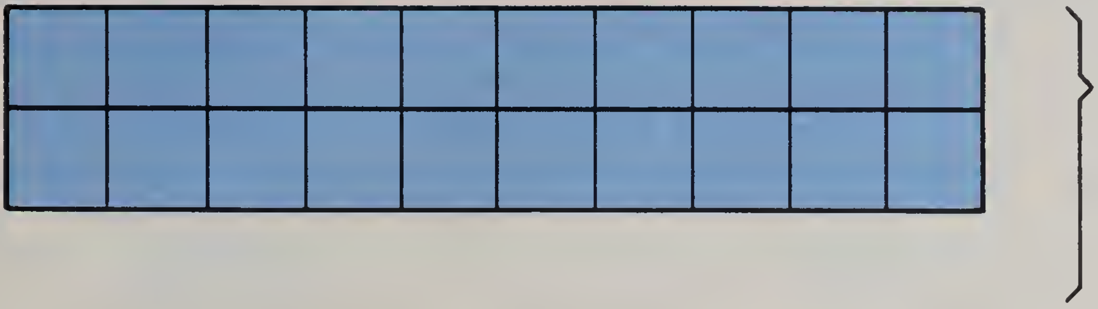
In doing the work on page 63, the pupils should notice that each blue strip plays the same role as a bundle of 10 sticks.

Headline Story: Carl had 3 dimes and ____ nickels. He had ____¢. (Make a chart.)



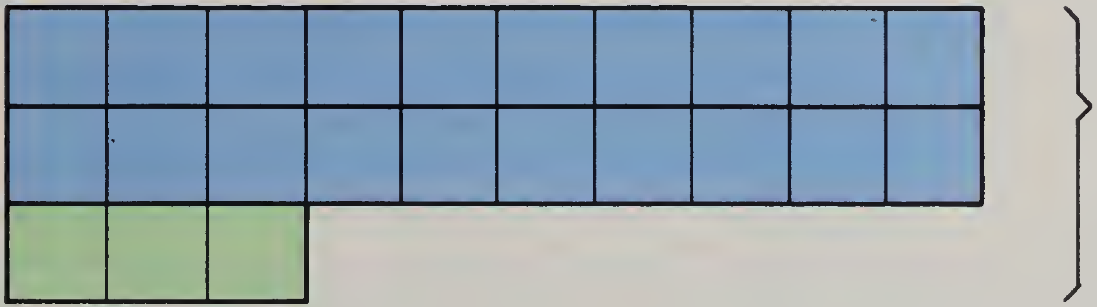
10 7

17



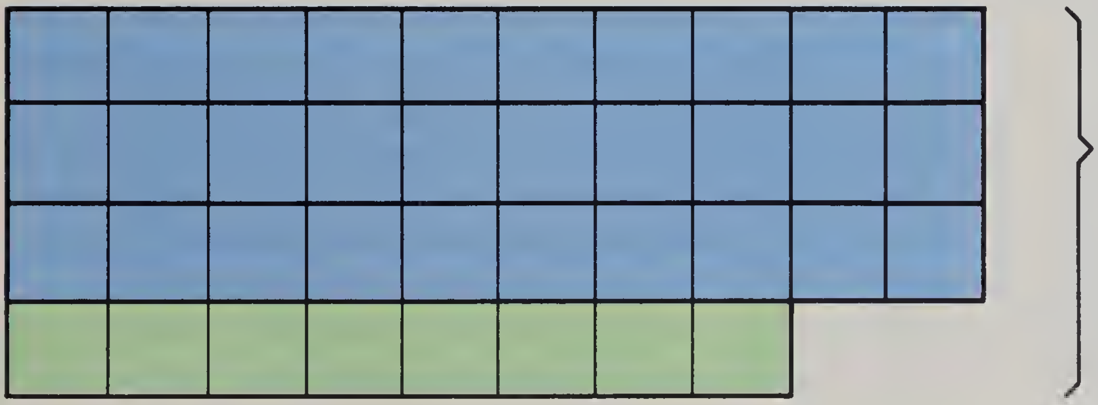
20 0

20



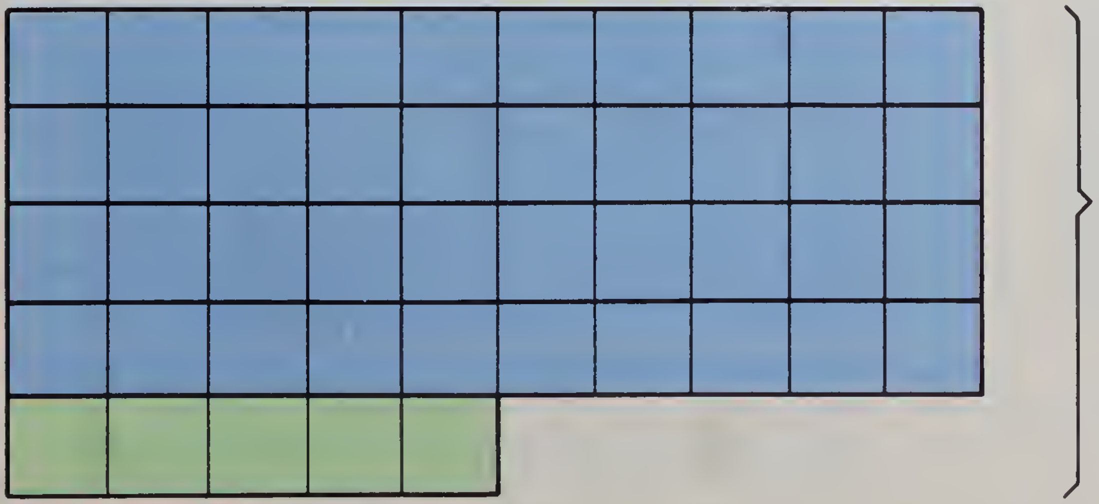
20 3

23



30 8

38



40 5

45

Notes on Pupil Page 64

Tasks: *Pupils write numbers in the colored cells as suggested by the patterns.*

Purpose: To continue instruction and practice in counting by 2's, 3's, 4's and 5's (readiness for multiplication and division), and to apply counting by 5's to finding the value of a group of nickles.

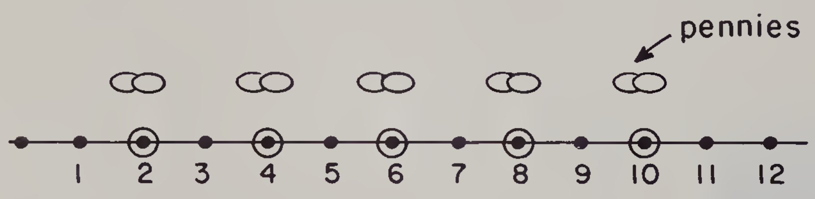
Unifying Ideas: Number and counting; multiplication and division.

Materials for Teacher: A number line on the floor; 25 pennies and 5 nickels.

Materials for Pupils: A number line on paper; a supply of toy money.

The Lesson:

Have a pupil stand at 2 on a number line on the floor and have another pupil stand at 4. Tell the pupils at their seats to circle the points on their number lines to show where their classmates are standing. Ask a third pupil to stand at a point the same distance from 4 as 4 is from 2. The pupils at their seats should circle the point for 6. Have each of the pupils who are standing say, in turn, the number he is standing on (2, 4, 6), while those at their seats follow this on their papers. In the same way, extend the pattern to 8, then 10. Now give 2 pennies to each of the pupils who are standing. Ask the other pupils to lay 2 pennies on each point of their number line which matches a point where a pupil is standing (they have these points circled).



Ask for an easy way to count the pennies. Have a pupil touch the hands holding the pennies as he counts, "2, 4, 6, 8, 10," while pupils at their seats touch their groups of 2 pennies and count silently with him. Have another pupil count the pennies again, collecting them as he goes along, while those at their seats do the same thing with their pennies. Everyone should check to see that there are 10 pennies in all. Discuss the value of the pennies in terms of nickels or dimes. Write on the board:

10 pennies 10 coins 10¢

Direct attention to the number line again. Ask, "What numbers did we skip?" Point out that you have been counting by 2's and this involves skipping one number each time.

Conduct similar activities in counting by 3's, 4's, and 5's. In using money with the 5's, first give each of the pupils standing on the number line five pennies, then (having collected the pennies) one nickel. Be sure all pupils see that one nickel has the same value as five pennies, so they can count the nickels as 5's.

Page 64 shows another way of recording what the pupils just did with the number line. White cells correspond to numbers they skipped. Encourage pupils to write only the numbers that belong in the colored cells. Of course, pupils who need to fill in the white cells in order to find numbers for the colored cells should be encouraged to do so. Note that the horizontal strips in the lower left-hand corner of the page summarize corresponding activities elsewhere on the page and furnish additional practice in this kind of counting.

Be sure to review the accompanying booklet, *Games and Enrichment Activities*, for suggestions you may wish to incorporate in your daily lesson plan.

Headline Story: Can you have exactly 23¢ with 5 coins? . . . 6 coins? . . . 14 coins? . . . Etc.

A.

1	2
	4
	6
	8

B.

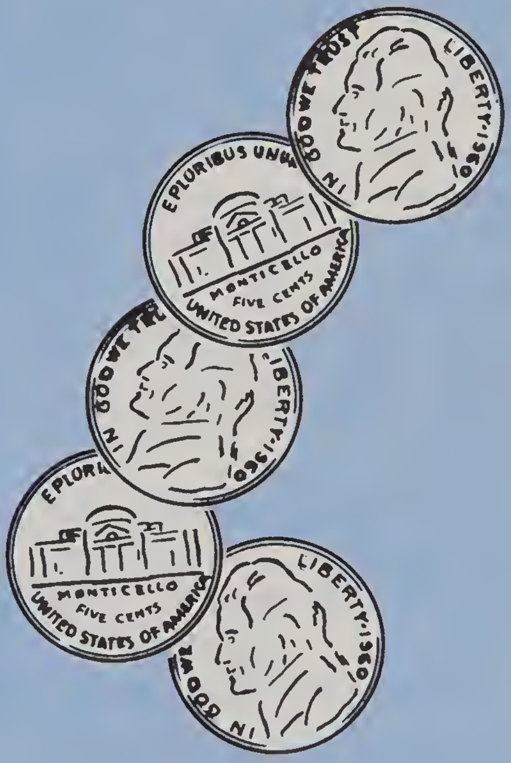
1	2	3
		6
		9
		12

C.

1		3	4
			8
			12
			16
			20

D.

1			4	5
				10
				15
				20
				25


IN GOD WE TRUST
LIBERTY 1960
E PLURIBUS UNUM
MONTICELLO
FIVE CENTS
UNITED STATES OF AMERICA

5 coins

25 ¢

A	2	4	6	8	
B	3	6	9	12	
C	4	8	12	16	20
D	5	10	15	20	25

Notes on Pupil Page 65

Tasks: *Pupils supply missing numbers in the colored squares and draw pictures of members of appropriate sets where pictures are not given.*

Purpose: To teach some of the addition facts with sums of 10.

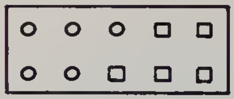
Unifying Ideas: Sets; addition and subtraction; structure.

Materials for Teacher: Ten cards, just alike, with the pattern shown below, and a pair of scissors.

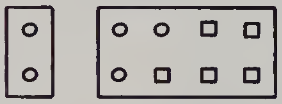
Materials for Pupils: Five of each of two kinds of counters; paper to write on.

The Lesson:

Your ten cards should all look like this:

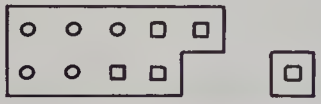


Hold up one of your cards. Ask the pupils to arrange their counters in the same pattern. Help them to notice how you have used two shapes of pictures on your cards and ask them to use their two kinds of counters in the same way. Cut your card into two pieces and hold one piece in each hand.



Have the pupils separate their counters to show how you have cut your card. How many are in each set? How many in all? Show the two pieces of your card separately, then together, as you say, "Two plus eight equals ten." Write on the board while the pupils write on their papers: 2 + 8 = 10.

Cut a second card another way, possibly like this:



Have the pupils duplicate these patterns and continue as before.

Repeat this activity, with needed alterations, until you have used all the numbers 1-9 for the left-hand part of your card. Hold up your remaining card and ask, "What two number facts or sentences are pictured by this card?" Elicit:

10 + 0 = 10 and 0 + 10 = 10.

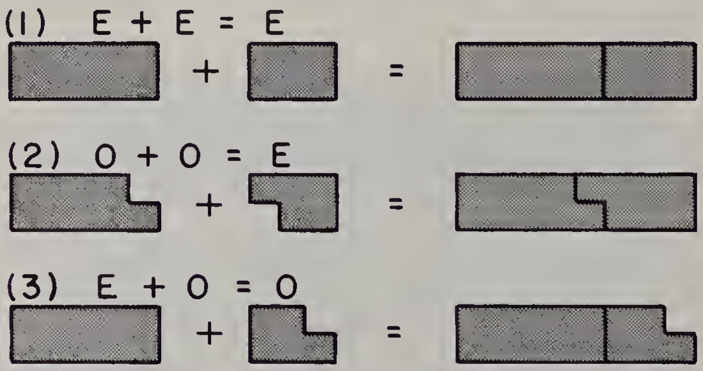
To the right of 2 + 8 = 10, write on the board the following for pupils to complete.

2 + 8 = 10 $\underline{8 + 2}$ =

Such studies of regrouping numbers should lead to three kinds of records: (1) a random grouping, (2) a pattern of numbers in serial order, and (3) a pattern of pairs showing the commutative principle for addition.

(1)	(2)	(3)
2 + 8 = 10	0 + 10 = 10	0 + 10 = 10
7 + 3 = 10	1 + 9 = 10	10 + 0 = 10
4 + 6 = 10	2 + 8 = 10	1 + 9 = 10
10 + 0 = 10	3 + 7 = 10	9 + 1 = 10
⋮	⋮	⋮
5 + 5 = 10	10 + 0 = 10	5 + 5 = 10

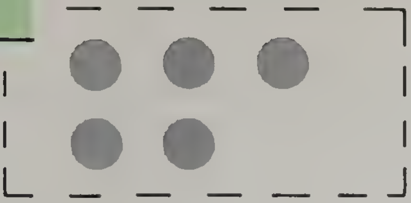
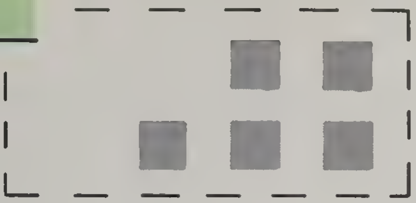
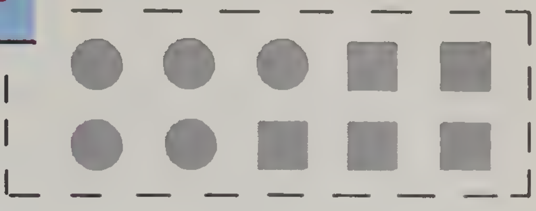
This type of lesson uses physical representations for numbers which make it easy to visualize the facts that (1) the sum of two even numbers is an even number, (2) the sum of two odd numbers is an even number, and (3) the sum of an even and odd number is odd. You may wish to explore this relationship.



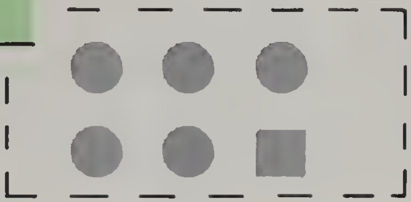
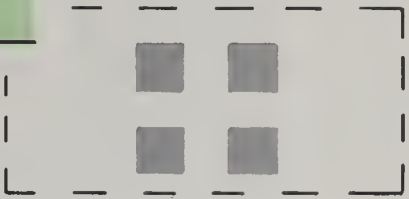
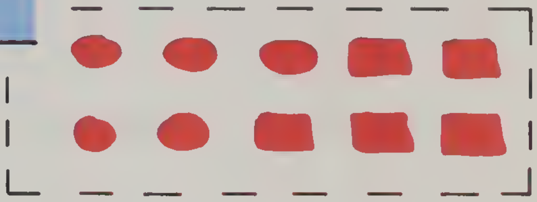
Headline Story: Make up stories to fit this headline:

10 = □ + △

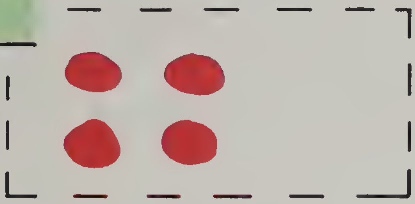
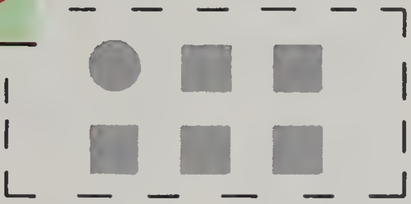
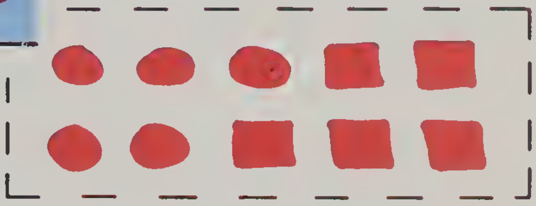
5 + 5 = 10

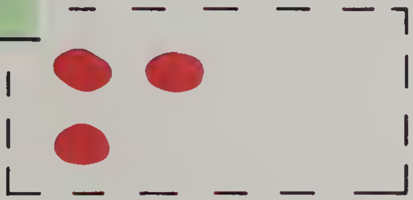
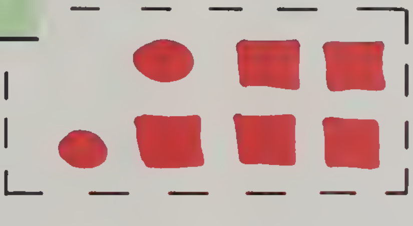
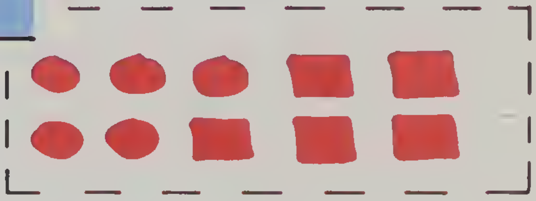
6 + 4 = 10

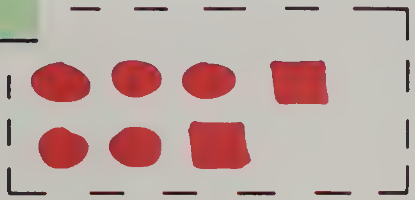
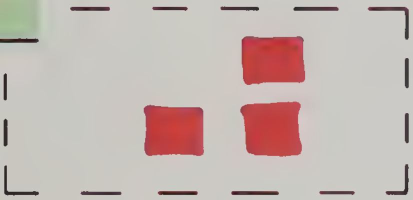
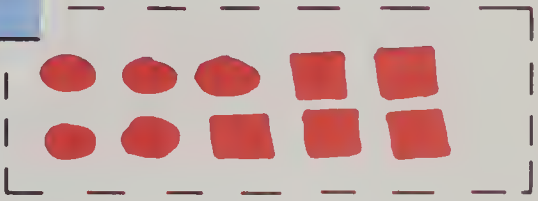
4 + 6 = 10

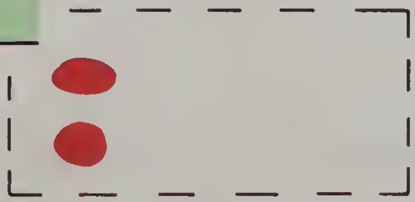
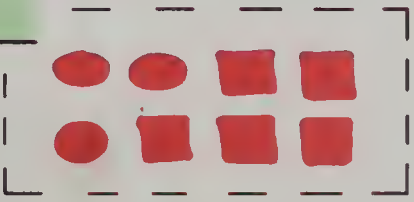
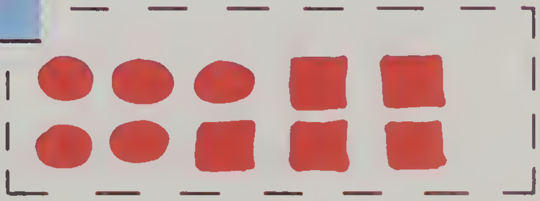
3 + 7 = 10

7 + 3 = 10

2 + 8 = 10

Notes on Pupil Page 66

Tasks: *Pupils draw arrows to show indicated jumps on the number line, circle the number which goes with the "landing point," and supply missing numbers in the frames.*

Purpose: To review the analogy between jumps forward on the number line and addition, to review some number facts, and to show a way of discovering new facts.

Unifying Ideas: Addition and subtraction; structure.

Materials for Teacher: No special materials are needed.

Materials for Pupils: Number lines on paper, such as those suggested for page 42.

The Lesson:

Have the pupils write numbers, beginning with 0, for the points of three of their number lines. Ask them to pretend their counters are frogs, and pose problems such as: "A frog starts at 0 and jumps 3 spaces. Act out this story with your frog and the first number line on your paper. Take your pencil and circle the number that goes with the point where he lands. Now draw a curved arrow that could be a picture of his jump. Where did he start? How far (3 spaces) did he jump? Where did he land?" As the pupils answer these three questions, write on the board the corresponding components of:

$$0 \ \overset{\curvearrowright}{3} \rightarrow 3$$

To the right of this, write: $0 + 3 = \square$ and ask the pupils what number should be used in the frame. Do two more similar problems, having the frog start at 1 and 2 and jump not more than 6 spaces. Keep the two kinds of record each time.

Have the pupils number two more lines, starting with 3. Pose two more problems, first having the frog start at 4 and jump 3 spaces, then start at 3 and jump 4 spaces. Compare the records of these two problems.

Have the pupils number one more line, starting with 5. This time, suppose the frog starts at 6 and jumps 7 spaces. When the pupils have worked this out, write all of the following on the board:

$$6 \ \overset{\curvearrowright}{7} \rightarrow 13 \qquad\qquad 7 \ \overset{\curvearrowright}{6} \rightarrow \square$$
$$6 + 7 = 13 \qquad\qquad 7 + 6 = \square$$

Ask the pupils to supply the missing numbers. Many of them should be able to do this without going through the motions and drawing the pictures.

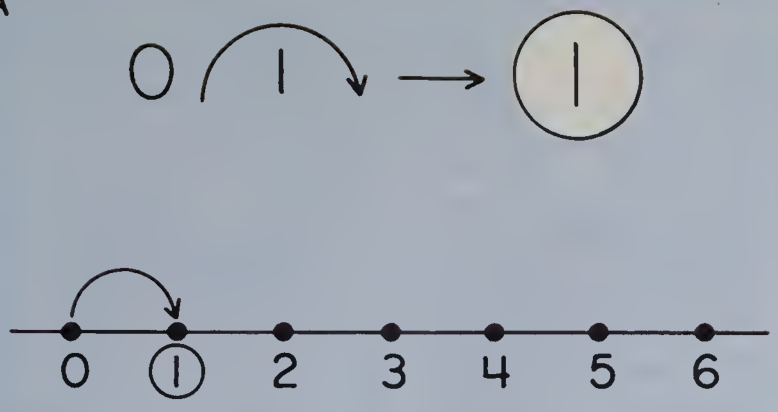
Assign page 66 for individual study. When all have finished, discuss the page. Direct the pupils' attention to section D. Who can tell where a frog would land if he started at 4 and jumped 2 spaces? Those who cannot tell should act out the story. Then they should think about what is in section D and what they just discovered. Ask similar "commutativity" questions related to sections G, I and J.

For those who wish to explore this relationship in another context, consider filling in and extending the following record:

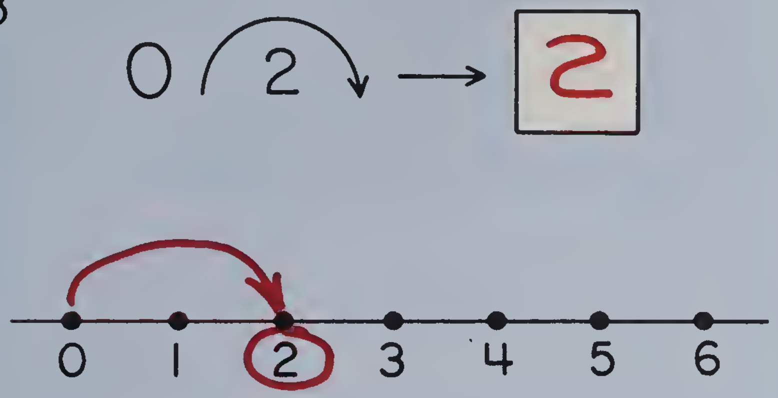
Start	3	2	4	3	3	5	
Size of jump	2	3	3	4			
Land	5				8	8	

Headline Story: Eve lived 3 blocks from school. She walked 4 blocks past the school. How far did she walk?

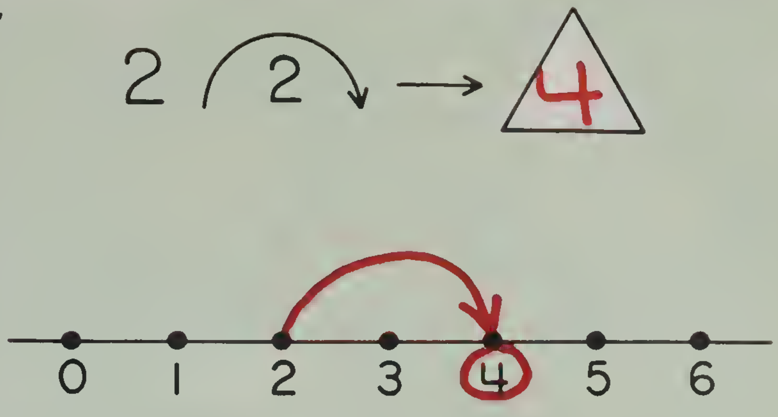
A


0 1 → 1
0 1 2 3 4 5 6

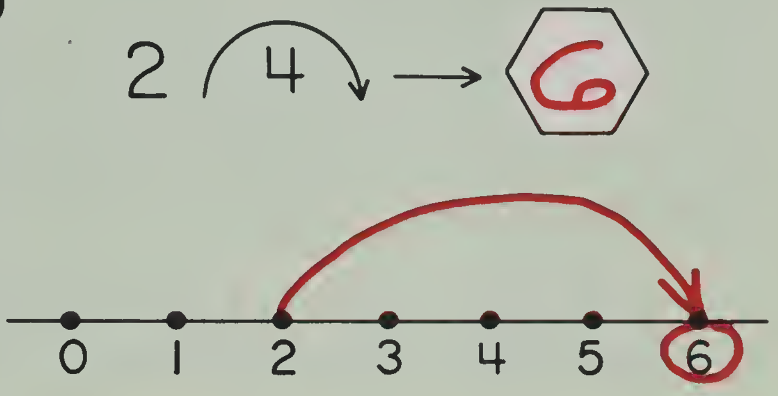
B


0 2 → 2
0 1 2 3 4 5 6

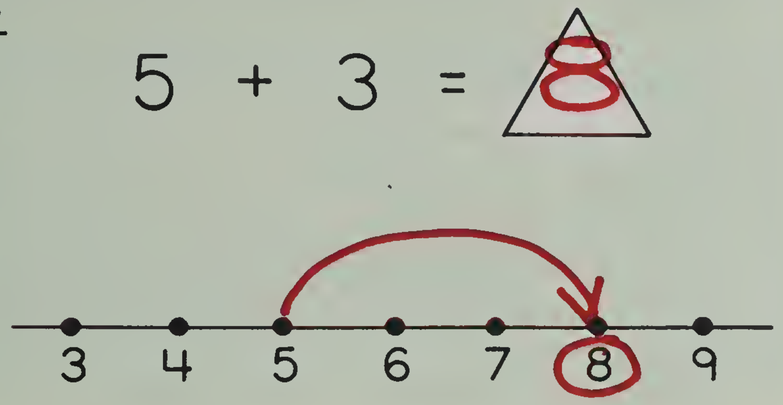
C


2 2 → 4
0 1 2 3 4 5 6

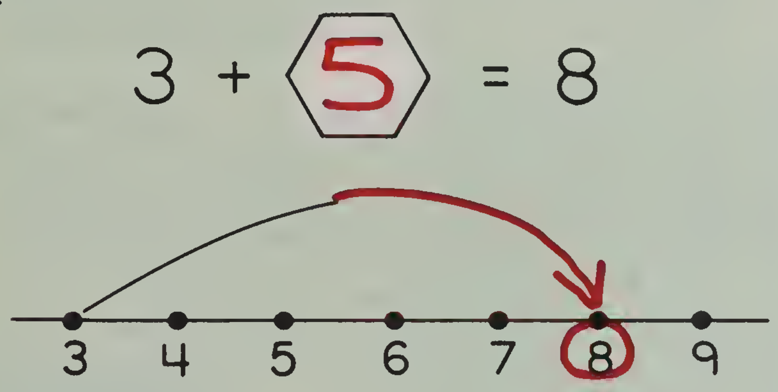
D


2 4 → 6
0 1 2 3 4 5 6

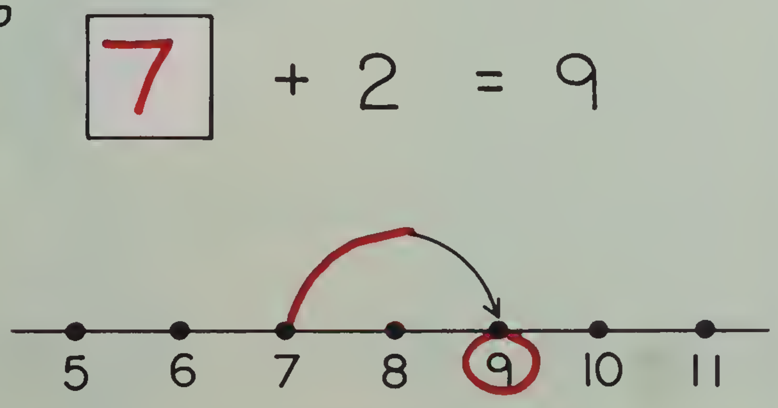
E


5 + 3 = 8
3 4 5 6 7 8 9

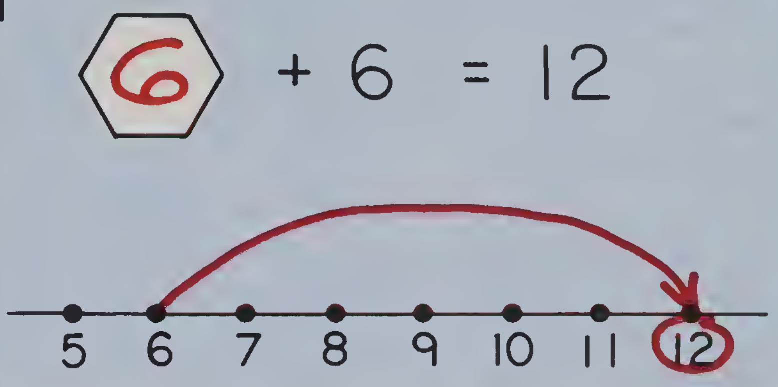
F


3 + 5 = 8
3 4 5 6 7 8 9

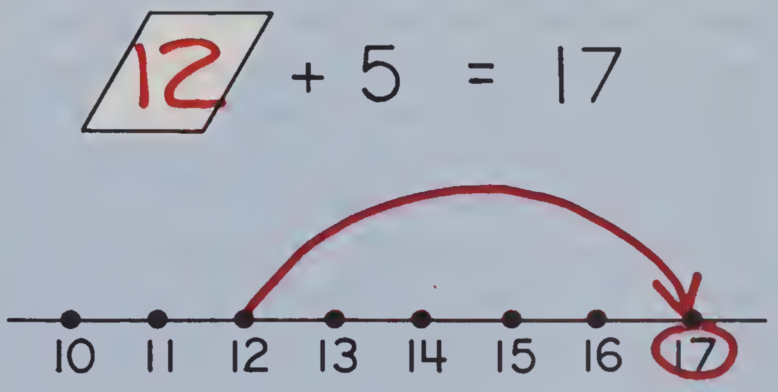
G


7 + 2 = 9
5 6 7 8 9 10 11

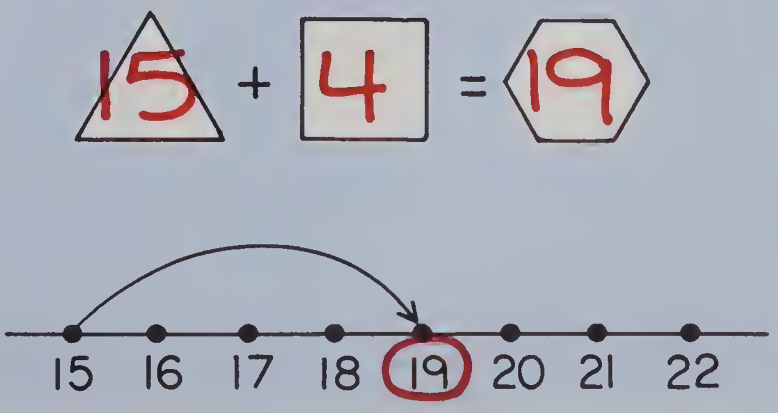
H


6 + 6 = 12
5 6 7 8 9 10 11 12

I


12 + 5 = 17
10 11 12 13 14 15 16 17

J


15 + 4 = 19
15 16 17 18 19 20 21 22

Notes on Pupil Page 67

Tasks: *Pupils complete the records of the number of pennies and nickels shown in the pictures at the top of the page and of their values in cents.*

Purpose: To review simple counting and counting by 5's and to apply these skills to finding the total value of a number of coins.

Unifying Ideas: Number and counting.

Materials for Teacher: A supply of nickels and pennies in real money.

Materials for Pupils: One nickel and one penny for each group of two or three pupils.

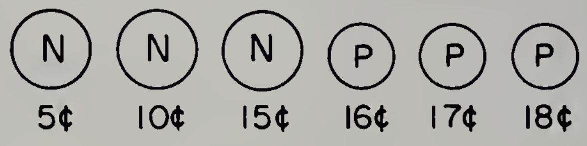
The Lesson:

Your pupils should examine carefully and discuss both sides of a nickel and a penny so that they have multisensory impressions of relative sizes and details of designs.

Ask your pupils to identify the pictures of nickels and pennies on page 67 in their books. Those who have no trouble with this and who are good at telling the value of coins may be assigned this page for individual study while you work with the rest as described below.

Tell the pupils you are going to hold only pennies in the hand on their left (your right hand as you face them) and only nickels in the other hand. Ask them how many pennies and how many nickels you should hold so that the coins in your hands will match picture A. Hold out your hands, displaying the coins, and ask, "How many coins do I have in both hands together?" Show them where to write 5 in the record, and be sure everyone sees what the 4, 1, and 5 have to do with the 4 pennies and 1 nickel you are holding. Then ask, "How much is all this money worth?" Discuss several ways of finding this value and decide upon a way that may be the most efficient (find the value of all the pennies and find the value of all the nickels, and add). Complete the record in row A.

Conduct similar activities for pictures B, C, and D. Count by 5's, or add 5's, to find the value of the nickels. If anyone has trouble adding the 3¢ and 15¢ in the record for B, lay out the coins and have him count out their total value like this:

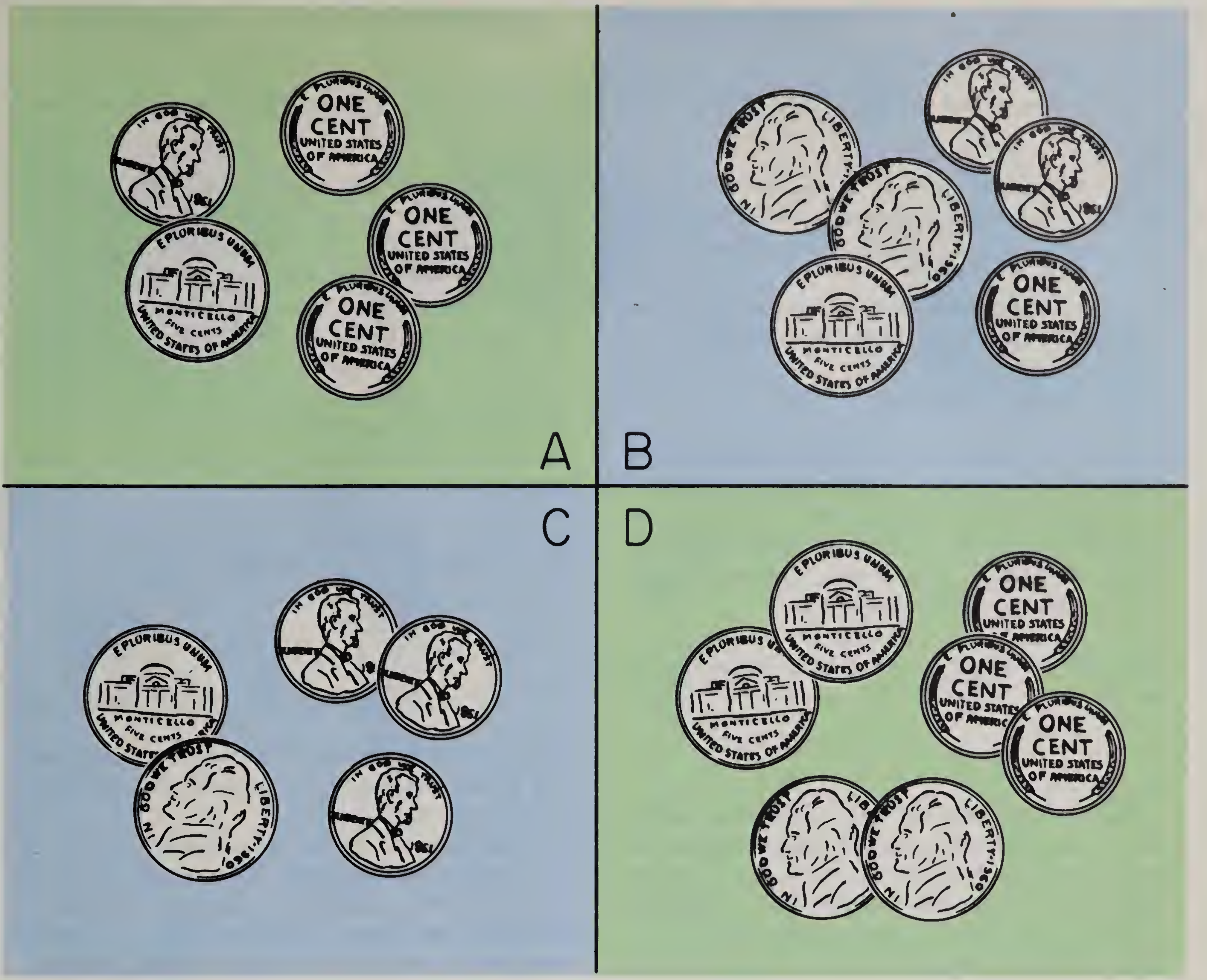


Discuss the completed page with your pupils. Do any two boxes show the same number of coins? (A and C) Are the coins shown in these two boxes worth the same amount? Are the coins in any two boxes worth the same amount? Which box shows the greatest number of coins? What is the largest amount of money you could have if you could have just 7 coins and each coin had to be either a nickel or a penny? (35¢)

Interesting extensions of this activity are the games, "How Many Ways?" and "Fewest Coins." Keep a record of how many ways we can make 5¢, 10c, 15¢, etc. Which way uses the fewest coins?

	How Many Ways	Fewest Coins
5¢	2-(5 pennies or 1 nickel)	1 nickel
10¢	4-(10 pennies; 2 nickels; 5 pennies, 1 nickel; 1 dime)	1 dime
15¢		

Headline Story: How many different combinations amount to 18¢?



	COINS			¢		
	Pennies	Nickels	Total	Pennies	Nickels	Total
A	4	1	5	4¢	5¢	9¢
B	3	3	6	3¢	15¢	18¢
C	3	2	5	3¢	10¢	13¢
D	3	4	7	3¢	20¢	23¢

Notes on Pupil Page 68

Tasks: *Pupils write letters in the table to show which of three items of the same shape is largest, which is in-between, and which is smallest.*

Purpose: To increase pupil awareness of relative sizes and use of the appropriate vocabulary.

Unifying Ideas: Geometry.

Materials for Teacher: Three cards, folded so that they will stand up on a table top, and labeled *smallest*, *largest*, and *in-between;* a collection of paper cutouts, rods, boxes, and other objects of various sizes to be used in sets of three in which relative sizes are readily apparent.

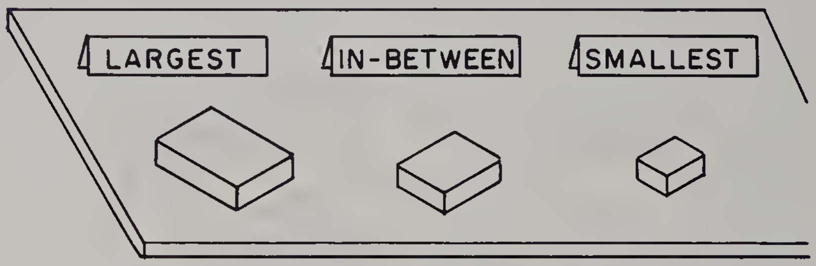
Materials for Pupils: No special materials are needed.

The Lesson:

Hold up two objects, very much alike except for size, and ask your pupils to describe these objects. Lead the discussion to the point of talking about how to tell these objects apart. Since the difference in size is the major distinguishing feature, emphasize applicable words of comparison, eventually deciding upon *large* and *small*. Discuss pairs of objects in the same way until the pupils understand the idea of relative sizes of two things and are comfortable with the vocabulary.

Display three objects, again very much alike except for size, and ask the pupils what they see. Then you might say something like, "I am thinking of one of these boxes. It is the largest box. Who is sure which box I am thinking of? Show me. Now I'm thinking of the smallest box. Which box is that? What can we say about the other box?" Approve of suggestions like "in the middle" or "medium," but decide eventually on the phrase *in-between*. (The notion of "betweenness" is important in mathematics.) Discuss three more objects in the same way.

Show the pupils the cards you have prepared and teach them to read the words. Set the cards up on the table top, in the order shown below, and put three objects in place:



Present other collections of three objects and have the pupils place them in order of size according to the words on the cards.

Assign page 68 for individual study. Explain to the pupils that, since they cannot move the pictures to places in the table, they are to write in the table the letter which goes with the picture they wish to use. In a few cases, it may be necessary to do some work on the chalkboard or a flannel board to make the record-keeping clear. Use 2x2 or 3x2 tables.

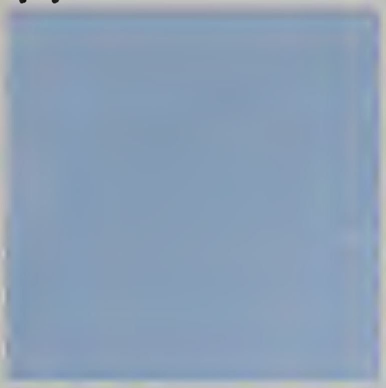
	Largest	Smallest
□		
○		

	Largest	In-Between	Smallest
△			
⬡			

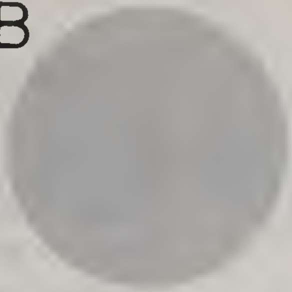
Headline Story: Bill is 2 years older than Alice and 2 years younger than Jane.
He is ____ years old? (Extend.)

LARGEST — IN-BETWEEN — SMALLEST

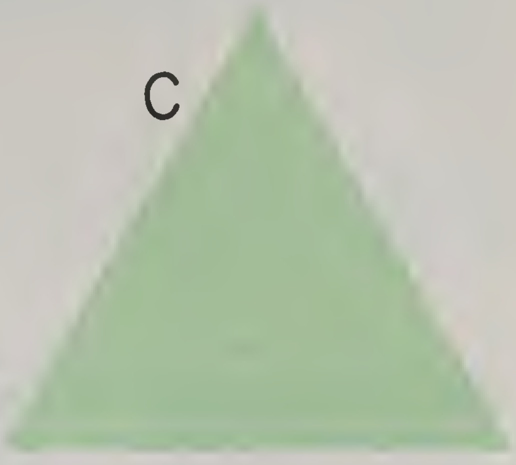
A



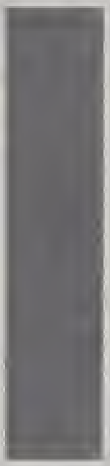
B



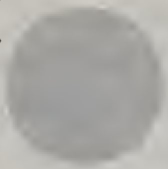
C



D

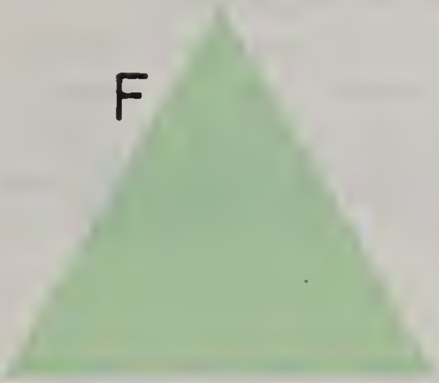


E

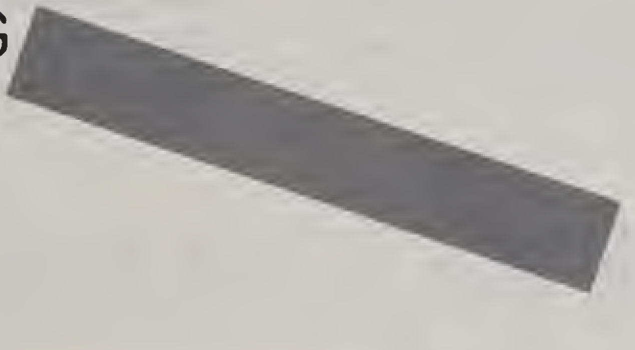


	LARGEST	IN-BETWEEN	SMALLEST
🟦	A	H	J
●	L	B	E
▲	C	F	K
▬	I	G	D

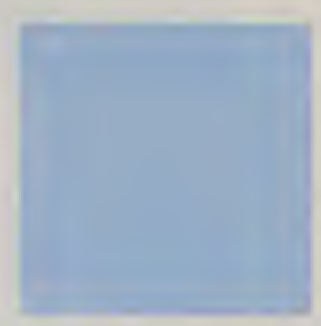
F



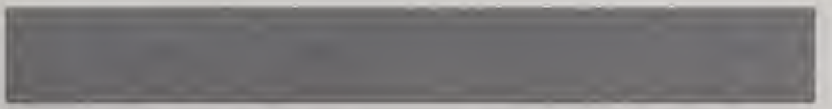
G



H



I



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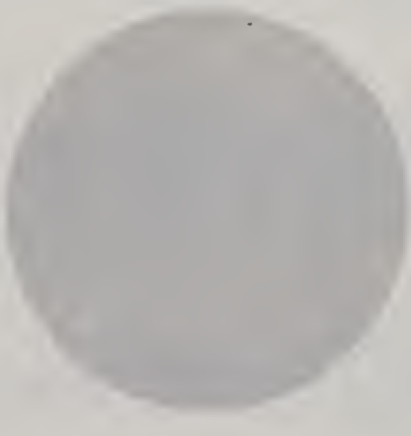
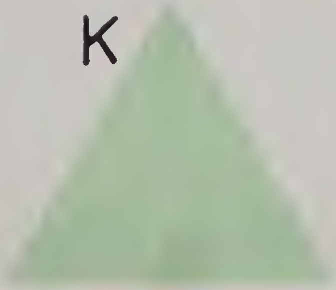
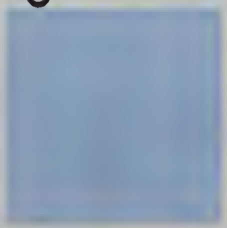


K



J



Notes on Pupil Page 69

Tasks: Pupils count the number of green beads on each string and record the count; they color enough blue beads on each string so that the total number of blue beads will match the given count.

Purpose: To review counting, singly and by groups, and to explore other number relations.

Unifying Ideas: Number and counting.

Materials for Teacher: A piece of fairly stiff wire and some wooden or plastic beads, if necessary.

Materials for Pupils: A blue crayon.

The Lesson:

One way to treat this lesson is to assign page 69 to the class with no more introduction than an explanation of what the pupils are to do. Most of them should be able to do the work without help. Use your materials to help those who have difficulty.

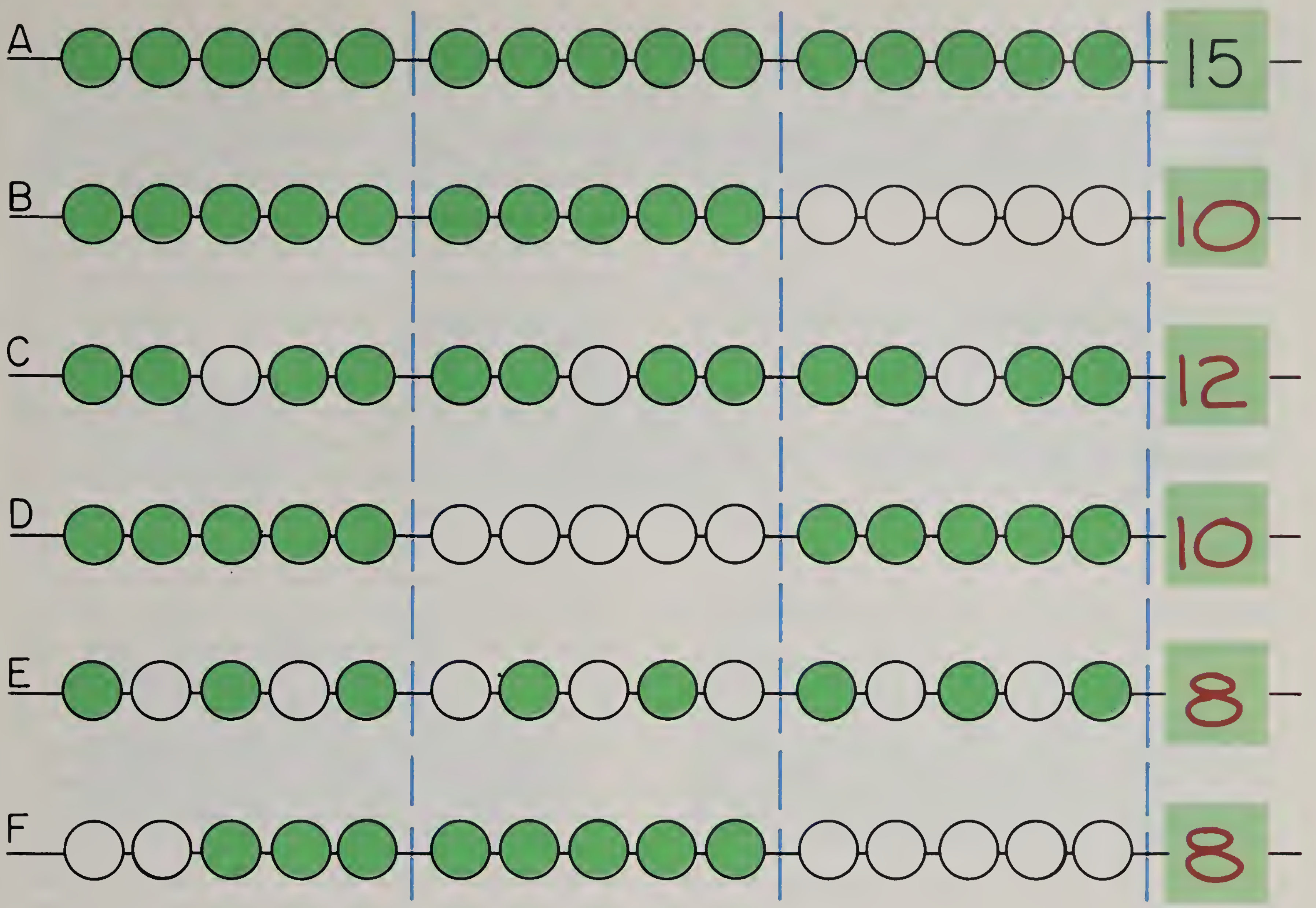
Try to notice how different pupils appear to be arriving at their answers. Those who finish quickly might be asked to make up other exercises like those on this page. From what you know of your pupils, you might ask selected individuals to make up exercises in which the colored beads could be counted by 3's or 4's, or in which the colored beads would represent specified addition facts.

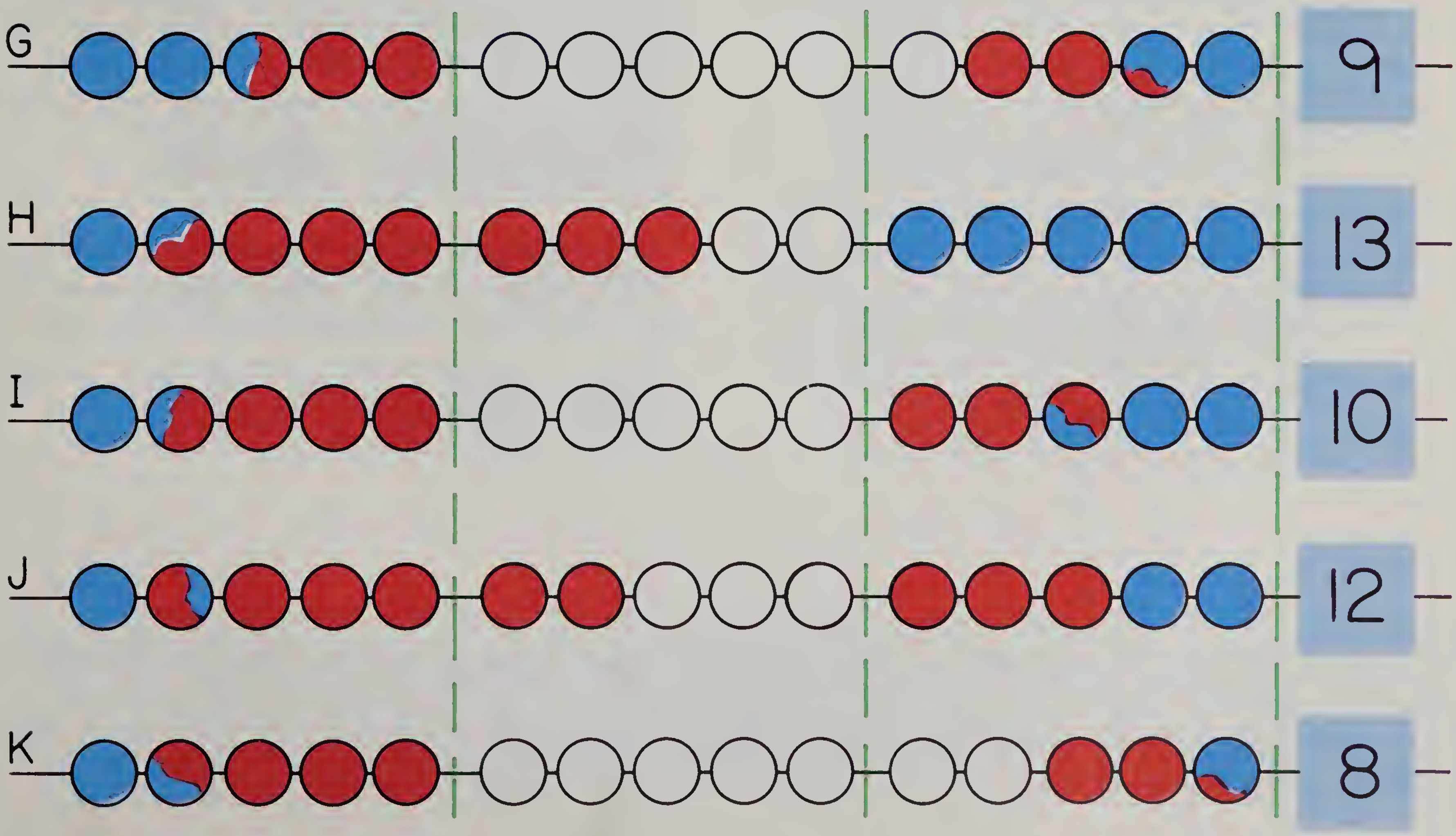
When all pupils have finished the page, discuss it with the class. Ask how they found their answers. Who counted by 5's in the first two rows? Who added 5's? Did anyone count some other way in rows A and B? For row C, who counted by 2's? Did anyone subtract 3 (the number of white beads) from 15 to get the 12? If so, this is very mature performance. Who counted by 5's, or added two 5's, in row D? Did anyone do row E by adding 3 + 2 + 3? In what other ways could one find the answer in row E? For row F, how many pupils added 3 and 5? Did anyone count the green beads by 2's? Did anyone use some other method?

For each of the rows on the bottom half of the page, there are many correct responses. For instance, for row G, any scheme which colors the rest of the two beads which were partially colored to begin with, and 4 other beads anywhere on the string, is a correct way to have 9 blue beads in all. Did anyone complete row G so that there were 3 blue beads in each section? Did anyone color the beads in the last three rows so that they could easily be counted by 2's? Did anyone color the last three rows so that the blue beads could be seen as two 5's, two 6's, and two 4's, respectively?

It is desirable that even the most immature pupils see that there are several ways to arrive at correct answers, and pupils who figure out short-cuts or who see ways to make a simple task more challenging and interesting are to be commended.

Headline Story: On a string of 15 beads, 8 are red. The rest are blue. How many blue beads?


A
15
B
10
C
12
D
10
E
8
F
8


G
9
H
13
I
10
J
12
K
8

Tasks: Pupils supply, in the blue cells, missing items in number sequences and in simple geometric patterns.

Purpose: To test pupil knowledge of simple number sequences, their ability to use numbers associated with measurement, and their facility in completing patterns.

Unifying Ideas: Number and counting; geometry.

Materials for Teacher: No special materials are needed.

Materials for Pupils: No special materials are needed.

The Lesson:

Tell the pupils that they are to write or draw something in each blue space on the page. Be ready to help the rare pupil who does not see what to do, but help him by asking questions, not by telling him.

After you have checked the results, group the pupils who made the same kinds of mistakes so that you can give them appropriate remedial work.

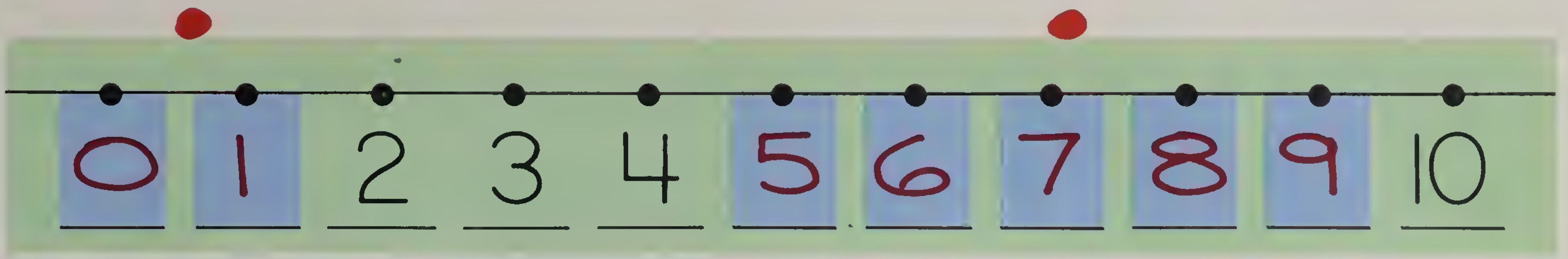
Those who made mistakes in the top and bottom items are probably in the most trouble, but if they had correct responses elsewhere on the page, you will have to speak with them individually to try to find out what their trouble is. Can they count by rote? Can they write the numbers in simple sequence starting with 1? Are they disturbed when asked to start with 0? Do they get confused when the first number given is something other than 0 or 1? Is it difficult for them to count backward? For these pupils, and indeed for others also, it is desirable to have in the classroom two labeled number lines, one starting at 0 and ascending to 100 from left to right, and one starting at 100 and descending to 0 from left to right. Put these number lines along a baseboard, if necessary.

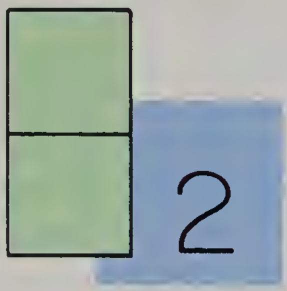
Seize every opportunity, even in the middle of a story period (How many hats has Bartholomew lost so far?), to ask these pupils to use the number lines in counting, in telling what number comes just before or just after some given number, in counting for games like "Hide-and-seek," and so on.

Those who had trouble with the "tiling" problems would probably benefit from tactile experiences with linoleum squares, ceramic tiles, or squares cut from heavy cardboard. Space perception can be developed, but this development is a slow process in people who lack it to a significant degree. These same pupils may need practice in simple counting. The left-hand block just below the middle of the page is a variation of the exercises which precede it and, furthermore, it involves addition facts with sums of 5. The right-hand block in the same row uses two designs in such a way that no two adjacent tiles are of the same design, and it furnishes some practice in writing + and − signs. Pupils who do not see the pattern in designs such as this should be encouraged to make designs of their own which other pupils then criticize. Usually, approval is given only to designs with some sort of obvious pattern.

Be sure to review the accompanying booklet, *Games and Enrichment Activities*, for suggestions you may wish to incorporate in your daily lesson plan.

Headline Story: Extend this pattern: 18 27 36 __ 54 __ __ 81

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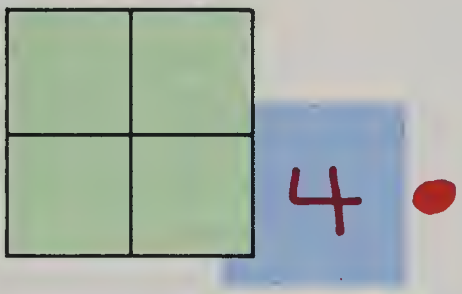
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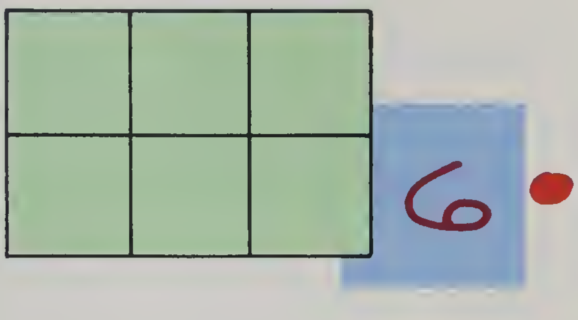
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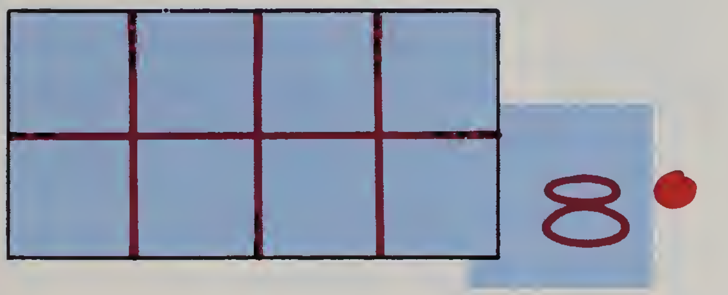
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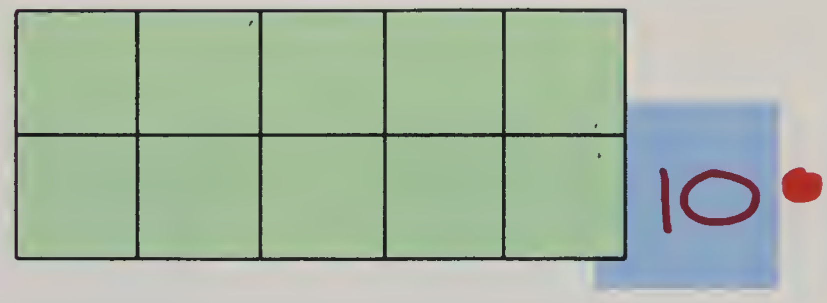
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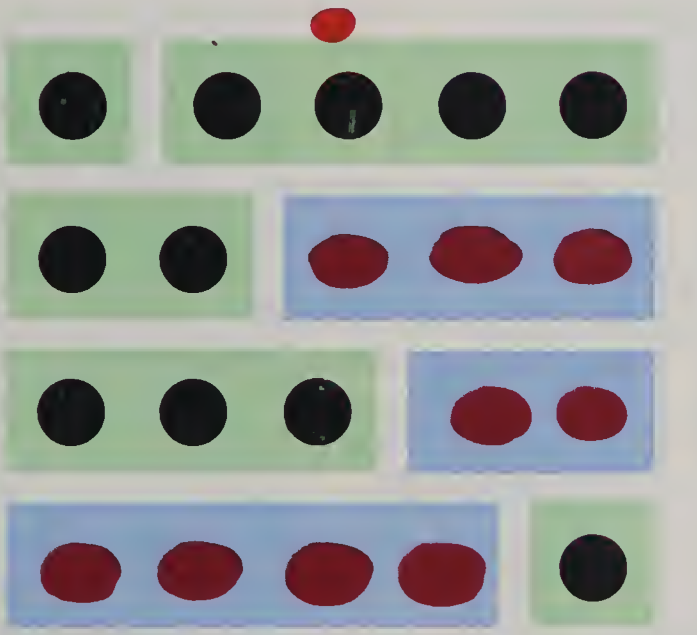
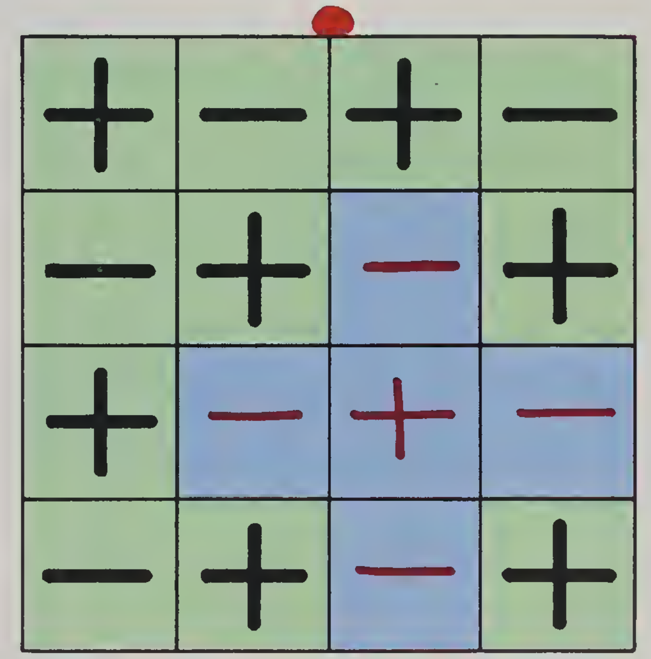
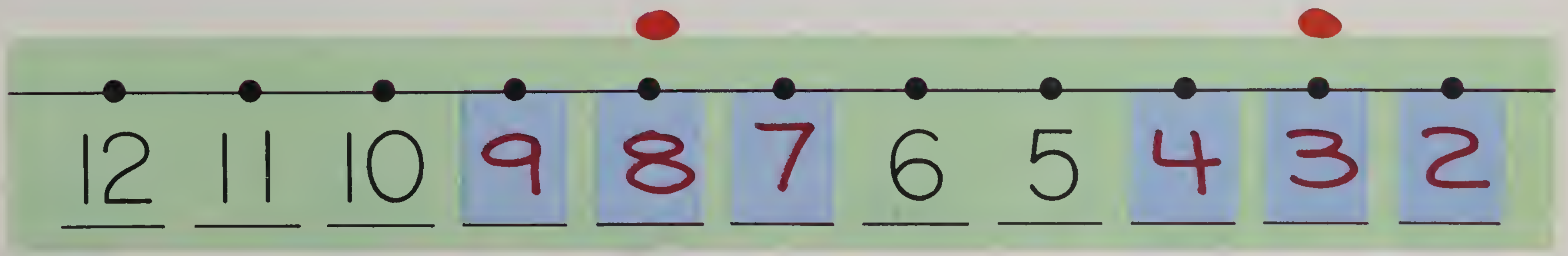




+	−	+	−
−	+	−	+
+	−	+	−
−	+	−	+



Notes on Pupil Page 71

Tasks: *Pupils supply numbers for the tags and missing items in the addition sentences.*

Purpose: To test pupil ability to associate numbers with equally-spaced points on the number line and their knowledge of addition facts with sums to 10.

Unifying Ideas: Number and counting; addition and subtraction.

Materials for Teacher: No special materials are needed.

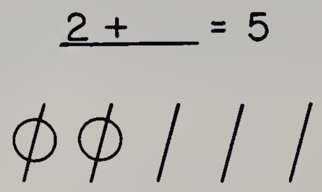
Materials for Pupils: No special materials are needed.

The Lesson:

Tell the pupils they are to write a number, wherever one is missing, but the number must make sense in terms of what goes with it. Elicit the fact that, in the examples which look like a capital letter L written backward, adding across up and down should give the same answer. Then assign page 71 for individual work.

By this time, your pupils should all know the addition facts with sums to 6, at least. However, many of the exercises on this page involve finding missing addends. If a pupil did not respond correctly to 2 + ___ = 5, for instance, you will have to find out whether he knows the sum of 2 and 3. If he does not, the first step in helping him is to teach him that. The next step is to work the same exercises with counters and with a number line until he understands what is required in the exercises and has ways of finding the answers.

In working with pupils who need remedial work, you may proceed somewhat as follows, treating many examples at each stage and having the pupils act out each step as you proceed: "When working with counters, the number sentence 2 + 3 = 5 tells you, 'Start with 2 counters, put 3 more with them, and you have 5 in all.' . . . Suppose one of these numbers is missing. How will you find it?" If the sum is missing, the problem is relatively easy. Missing addends are harder to find. "The open sentence 2 + ___ = 5 says, 'Start with 2 counters, put enough counters with them so that you have 5 in all.' How many would you need?" Pupils who do not see at all how to find this number may be directed to make tally marks spaced out to represent the number they want to have in all, lay counters on tally marks to represent the known addend, and count the uncovered tally marks to find how many more counters they need.



"In working with 2 + 3 = 5 and a number line, the 2 says, 'Start with 2,' the + 3 says, 'Jump forward 3 spaces,' and the = 5 says, 'You land at 5.' " In the open sentence, a missing element represents either the starting point, the size of the jump, or the landing point. Use number lines on the floor, the sand table, the chalkrail, the pupils' desks, sheets of paper, and eventually the pupils' imaginations to provide a variety of multisensory experiences with this concept.

Headline Story: Candy bars cost 5¢ each. How much will 3 bars cost?

0 1 2 3

0 2 4 6 8 10 12

5
+1
6

2 + 4 = 6

3 0 2 0 3
+1 +4 +2 +4 +1
4 4 4 4 4

1
+7
8

4 + 4 = 8

6
+0
6

3 + 3 = 6

5 + 2 = 7

5 + 0 = 5
2 + 3 = 5
4 + 1 = 5
3 + 2 = 5
1 + 4 = 5

5
+5
10

9 + 1 = 10

6 + 1 = 7

4
+3
7

2 + 5 = 7

0 1 2 3 4 5

0 3 6 9 12 15

Notes on Pupil Page 72

Tasks: *Pupils color appropriate coins in each column to indicate the amount of money listed at the top of the column; at the bottom of each column, they record the count of the coins they colored.*

Purpose: To provide more practice in dealing with nickels and pennies, and to introduce, for those pupils who can grasp it at this stage, the idea of the game, "Fewest Coins."

Unifying Ideas: Number and counting.

Materials for Teacher: Three nickels and five pennies in real money.

Materials for Pupils: Three nickels and five pennies in real or toy money; crayons.

The Lesson:

Show the pupils the coins you have. Ask them to put their coins on their desks with all the pennies in one place and all the nickels in another. Review the value of each of the coins and have the pupils count the money they have. Tell them you are going to play a game. You will hold only nickels in one hand and only pennies in the other. You will tell them how much money you are holding, and they are to show with their coins how many nickels and how many pennies make that amount. Tabulate results (see Table at end). Choose at first amounts of money for which there is only one possible combination of the coins you have, then amounts you could have in two ways. In these latter examples, circle the combination which takes the fewest coins.

Those pupils who see immediately that they cannot tell exactly what coins you are holding to make 5¢, 10¢, or 15¢ might enjoy playing the same game, given five more pennies and two dimes. There are four ways to have ten cents in pennies, nickels, or dimes, for example, and it usually amuses pupils to figure out "the lazy man's way."

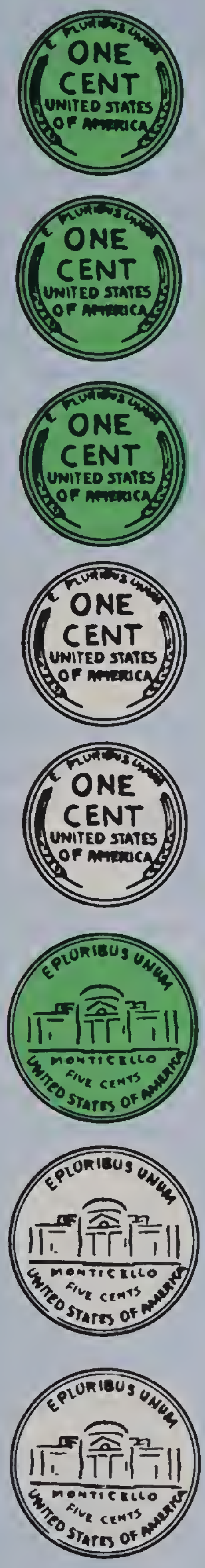
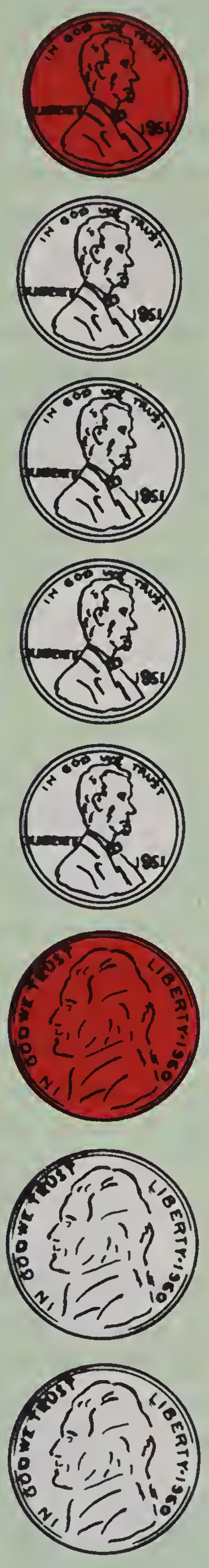
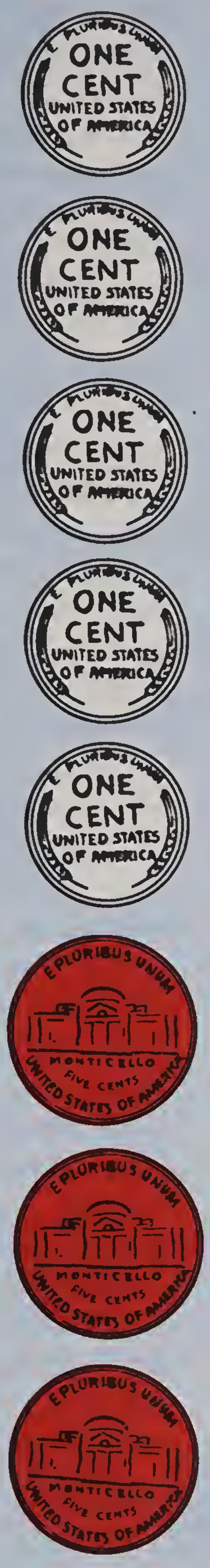
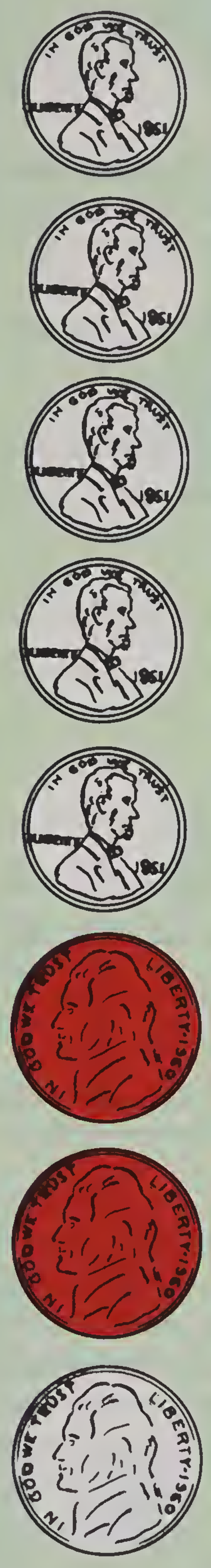
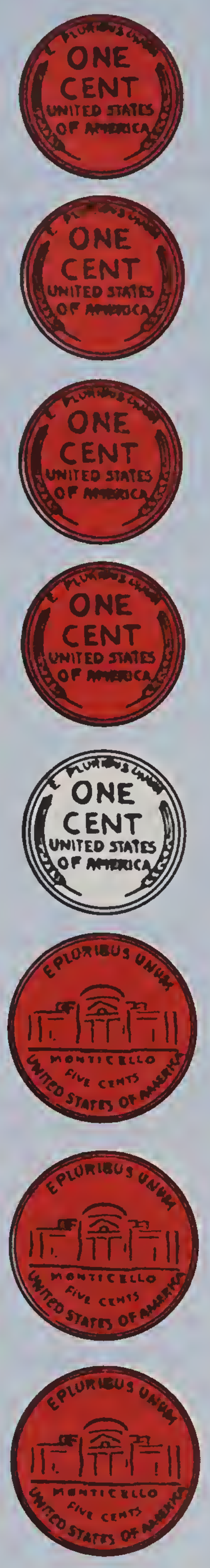
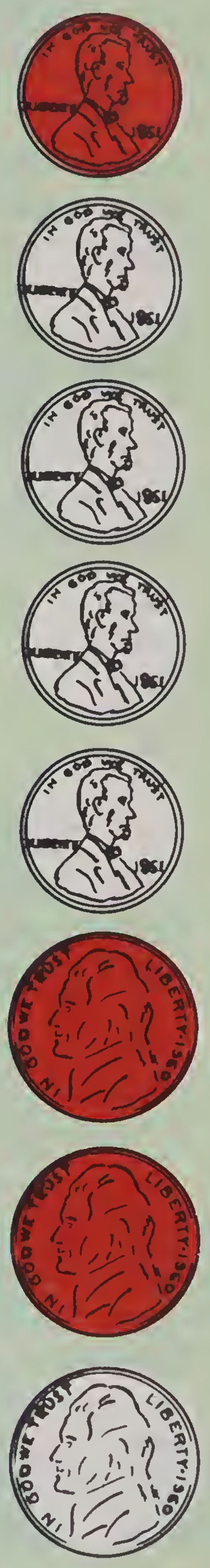
Discuss column A on page 72 with your pupils and explain what they are to do with the other columns on the page. Consider correct a response of 5 pennies and 2 nickels for column C or a response of 5 pennies and 1 nickel for column D.

Playing store is a particularly effective way to learn the value of coins, the use of money, and a great many number facts.

Amount	Pennies	Nickels
6¢	1	1
9¢	4	1
2¢	2	0
. . .	. . .	. . .
5¢	5	0
	0	1
15¢	5	2
	0	3

Headline Story: With nickels and pennies, how many ways can you make 11¢ ?

Color in FEWEST NUMBER OF COINS for each amount shown at top of column.

A	B	C	D	E	F
8¢	6¢	15¢	10¢	19¢	11¢
					
3 1¢	1 1¢	0 1¢	0 1¢	4 1¢	1 1¢
1 5¢	1 5¢	3 5¢	2 5¢	3 5¢	2 5¢

Notes on Pupil Page 73

Tasks: Pupils supply missing numbers in the equations and write the time shown on the clockface. They draw arrows to indicate jumps on the number line if they need help in finding the missing numbers.

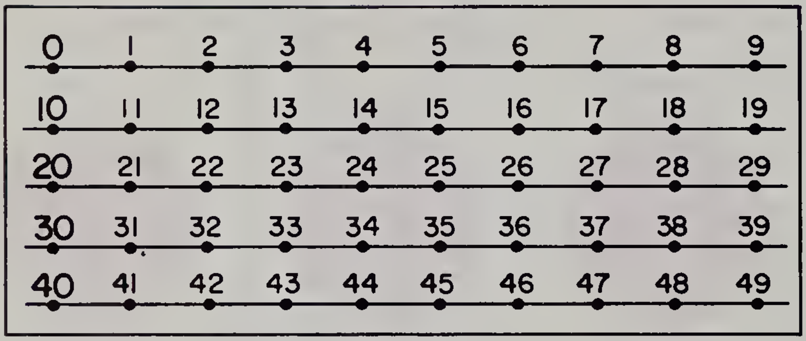
Purpose: To develop further pupil understanding of the decimal system of numeration and of telling time.

Unifying Ideas: Numeration; addition and subtraction; structure.

Materials for Teacher: Number line chunks with all the numbers which are multiples of 10 a different color from the rest; a clockface.

Materials for Pupils: No special materials are needed.

The Lesson:



Have the pupils start with 0 and count up to the largest number shown on your number line, just to review the words. Then have them count again, but this time you point to corresponding numbers as they say the number words and develop a rhythm so that they "feel a bump" when they come to 10, 20, 30, and so on. (You want them to sense the additive structure of the numeration system. They may not be able to tell you, but they should know that 20 + 5 is what *twenty-five* means, that 40 + 7 is what *forty-seven* means and so on.)

Now act out, on the number line, stories which go with appropriate number sentences as you write these sentences on the board. For instance, "I start at 20 (point to 20 on the number line; write 20 on the board); I jump 3 spaces (jump 3 spaces; write + 3); I land at (write =) twenty-three (write 23)." Read what you have written, as follows: "Twenty plus three, twenty-three." Do several more examples in the 20's, then others in different decades until the pupils are able to respond correctly with little time for thought.

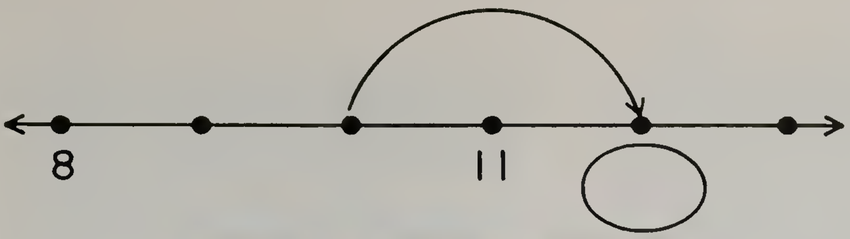
Next, act out sentences like 20 + ◇ = 26 until the pupils see that the second digit in the number symbol associated with the landing point gives the answer away.

Act out sentences like ◇ + 2 = 32 until the pupils see that the first digit in the number associated with the landing point tells them the missing number, but this time they have to know that a 3 in that position indicates 3 tens, and another name for 3 tens is the symbol "30."

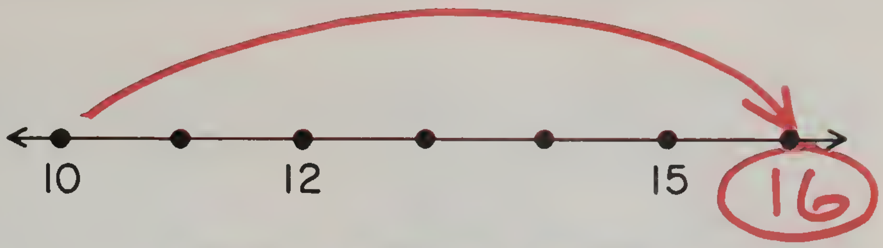
Use your clockface to review telling time by whole hours and by half hours. The pupils should note carefully the positions of the two hands at these times. For instance, at 4 o'clock, the hour hand points directly at 4 and the minute hand points directly at 12. At 4:30, the hour hand is halfway between 4 and 5 and the minute hand points directly at 6.

Assign page 73 for individual study. Pupils who are aware of the commutative principle of addition will have no trouble with the more difficult exercises at the bottom of the page. At this stage, do not worry about the others.

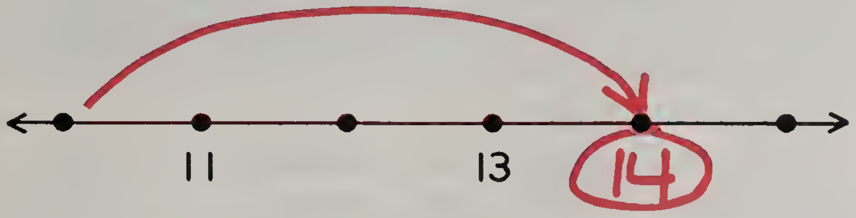
Headline Story: How many hours between 12:30 P.M. and 2:30 P.M.?



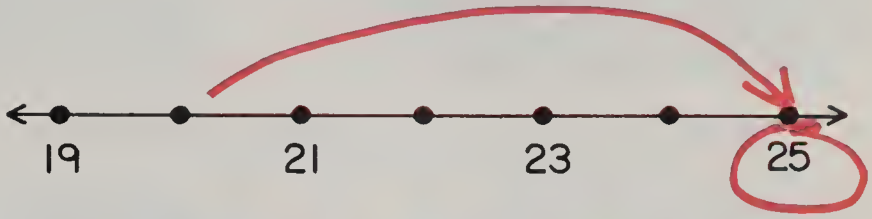
10 + 2 = 12



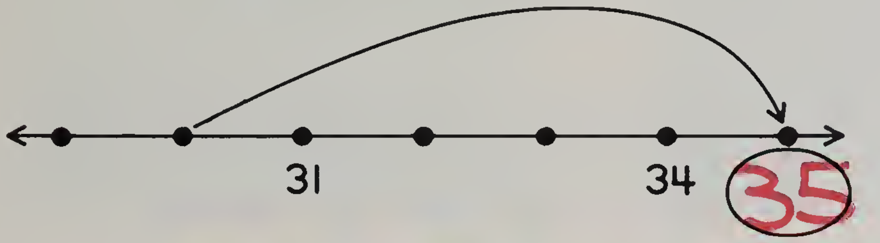
10 + 6 = 16



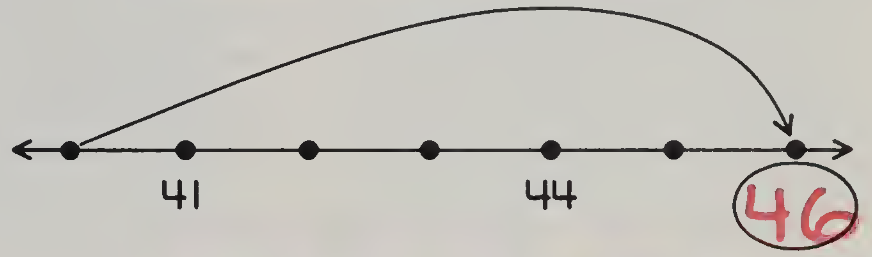
10 + 4 = 14



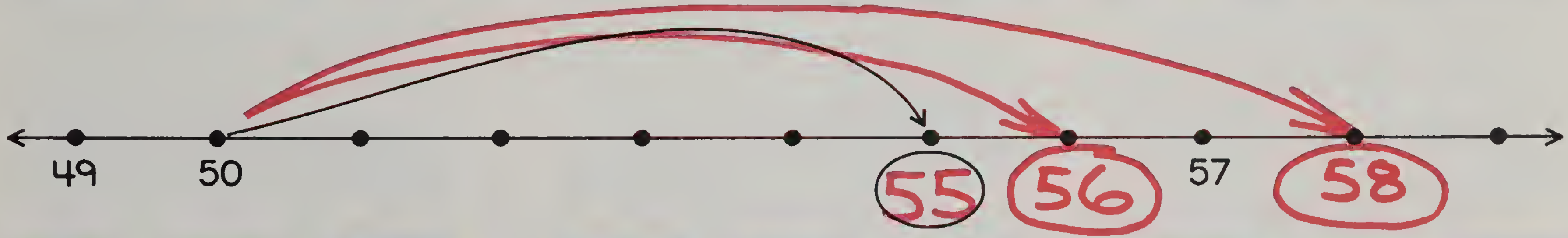
20 + 5 = 25



30 + 5 = 35



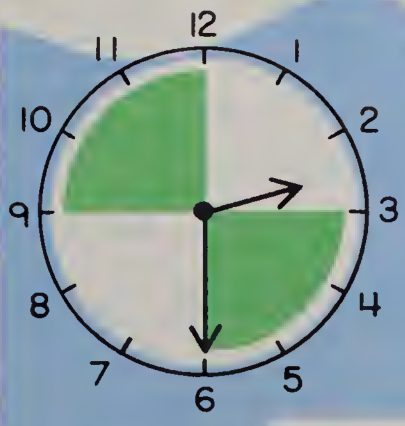
40 + 6 = 46


49
50
55
56
57
58

50 + 5 = 55

50 + 6 = 56

50 + 8 = 58


12
1
2
3
4
5
6
7
8
9
10
11

The time is 2:30

20 + 6 = 26

6 + 20 = 26

4 + 30 = 34

40 + 7 = 47

57 = 7 + 50

57 = 50 + 7

Notes on Pupil Page 74

Tasks: *Pupils examine each picture and either record the missing information in the table at the bottom of the page or complete the picture based on the information in the table.*

Purpose: To anticipate the array or row-column representation for multiplication.

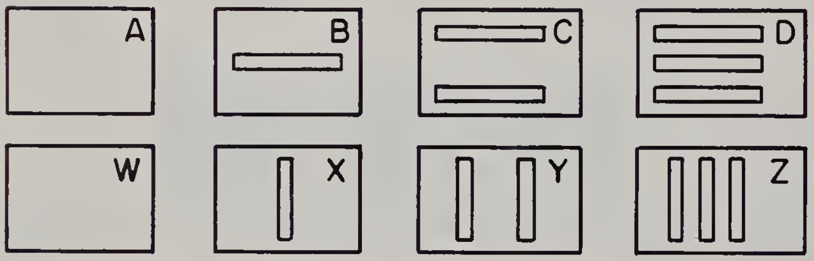
Unifying Ideas: Multiplication and division.

Materials for Teacher: A demonstration size set of the cards described in "The Lesson."

Materials for Pupils: A set of the cards and long, narrow objects described in "The Lesson" for each pupil to use at his desk; a supply of counters.

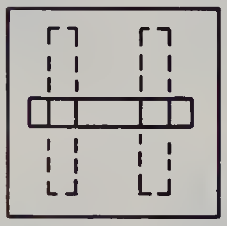
The Lesson:

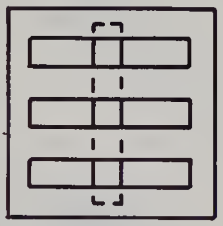
A series of eight cards—4 in one color and 4 in another color—helps provide a dramatic demonstration of the ideas behind this page. One card in each set is blank; others have slots as indicated.

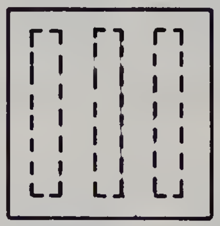


What happens when we place a card from one group on top of one from the other group? The cards are labeled with letters to facilitate talking about them.

For example: combine B with Y, D with X, and A with Z. The results are:


B and Y

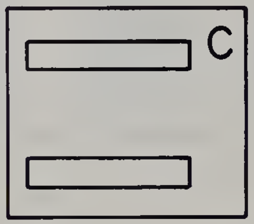

D and X


A and Z

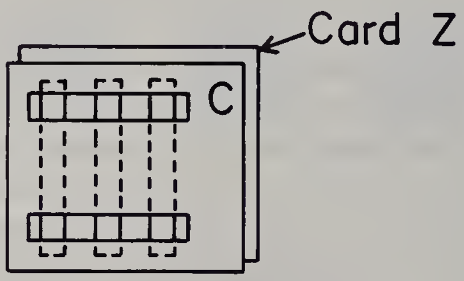
B and Y: two holes through the pair of cards.
D and X: three holes through the pair of cards.
A and Z: no holes through the pair of cards.

This activity will focus attention on the question we are interested in on page 74—how many times do the lines cross in each example? The results, of course, anticipate the first 16 multiplication facts—including those in which one or both of the factors are zero.

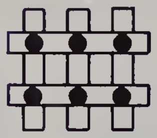
Division is anticipated by a situation such as the following: If we have selected card C,

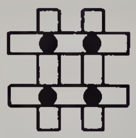


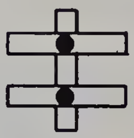
which card in the group with vertical slots must we use so that there are 6 holes through the 2-card sandwich? . . . Card Z, with 3 vertical slots.

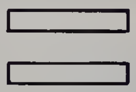


The diagrams below show a possible arrangement using pencils, rods, and strips of paper. Each of the nine situations on page 74 can be similarly constructed. Pupils place counters on each intersection of strips.

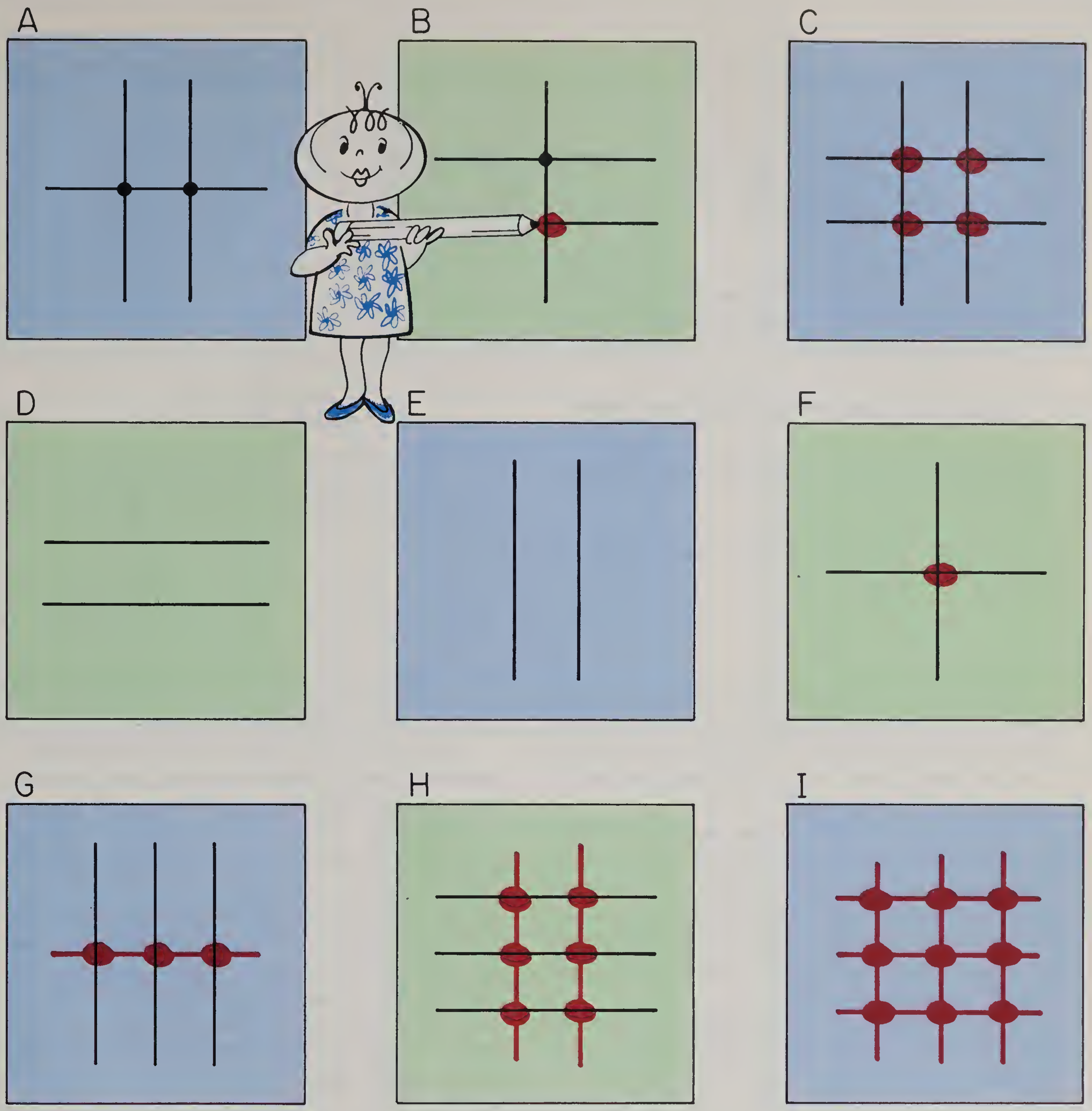

6


4


2


0

Headline Story: Three children were given 2 cookies each. How many cookies altogether?



	A	B	C	D	E	F	G	H	I
— Lines	1	2	2	2	0	1	1	3	3
| Lines	2	1	2	0	2	1	3	2	3
+	2	2	4	0	0	1	3	6	9

Notes on Pupil Page 75

Tasks: Pupils complete the drawings, where necessary, put dots on all the places where two lines cross, and complete the record at the bottom of the page.

Purpose: To continue developing the concept of the product of pairs of numbers chosen from 0, 1, 2, 3, and 4.

Unifying Ideas: Sets; multiplication and division.

Materials for Teacher: Any of the items described below.

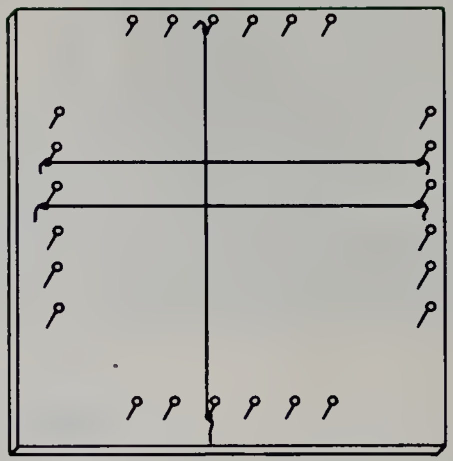
Materials for Pupils: A piece of paper folded to make nine sections.

The Lesson:

This lesson is obviously an extension of the one on the preceding page, so you may make two more cardboard squares with 4 slits each and follow the same procedures, or you may use one or more of the following ideas.

If you have a sand table, you may play "Roads in the Desert" with the stipulations that all roads are straight and that there is a town at every crossroad, but nowhere else. Have the pupils draw pictures of the roads considered in each example, put dots to show towns, and keep a record of the roads (each way) and the towns.

Get a square or rectangular piece of wood and drive in small nails at equal intervals on all four sides:



Use string, heavy thread or rubber bands to show the numbers you are considering. The pupils count the number of strings each way and the number of places where the strings cross. They keep a record, as usual.

Use a corkboard, string, and thumbtacks in the manner described in the preceding paragraph.

When you think your pupils are ready for the more difficult aspects of these situations, pose several problems equivalent to the following and use a variety of combinations of numbers:

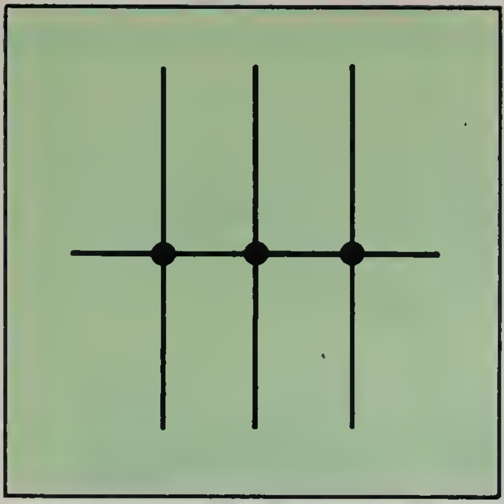
"There are 2 roads going in the same direction across the desert. How many roads going the other way would we have to build if we want 2 towns?"

Assign page 75 for individual study. Pupils who find this lesson easy may be able to answer questions involving 5 or 6 elements in one of the sets, or they may be interested in problems which have more than one correct solution. For instance, referring to the "Roads in the Desert" activity, "We want to have 4 towns. How should we build our roads?" or, "We have 3 roads all going the same way. We don't want any towns. How many more roads can we build?"

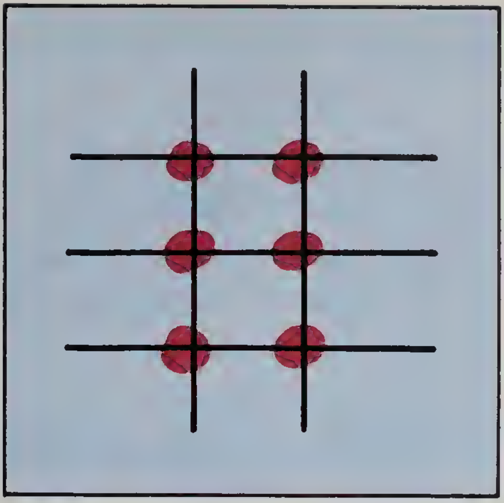
Headline Story: Tell stories to fit these headlines:

$10+10=\square$, $10+\_\_\_=30$, $\_\_\_+10=40$

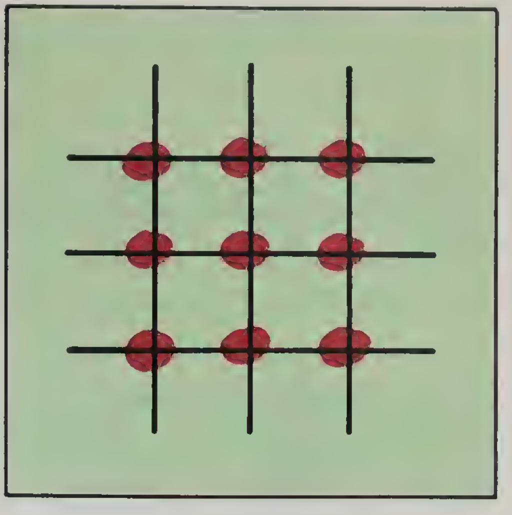
J



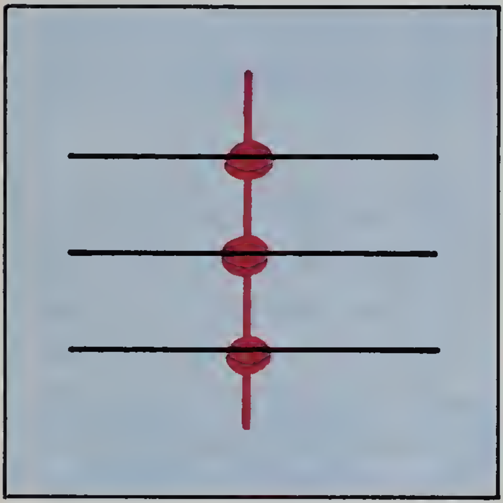
K



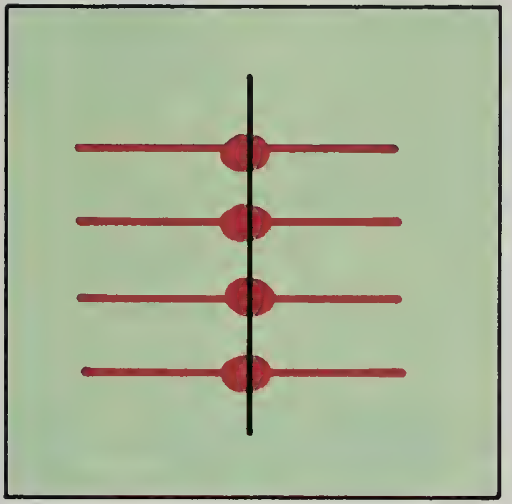
L



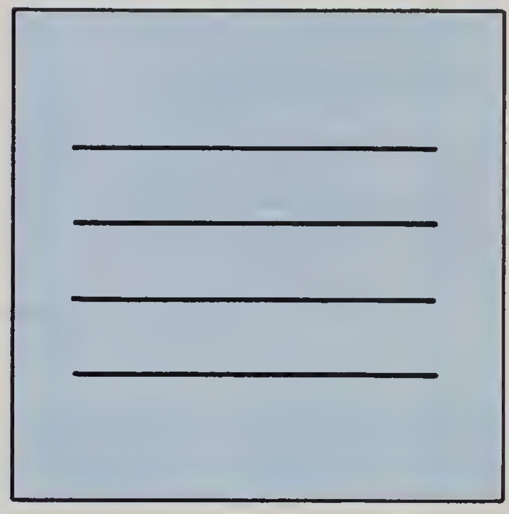
M



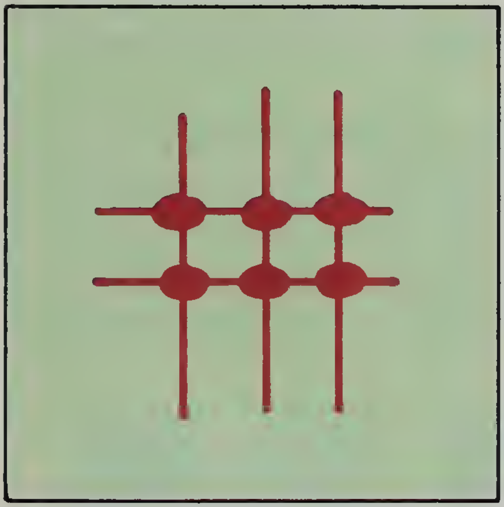
N



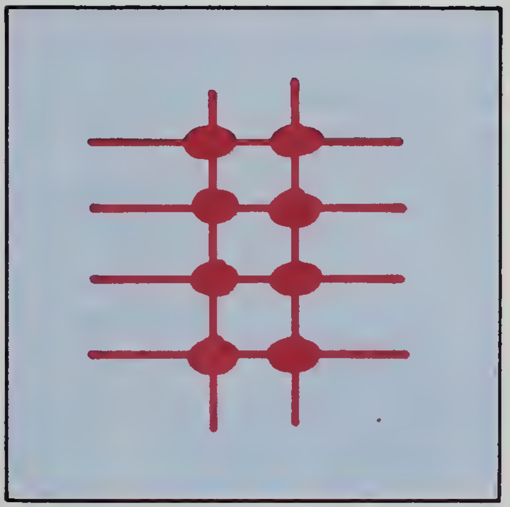
O



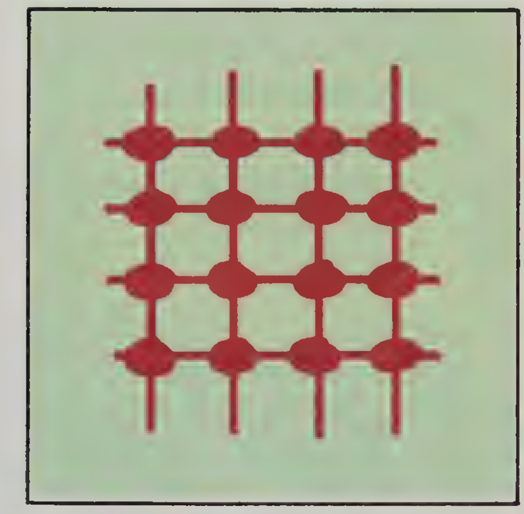
P



Q



R



	J	K	L	M	N	O	P	Q	R
— Lines	1	3	3	3	4	4	2	4	4
| Lines	3	2	3	1	1	0	3	2	4
+	3	6	9	3	4	0	6	8	16

Tasks: *Pupils discover the pattern for each row suggested by the first two number line jumps and corresponding entries in the blue and green blocks beneath each of the first three number lines. (The fourth row contains one less clue than the others.) For each of the four situations they must make use of their discoveries to extend each number pattern.*

Purpose: To provide pupils with more experiences with the operations of addition and subtraction via a number line model and number patterns.

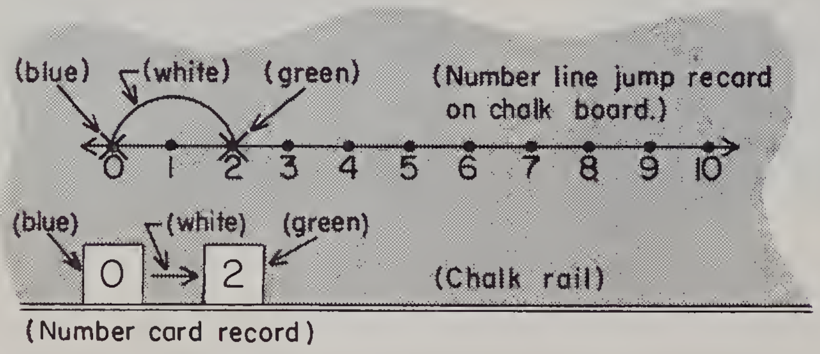
Unifying Ideas: Addition and subtraction; functions and relations.

Materials for Teacher: Demonstration size sets of blue and green number cards (1-10); blue, green, and white chalk.

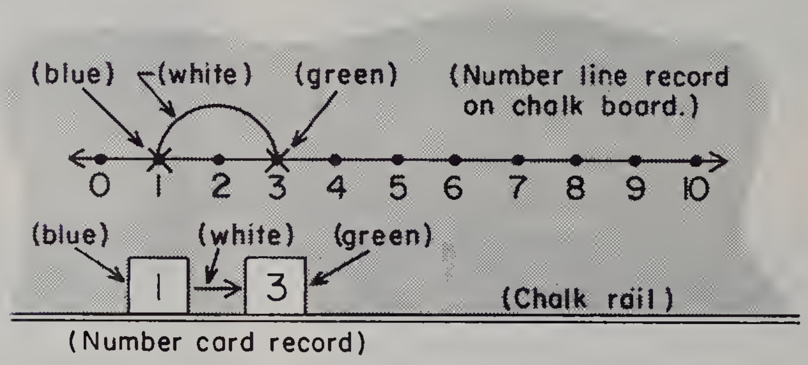
Materials for Pupils: No special materials needed.

The Lesson:

Do some silent teaching to start the lesson off by drawing a number line on the board. Place an "X" with blue chalk on the point 0. Draw a white curved arrow from 0 to the point 2. Place an "X" with green colored chalk on the point 2. On the chalkrail and below the appropriate points on the number line place the blue 0-card and the green 2-card. Draw a white arrow between the two number cards. Here is a sketch of your actions:



Remove the cards and erase the jump record. Follow the same procedure in identifying the number line jump from 1 to 3 and the placement of the blue 1-card and the green 3-card.

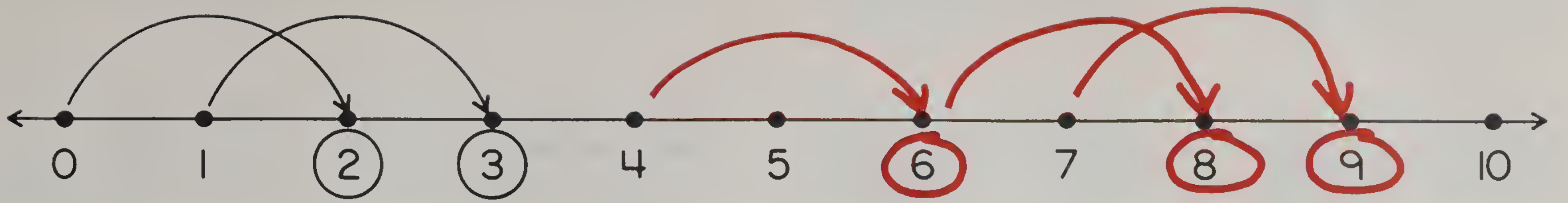


Remove the number card and the jump records. Now place a blue 4-card under the point 4 on the number line. Turn to the class and hold out the three colored pieces of chalk. If no one accepts your silent invitation, quietly select a "volunteer" to come to the board to sketch the chalkboard record of the number line jump. (Blue "X" on the 4; white curved arrow for a 2-space jump; green "X" on the 6.) Look to the class for their silent approval. Continue adding number pairs to this pattern until pupils discover how to extend the pattern easily.

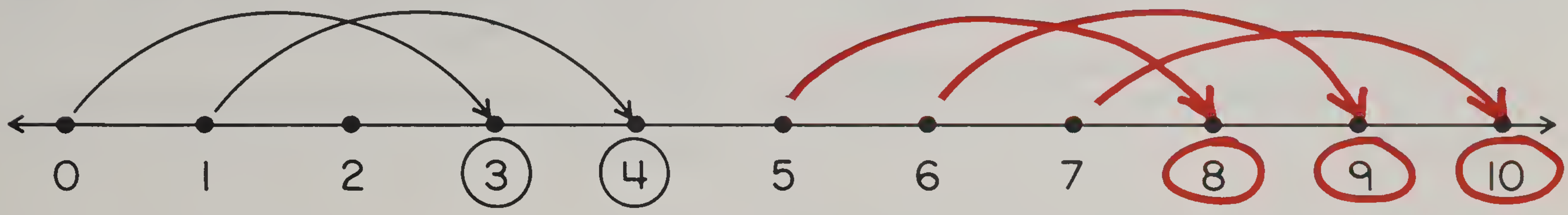
When all pupils have completed page 76, invite someone to play your silent role in presenting his rule for the class to discover. Assist him in developing the silent interaction between chalkboard sketches and the placement of the colored number cards on the chalkrail.

Be sure to review the accompanying booklet, *Games and Enrichment Activities*, for suggestions you may wish to incorporate in your daily lesson plan.

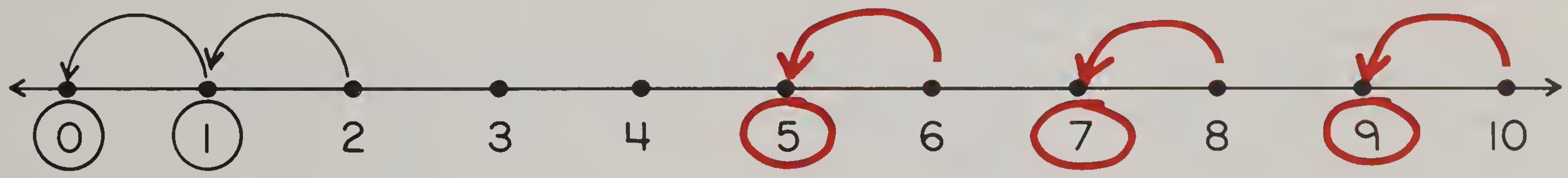
Headline Story: Katie is 3 years older than Lou. If Katie is __, Lou is __.


0
1
2
3
4
5
6
7
8
9
10

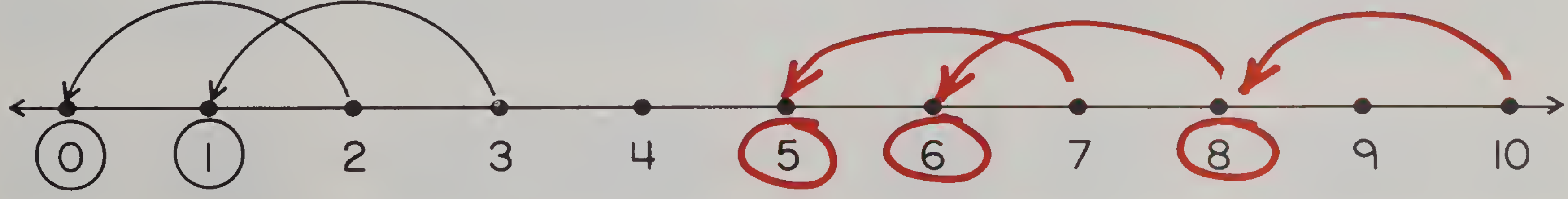
0 → 2   1 → 3   4 → 6   6 → 8   7 → 9


0
1
2
3
4
5
6
7
8
9
10

0 → 3   1 → 4   7 → 10   5 → 8   6 → 9


0
1
2
3
4
5
6
7
8
9
10

1 → 0   2 → 1   8 → 7   6 → 5   10 → 9


0
1
2
3
4
5
6
7
8
9
10

2 → 0   3 → 1   7 → 5   8 → 6   10 → 8

Notes on Pupil Page 77

Tasks: *For each of the three sections, pupils discover the rule for the number line jumps. They must use their discoveries to assist them in completing the number patterns started in the blue and green blocks.*

Purpose: To provide pupils with more experience with the operations of addition and subtraction via a number line model and number patterns.

Unifying Ideas: Addition and subtraction; functions and relations.

Materials for Teacher: Demonstration sets of blue and green number cards (1-20); blue, green, and white chalk.

Materials for Pupils: Several packs of blue and green number cards; counters and Cuisenaire® Rods; dittoed sheets of number lines and several number patterns as described below.

The Lesson:

Assign page 77 for independent study, taking note of those pupils who seem to need more experiences under your direct guidance.

Upon completing the page, pupil pairs might play a game using a pack of blue and a pack of green number cards and one of the ditto sheets containing what is shown in the diagram in the next column.

Team members work together to discover the rule of formation, then play as follows:

"A" randomly selects a blue number card.

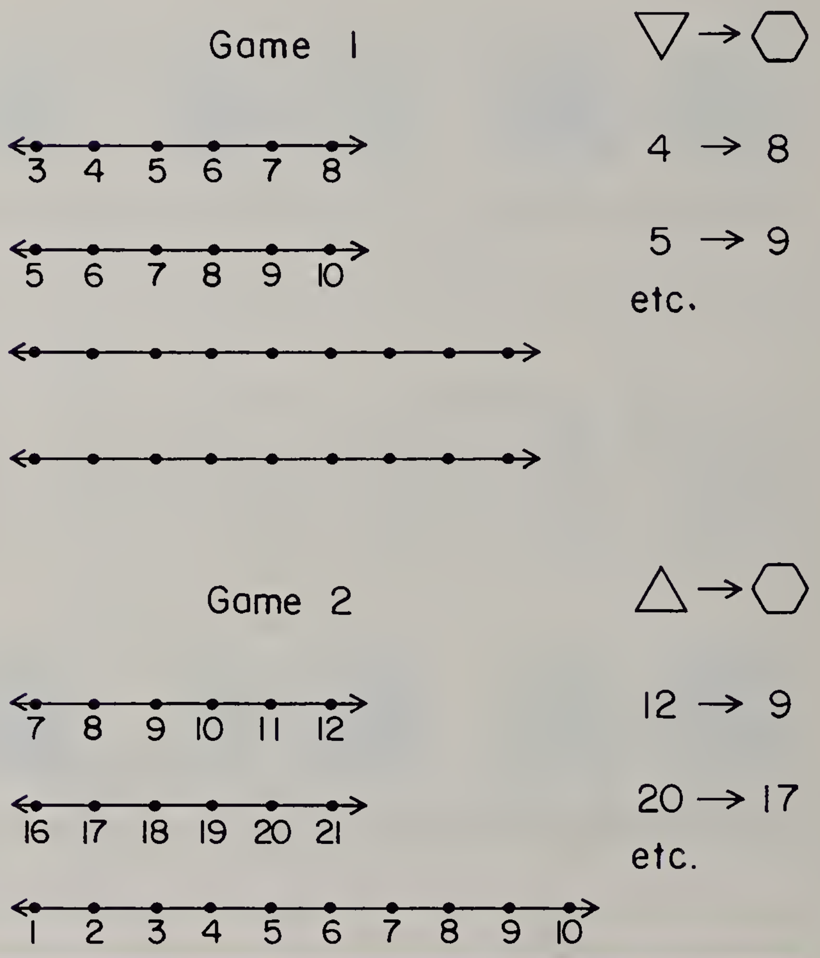
"B" must select the green number card that goes with "A's" selection in accordance with the rule; then randomly selects a green card.

"A" must select the blue card that goes with "B's" selection in accordance with the same rule. Each time a correct pairing is made, "A" and "B" make number line sketches of their pairing.

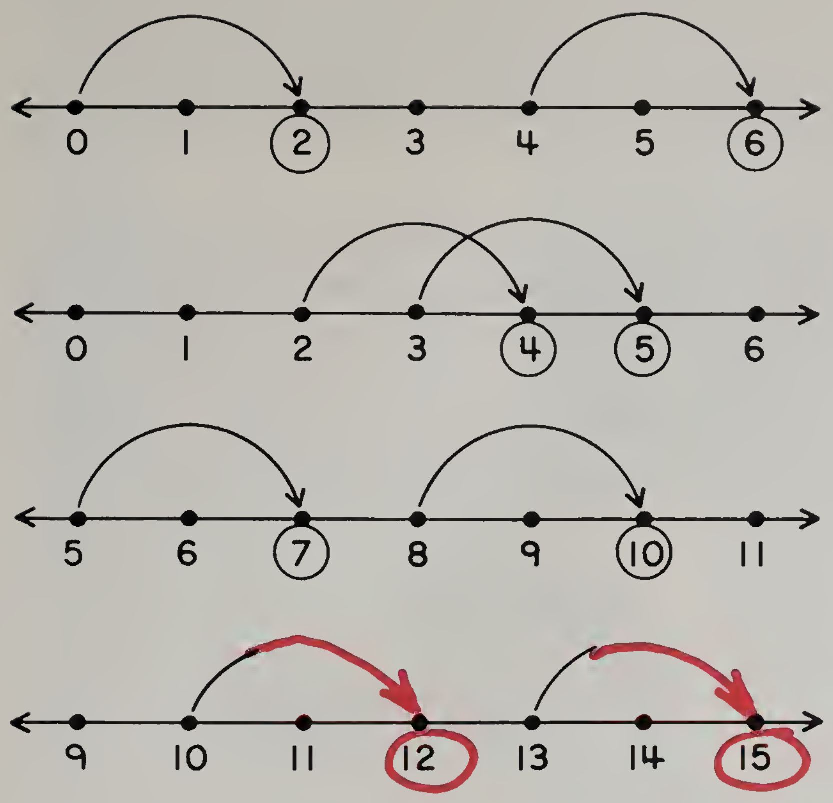
Further understanding of each pattern can be nurtured by having pupils find pairs of rods which follow the pattern.

Note the occurrence of sentences consisting of pairs of frames connected by arrows. Pupils will come to recognize such a sentence as a signal for a "What's My Rule?" game. In using it with number line jumps, you can write the starting number in the left frame and then write the landing number in the right frame. Instead of writing in the frames, you can write under them as is done on the pupil's page.

Have pupils invent their own games of "What's My Rule?"



Headline Story: Mondays fall on __, __, __, __ this month. So Wednesdays fall on __, __, __, __.

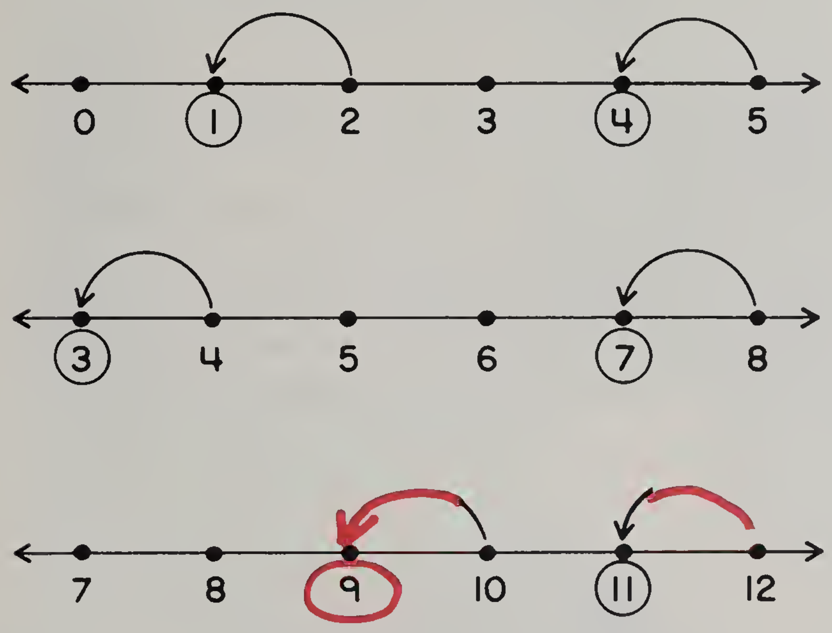

0 1 2 3 4 5 6
0 1 2 3 4 5 6
5 6 7 8 9 10 11
9 10 11 12 13 14 15

▽ → ⬡

0 → 2
3 → 5
5 → 7
10 → 12

▽ → ⬡

4 → 6
2 → 4
8 → 10
13 → 15

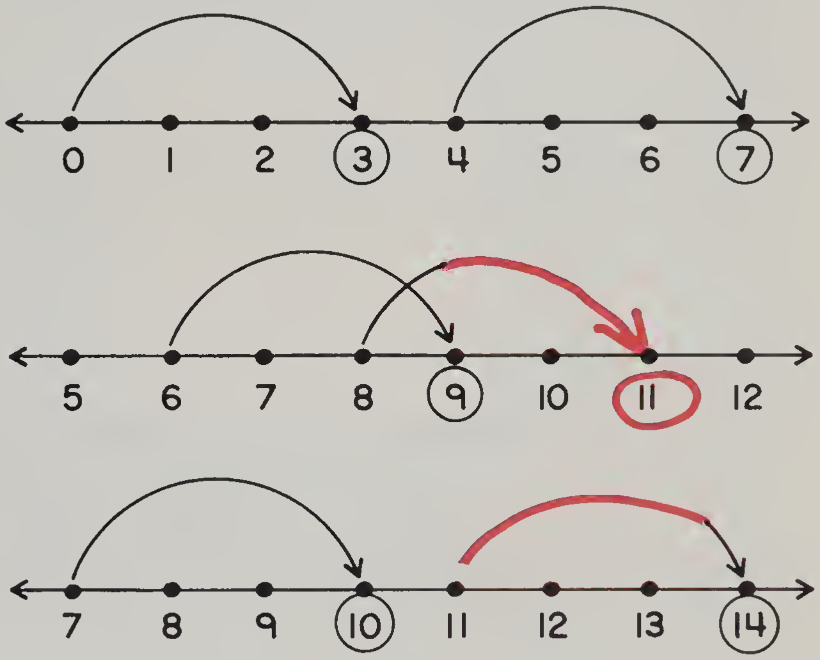

0 1 2 3 4 5
3 4 5 6 7 8
7 8 9 10 11 12

○ → △

2 → 1
4 → 3
10 → 9
14 → 13

○ → △

5 → 4
8 → 7
12 → 11
17 → 16


0 1 2 3 4 5 6 7
5 6 7 8 9 10 11 12
7 8 9 10 11 12 13 14

□ → ⬡

0 → 3
6 → 9
7 → 10
15 → 18

□ → ⬡

4 → 7
8 → 11
11 → 14
17 → 20

Notes on Pupil Page 78

Tasks: *Pupils examine the first Cross Number Puzzle carefully before completing the other three puzzles. They observe the connection between the number sentence labeled A and the green colored vertical and horizontal rows in the Cross Number Puzzle labeled A before writing appropriate number sentences for B, C, and D.*

Purpose: To provide pupils with further experiences with the "any-order—any-grouping" principle for addition via Cross Number Puzzles.

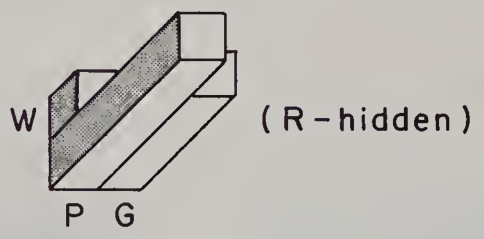
Unifying Ideas: Structure; addition and subtraction.

Materials for Teacher: No special materials are needed.

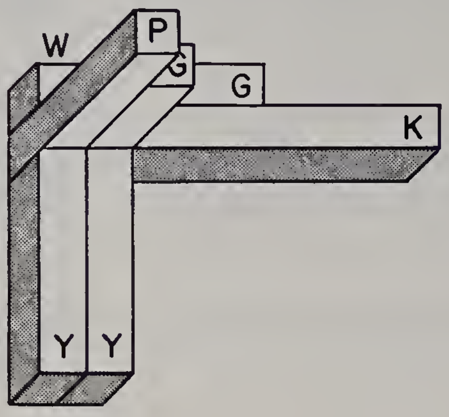
Materials for Pupils: Cuisenaire® Rods; a tag board for each pupil showing a copy of the Cross Number Puzzle form with each cell 1 square centimeter in area.

The Lesson:

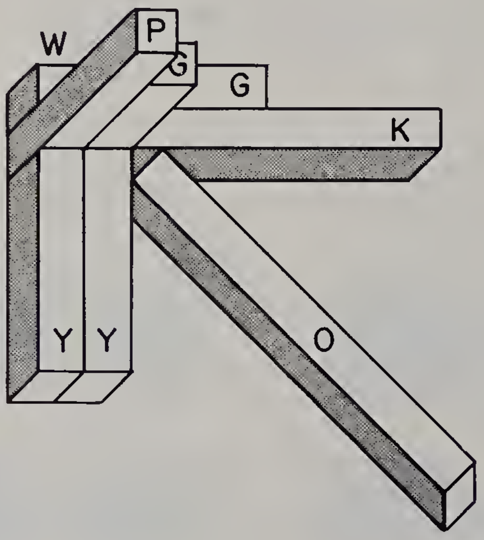
Have the class turn to page 78. Suggest that they stand the appropriate rods in the cells of the Cross Number Puzzle on their tag boards to comply with Cross Number Puzzle A. A sketch of their construction looks like this:



Ask them to place the white rod (1) on top of the red rod (2) and find the rod whose length is that of the vertical train (1 + 2). Have them place the light green rod at the right of the red rod in the first row and return the white rod to its original position. Similar actions upon the purple and light green rods will yield a black rod for the second row. Work on the first column will yield a yellow rod. The second column will also yield a yellow rod. The arrangement should look like this:



Now have them construct a train, using the rods representing the row sums, and find the rod whose length is that of the train (3 + 7). Have them form a train using the rods representing the column sums and find the rod whose length is that of the train (5 + 5). Have them compare the two orange rods and place one of them in the box representing the sum of the row and column sums as follows:



Headline Story: Complete the headline and tell a story that fits:

$$3+1+2+4=\triangle$$

A

	+		
+	1	2	3
	4	3	7
	5	5	10

B

	+		
+	4	2	6
	3	1	4
	7	3	10

C

	+		
+	2	3	5
	1	4	5
	3	7	10

D

	+		
+	4	3	7
	2	1	3
	6	4	10

A 5 + 5 = 3 + 7

B 7 + 3 = 6 + 4

C 3 + 7 = 5 + 5

D 6 + 4 = 7 + 3

Notes on Pupil Page 79

Tasks: *Pupils complete number sentences and draw number line jumps suggested by these sentences.*

Purpose: To help pupils gain more information about (a) the base ten system of numeration, (b) order on the number line, and (c) the operations of addition and subtraction via number line chunks and number patterns involving sums and differences containing 10 as one of the addends.

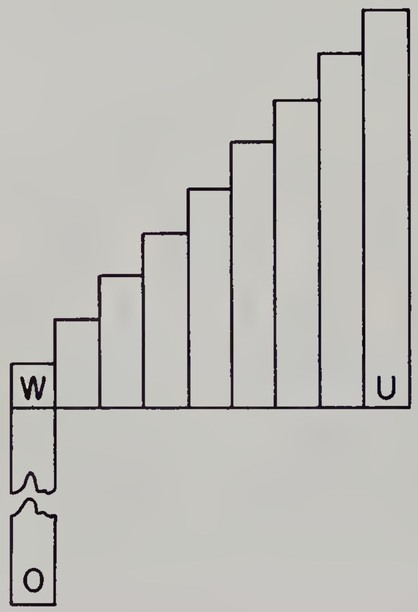
Unifying Ideas: Structure; numeration; addition and subtraction; functions and relations.

Materials for Teacher: Number line chunks (on masking tape); green, blue, red, and white colored chalk.

Materials for Pupils: Cuisenaire® Rods; paper and pencil to sketch number lines on.

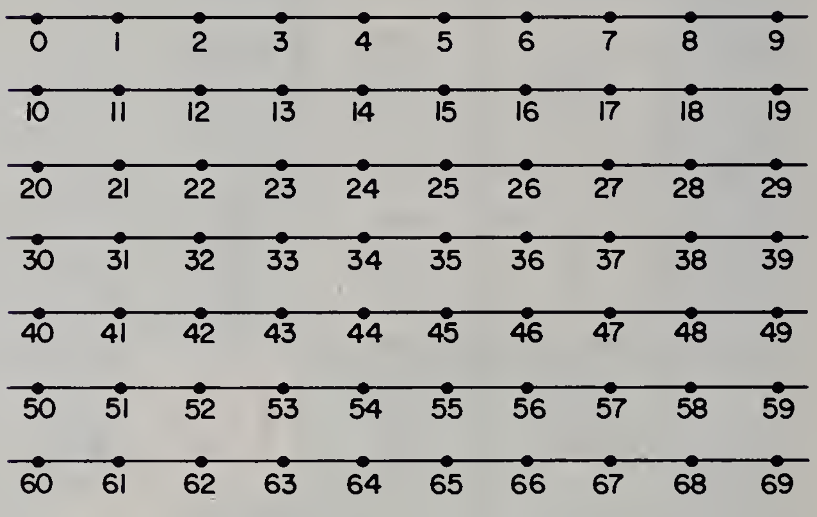
The Lesson:

Check to see that each pupil has ten orange rods and one rod of each of the other colors. Form four-member work teams. Have one member of each team construct a staircase from the white rod to the blue rod with the orange rod starting the next row as shown below:



Have pupils count up the staircase starting with zero. Without disturbing the staircase, have another team member use his set of orange rods to construct models for numbers 11 through 19, with his last orange rod tacked on to the first one to represent 20. Have the class count the teen numbers, pointing to each train in succession. The third team-member uses his orange rods to complete the next row and start the next. Have the class count the numbers 20 through 29, referring to each train in succession. The fourth team-member uses his orange rods to complete the row and start the next one. Finally the first team-member completes the last row with his orange rods.

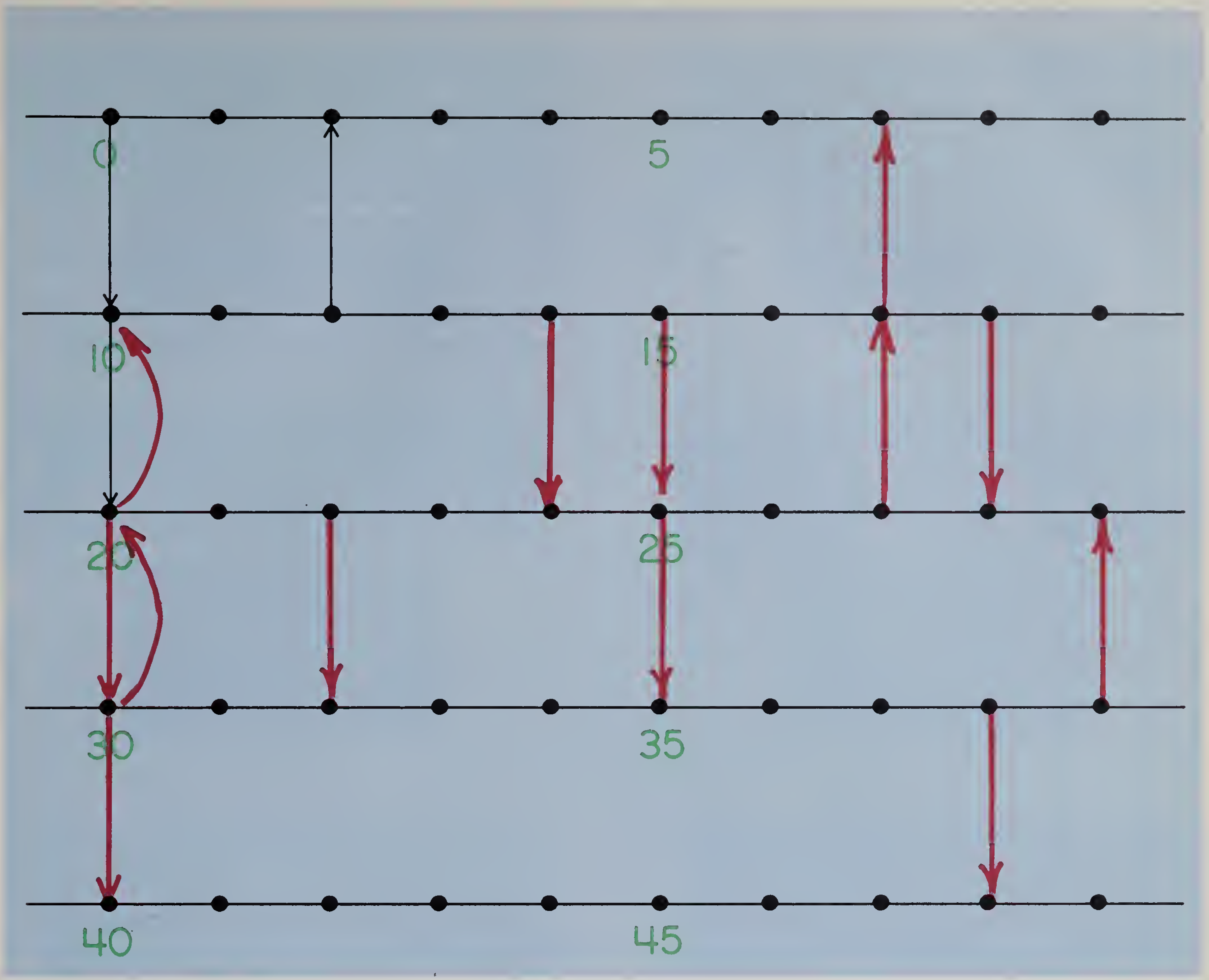
Put your masking tape number line chunks on the chalkboard: letting students point out why the chunks are arranged one above the other. (The chunks are too long for the board.) Pupils should copy the first four or five chunks on their own papers. Then write (with blue chalk) some incomplete number sentences such as those on the bottom half of the pupil's page. Have pupils complete the sentences with green chalk, and describe the addition of 10 by talking about jumps on the number line chunks. ("Start at 15, jump 10, land on 25.")



etc.

Now have the class turn to page 79 for individual study.

Headline Story: Make a headline to fit: Bill lost half of his 16 cards.



0 + 10 = 10

10 + 10 = 20

20 + 10 = 30

30 + 10 = 40

15 + 10 = 25

25 + 10 = 35

14 + 10 = 24

22 + 10 = 32

38 + 10 = 48

18 + 10 = 28

$$\begin{array}{r}12\\-10\\\hline 2\end{array}\qquad\begin{array}{r}20\\-10\\\hline 10\end{array}\qquad\begin{array}{r}30\\-10\\\hline 20\end{array}$$

$$\begin{array}{r}17\\-10\\\hline 7\end{array}\qquad\begin{array}{r}27\\-10\\\hline 17\end{array}\qquad\begin{array}{r}39\\-10\\\hline 29\end{array}$$

Notes on Pupil Page 80

Tasks: *Pupils write number sentences suggested by indicated jumps on the number line, or indicate jumps suggested by number sentences. Pupils complete the pattern in the bottom row.*

Purpose: To test pupil achievement.

Materials for Teacher: No special materials are needed.

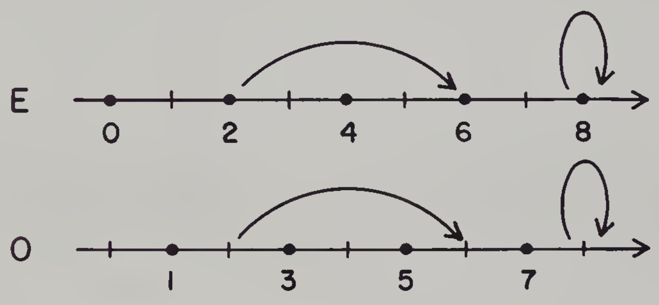
Materials for Pupils: No special materials needed.

The Lesson:

Draw a pair of number lines on the board, one with even-number labels and one with odd-number labels. Write mathematical sentences and indicate each on both number lines.

Begin with pairs of even addends:

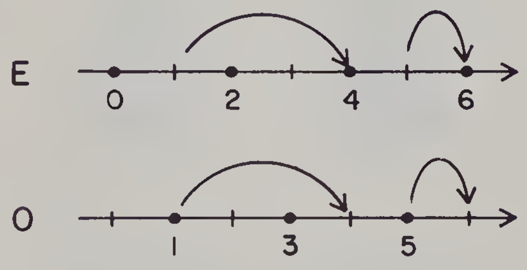
2 + 4 = 6　　8 + 0 = 8



Notice that when you begin with a pair of even addends, the starting and landing points are both labeled on the E (even) line and neither is labeled on the O (odd) line.

What happens when a pair of odd addends are considered?

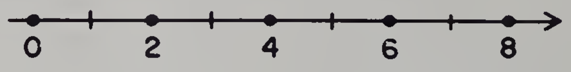
1 + 3 = 4　　5 + 1 = 6



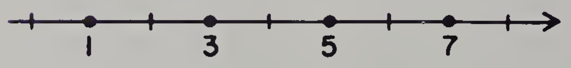
In the two examples we start on line E at an unlabeled point and end on a labeled point. The opposite is true on line O—we start at a labeled point and end on an unlabeled point.

Under each of the number lines, have pupils write number sentences that meet specified conditions.

1) Start and land on labeled points.

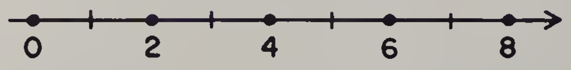


2 + 4 = 6
8 − 6 = 2
2 + 2 = 4
etc.

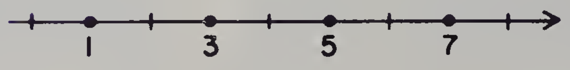


1 + 2 = 3
7 − 6 = 1
3 + 4 = 7
etc.

2) Start on a labeled point and land on an unlabeled point.

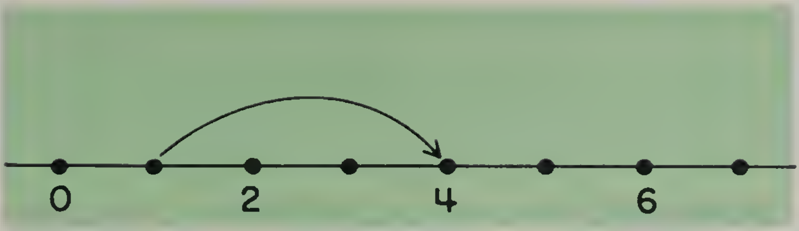


2 + 1 = 3
8 − 7 = 1
etc.

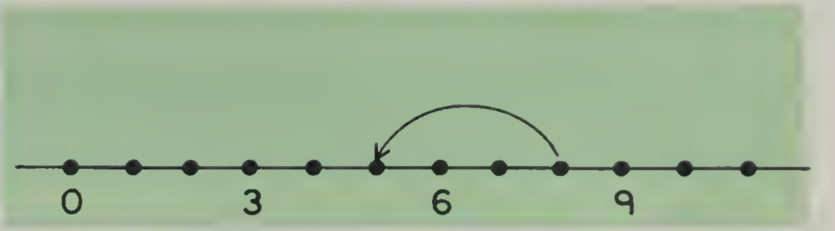


3 + 5 = 8
7 − 1 = 6
etc.

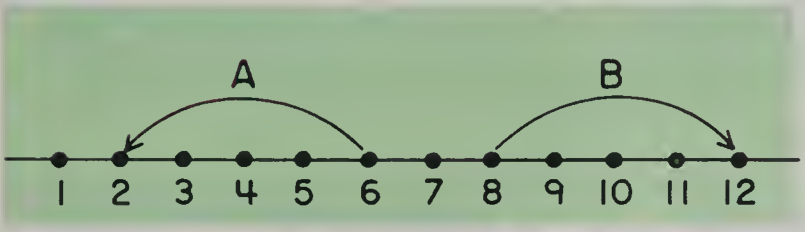
Headline Story: Two boys and 3 girls divide a pie evenly. Draw a picture.



1 + 3 = 4

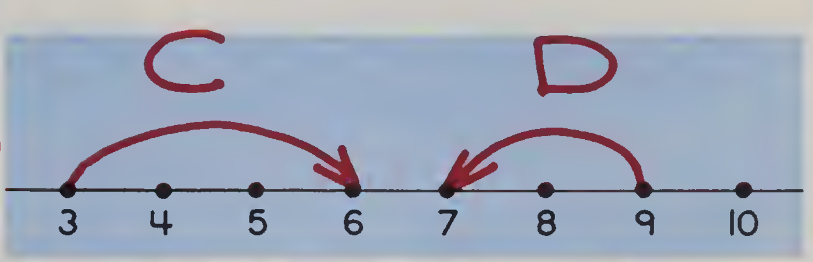


8 - 3 = 5


A
B
1
2
3
4
5
6
7
8
9
10
11
12

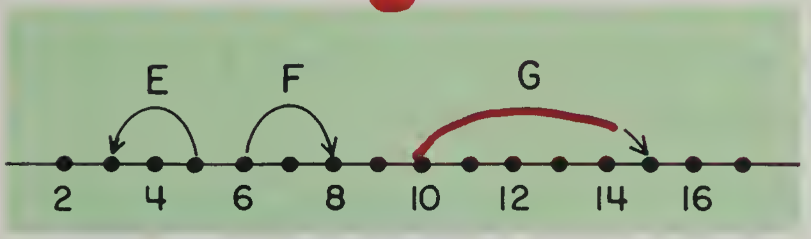
A. 6 - 4 = 2

B. 8 + 4 = 12


C
D
3
4
5
6
7
8
9
10

C. 3 + 3 = 6

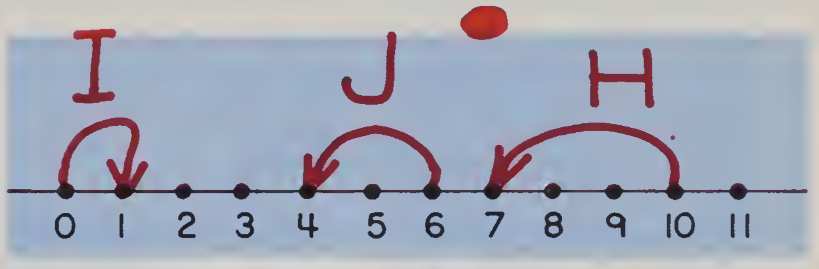
D. 9 - 2 = 7


E
F
G
2
4
6
8
10
12
14
16

E. 5 - 2 = 3

F. 6 + 2 = 8

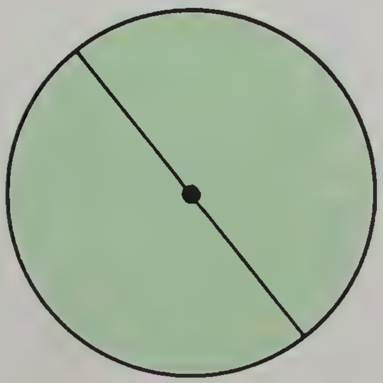
G. 10 + 5 = 15

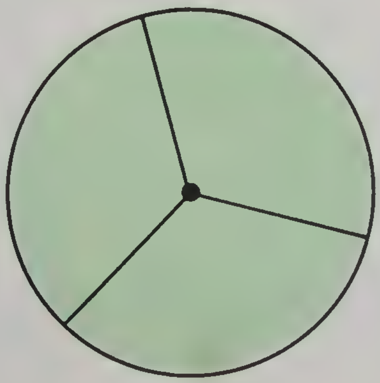

I
J
H
0
1
2
3
4
5
6
7
8
9
10
11

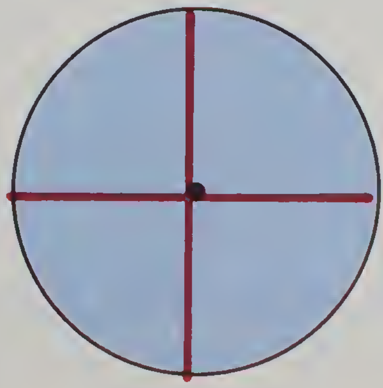
H. 10 - 3 = 7

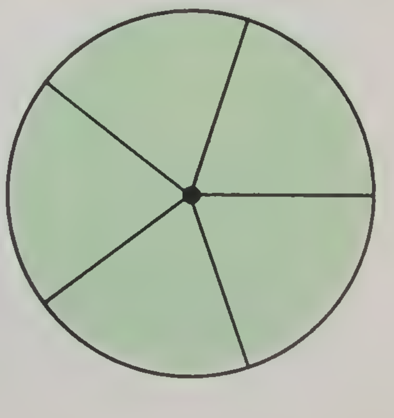
I. 0 + 1 = 1

J. 6 - 2 = 4









Notes on Pupil Page 81

Tasks:

Purpose: To test pupil achievement.

Materials for Teacher: No special materials are needed.

Materials for Pupils: No special materials are needed.

The Lesson:

Suppose we write pairs of true statements, one under the other, in this fashion:

$$\begin{array}{ccc} \text{(a)} & \text{(b)} & \\ 2+3=5 & 7+1=8 & \text{etc.} \\ 6+4=10 & 3+3=6 & \end{array}$$

Under each column of two numbers, write the sum of those numbers. Example (a), then, looks like this:

$$\begin{array}{ccccc} 2 & + & 3 & = & 5 \\ 6 & + & 4 & = & 10 \\ \hline 8 & & 7 & & 15 \end{array}$$

What do you notice about the results? You can write:

$$8 + 7 = 15$$

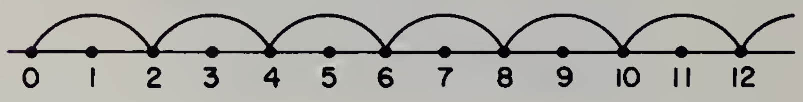
Will this happen with every pair of true statements of addition facts? (It will.)

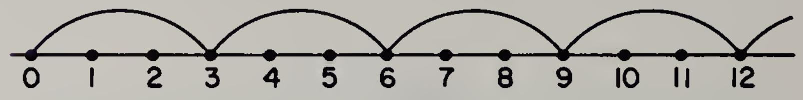
Experiment with pairs of subtraction facts written in a similar fashion:

$$\begin{array}{ccc} \text{(m)} & \text{(n)} & \\ \begin{array}{ccccc} 8 & - & 3 & = & 5 \\ 4 & - & 1 & = & 3 \\ \hline 12 & & 4 & & 8 \end{array} & \begin{array}{ccccc} 13 & - & 3 & = & 10 \\ 10 & - & 1 & = & 9 \\ \hline 23 & & 4 & & 19 \end{array} & \text{etc.} \end{array}$$

Is it true that 12 — 4 = 8 and 23 — 4=19? . . . etc.?

Indicate repeated jumps of 2 on a number line, starting from the point 0. Record the landing points. Do the same with jumps of 3 on another number line.





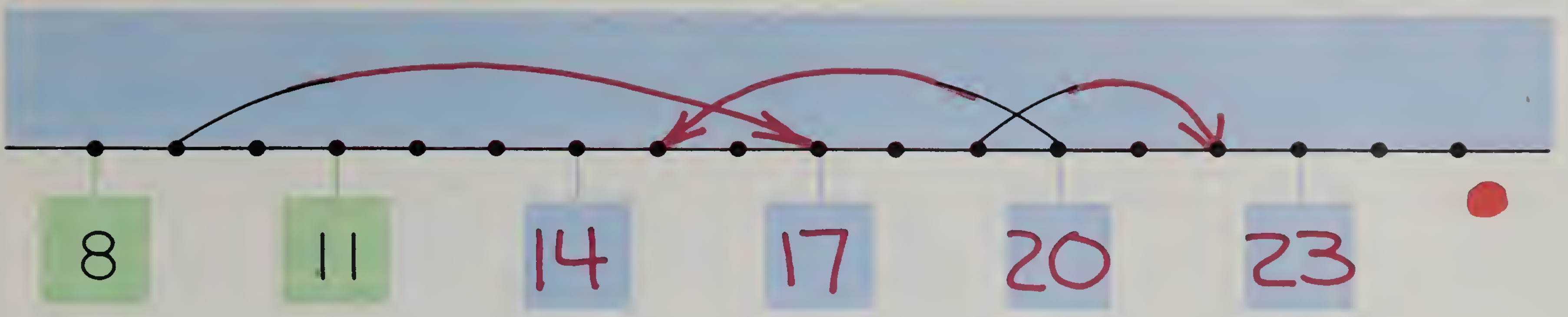
Jumps of 2 — 0 , 2 , 4 , 6 , 8 , 10 , 12,...
Jumps of 3 — 0 , 3 , 6 , 9 , 12,...

Suppose we add each column in our records. Our results are:

0 , 5 , 10 , 15 , 20,...

Extend each row. Does the pattern continue? When we add each column, do we get a record of repeated jumps? What size are the jumps?

Headline Story: It's 4:30. What time was it an hour ago? . . . 2 hours ago? . . . etc.


8
11
14
17
20
23

9 + 8 = 17

20 - 5 = 15

19 + 3 = 22

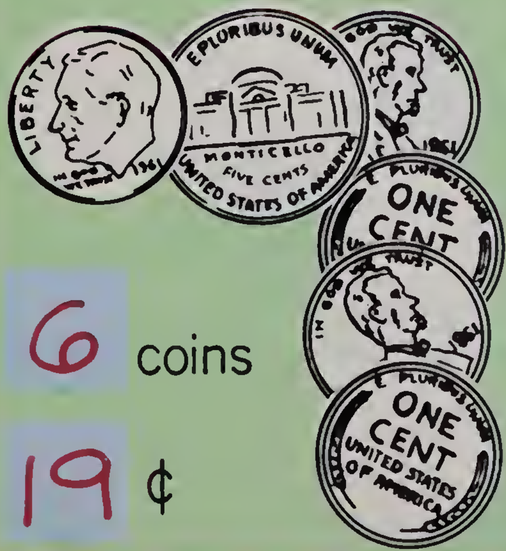
6	0	6
3	9	12
9	9	18

3	5	8
7	9	16
10	14	24

5	10
15	20
25	30
35	40
45	50
55	60
65	70
75	80

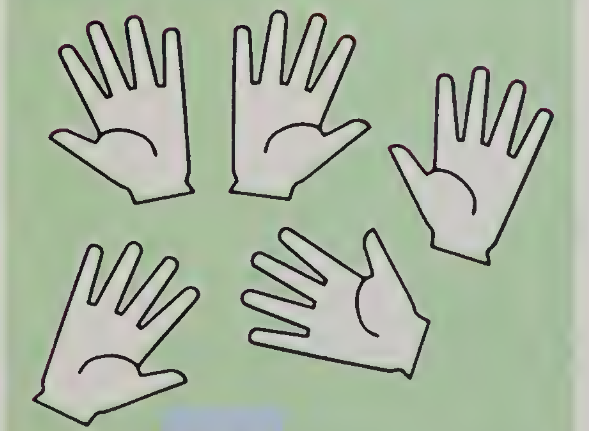
20	10	10
6	5	1
14	5	9

5	7	12
12	4	16
17	11	28

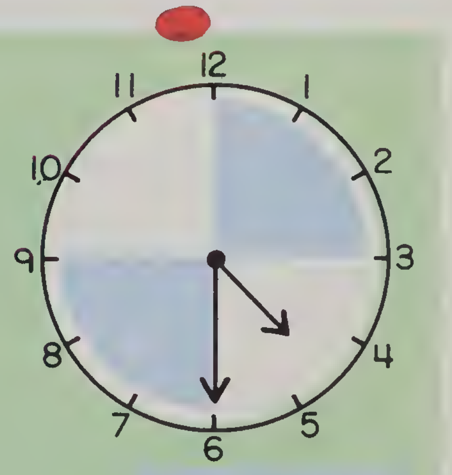


6 coins

19 ¢



25 fingers and thumbs


12
1
2
3
4
5
6
7
8
9
10
11

The time is 4:30

Notes on Pupil Page 82

Tasks: *Pupils color the interior of like rectangles the same color; establish and write the pattern of multiples of 5 based on the clue for rectangle A in section n; finally, they write the sums of various multiples of 5 found in sections O through S.*

Purpose: (a) To provide pupils with further experience in using numbers as measures of area and time and (b) to anticipate multiplication via simple addition situations involving multiples of 5.

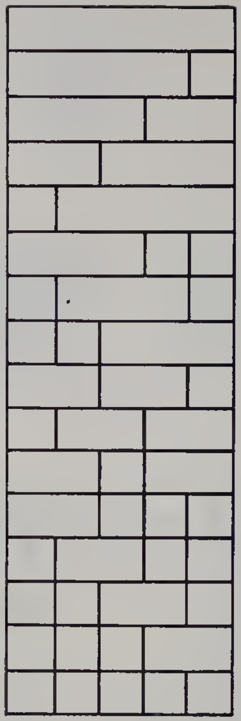
Unifying Ideas: Measurement; addition and subtraction; multiplication and division.

Materials for Teacher: A demonstration clockface with hour and minute hands.

Materials for Pupils: Cuisenaire® Rods.

The Lesson:

Have the pupils place a yellow rod on their desks. Ask them to construct as many different looking trains as possible which are as long as the yellow rod. Suggest that they proceed by first constructing trains using two rods, then three, then four, and finally, five rods in a train. Play some If-Then games, with the class referring to their patterns. *If* the white rod's name is 1, *then* how can we describe each row of the pattern? Proceed in a similar fashion naming the white rod 2, then 3, then 4, and finally 5. A summary of their construction and sentences when the white rod is named 5 is sketched below.



25 = 25
25 = 20 + 5
25 = 15 + 10
25 = 10 + 15
25 = 5 + 20
25 = 15 + 5 + 5
25 = 5 + 15 + 5
25 = 5 + 5 + 15
25 = 10 + 10 + 5
25 = 5 + 10 + 10
25 = 10 + 5 + 10
25 = 10 + 5 + 5 + 5
25 = 5 + 10 5 5
25 = 5 + 5 + 10 + 5
25 = 5 + 5 + 5 + 10
25 = 5 + 5 + 5 + 5 + 5

Pupils can now turn to page 82 for individual study. The breakout at the lower left of the pupil page can be handled somewhat as follows: Have each pupil hold a white rod in his hand. Using a clock or watch with a second hand sweep, give a signal for the class to squeeze the white rod as tightly as they can until you give them the signal to stop. After one minute of squeezing tell the class to relax. Ask how long they think they squeezed the white rod. Call the white rod "one minute." Have the class count out "sixty minutes."

Have them form as many "five minute" trains as possible. Have them replace the first five minute train by a yellow rod. Ask what the name of the yellow rod is if the white rod's name is one minute. As each succeeding train is replaced by a yellow rod, have them count by five minute intervals: e.g., five minutes, ten minutes, fifteen minutes . . . sixty minutes.

Headline Story: What time was it an hour and a half before noon?

m

5 Color A
10 B
15 C
20 D
25 E

n

15 C
20 D
5 Color A
25 E
10 B

o

10 5 5
10 10

p

20 10 10
20 20

q

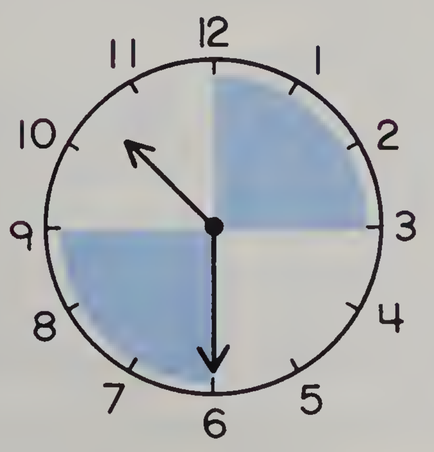
15 10 5
15 15

r

25 25
25 20 5

s

25 15 10
25 25


12
1
2
3
4
5
6
7
8
9
10
11

The time is 10:30

Notes on Pupil Page 83

Tasks: *Pupils complete number sentences (or fact teams) in each section. Each sentence must represent a condition suggested by the picture.*

Purpose: To provide pupils with further opportunities to explore the commutative property of addition and the operations of addition and subtraction, via fact teams.

Unifying Ideas: Structure; addition and subtraction; functions and relations.

Materials for Teacher: No special materials are needed.

Materials for Pupils: Two sets of colored counters or disks.

The Lesson:

Distribute four disks of one color and two of another color to each pupil. Ask the pupils to count the number of disks in each set of colored disks as well as how many in both sets together. Write and read aloud two number sentences to them ($4 + 2 = 6$, $2 + 4 = 6$), showing how each describes each collection of disks. Ask the pupils to take away the four disks of one color from their collection and tell how many are left. Write and read aloud the appropriate number sentence ($6 - 4 = 2$). After the pupils replace the set of four disks, have them remove the other two disks. Have someone tell the group the number sentence that describes the action.

Have pupils turn to page 83 and examine section A carefully. "What does the 3 refer to? The 2? How many dots in all? What number should we write in the *first* blue box?" Have someone tell the class the meaning of the second incomplete sentence. "What number should we write in the *second* blue box? If we have five dots and we color in three of them, how many are left uncolored? What number should we write in the *third* blue box? If we have five dots and we color two of them white, how many remain to be colored? What number should we write in the *fourth* blue box?" Have a pupil read each sentence of the fact team;

Three plus two equals five.
Two plus three equals five.
Five minus three equals two.
Five minus two equals three.

Have the class take note of the fact that sections A, B, C, D, and E have four-member fact teams while sections F, G, and H have only two-member fact teams. "What connection does the number of members of the fact teams for F, G, and H have with the addends of each sum for F, G, and H? What is the connection between the number of members of each fact team for A, B, C, D, and E and the addends of each sum for A, B, C, D, and E?"

Pupils who correctly complete the pages should form buzz groups to puzzle over the meanings of the following fact teams and report their observations to the rest of the class. (The letters refer to the sections of their page.)

①	②	③
$A + C = E$	$C + G = D$	$F + H = E$
$C + A = E$	$G + C = D$	$H + F = E$
$E - C = A$	$D - G = C$	$E - H = F$
$E - A = C$	$D - C = G$	$E - F = H$

Able students will respond well to questions which emphasize that adding a number is the inverse of subtracting a number and conversely. Start with questions which are easy to answer by computing, such as:

Bill has 4 counters. He adds 3 counters to these. Then he subtracts 3 counters. How many are left?
Start with 4. Add 3. How many altogether?
Subtract 3. Answer?
What is $5 - 4 + 4$? Etc.

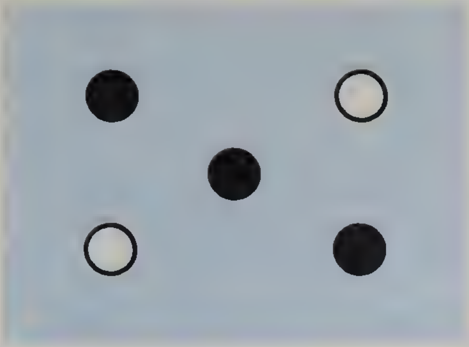
Then move to more difficult questions, such as:

Start with 59. Add 7.
Subtract 7. Answer?

Pupils who can answer these questions rapidly have caught on to the important relation between adding a number and subtracting that number.

Headline Story: Make a headline to fit: Paul spent 6¢ less than a quarter.

A



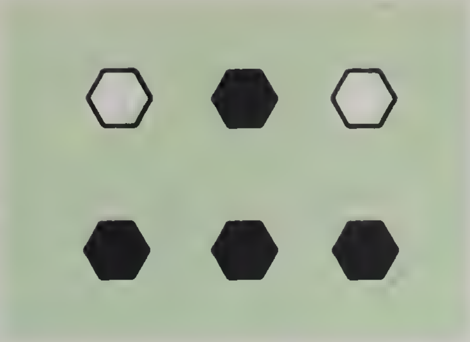
3 + 2 = 5

2 + 3 = 5

5 - 3 = 2

5 - 2 = 3

B



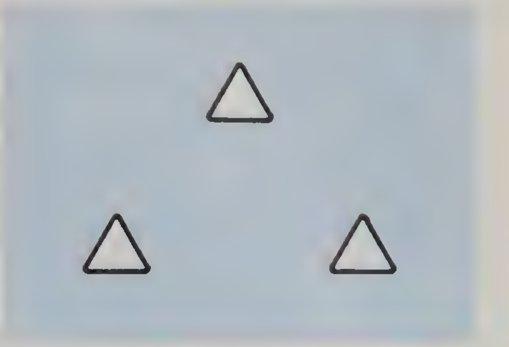
4 + 2 = 6

2 + 4 = 6

6 - 4 = 2

6 - 2 = 4

C



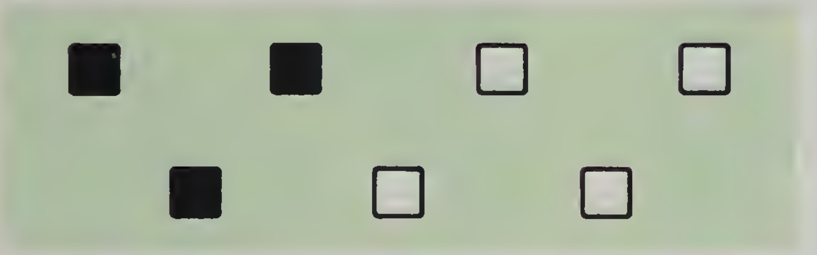
3 + 0 = 3

0 + 3 = 3

3 - 3 = 0

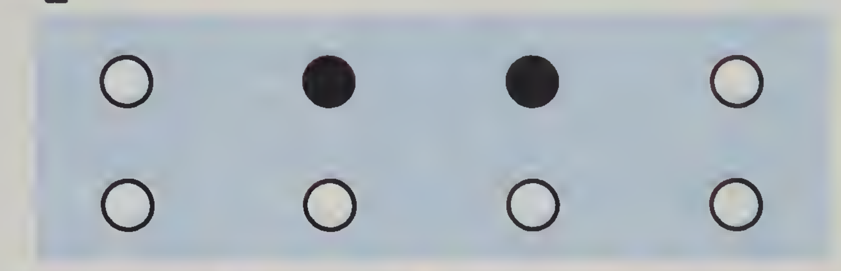
3 - 0 = 3

D



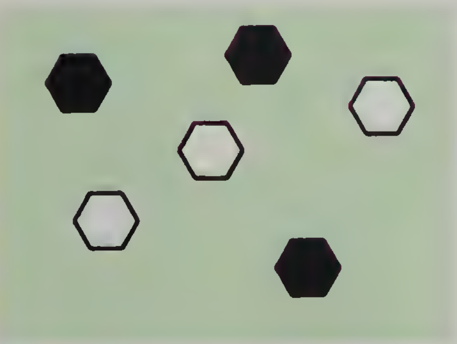
4	3	7	7
+ 3	+ 4	- 4	- 3
7	7	3	4

E



2	6	8	8
+ 6	+ 2	- 6	- 2
8	8	2	6

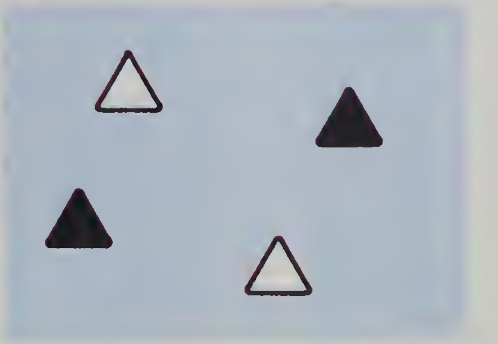
F



3 + 3 = 6

6 - 3 = 3

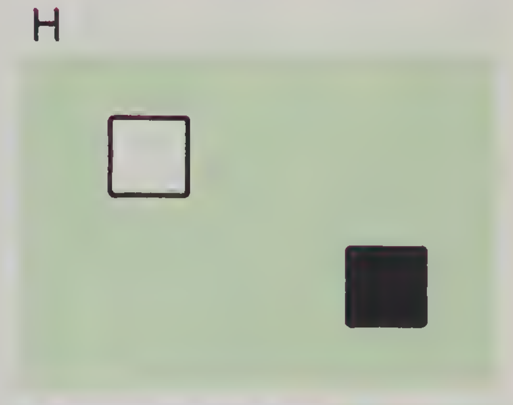
G



2 + 2 = 4

4 - 2 = 2

H



1 + 1 = 2

2 - 1 = 1

Notes on Pupil Page 84

Tasks: *Pupils examine each picture and complete each sentence of the fact team related to the picture.*

Purpose: To provide further experiences with the commutative property of addition and the operations of addition and subtraction, via fact teams.

Unifying Ideas: Structure; addition and subtraction; measurement.

Materials for Teacher: No special materials are needed.

Materials for Pupils: Cuisenaire® Rods.

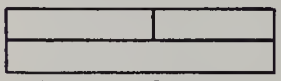
The Lesson:

Have the class turn to page 84 for independent study. Those requesting help can be guided with Cuisenaire® Rods used in either of two ways to work out each situation.

1) *Use of rods as counters.* The first picture shows five counters of one color and three of another. Pupils can match, say, five red rods with the five shaded disks and three light green rods with the three unshaded disks and count each collection and then the total collection. They might place the five red rods in front of them first and then include the three light green rods and count the total collection. This action is represented by the first sentence. Pupils who need more sensory data involving actual actions should be encouraged to use concrete materials to count their way through the other problem situations displayed on the page.

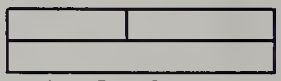
2) *Use of lengths of rods to represent counts.* A count of the objects displayed in the third picture shows five rectangular counters and four disks. Pupils can select a yellow rod for the five counters and a purple rod for the four disks. A train of the yellow and purple rods is as long as a blue rod. This triplet of rods can represent the complete fact team as follows:

a.



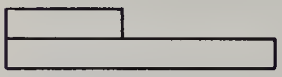
5 + 4 = 9

b.



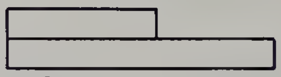
4 + 5 = 9

c.



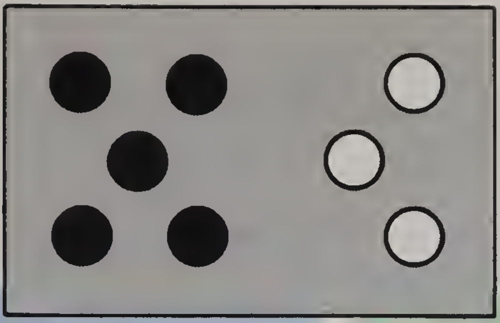
9 − 4 = 5

d.



9 − 5 = 4

Headline Story: Make a headline to fit: Bob had 18 pennies. Half were in each of two pockets.

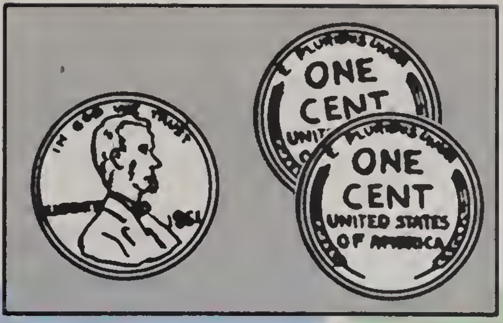


5 + 3 = 8

3 + 5 = 8

8 - 5 = 3

8 - 3 = 5

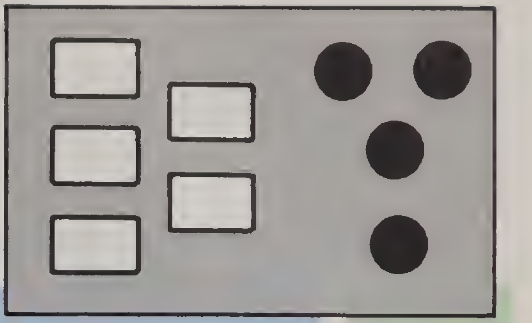

IN GOD WE TRUST
ONE CENT
UNITED STATES OF AMERICA
ONE CENT
UNITED STATES OF AMERICA

1 + 2 = 3

2 + 1 = 3

3 - 2 = 1

3 - 1 = 2

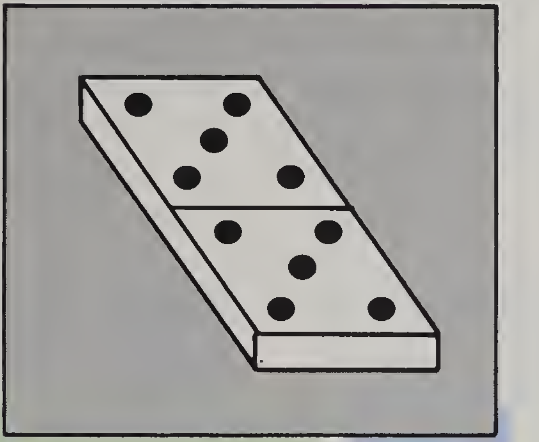


5 + 4 = 9

4 + 5 = 9

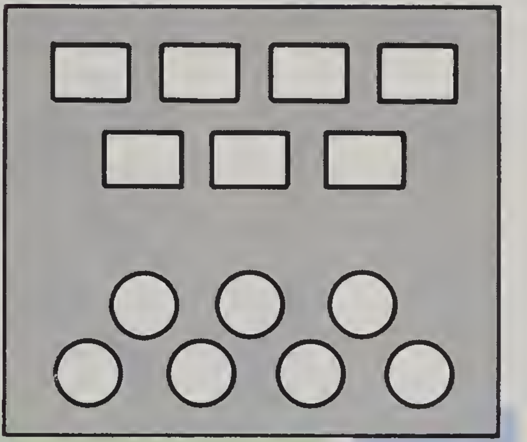
9 - 5 = 4

9 - 4 = 5



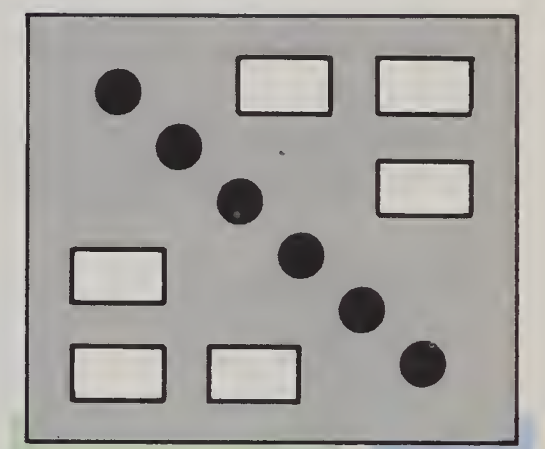
5 + 5 = 10

10 - 5 = 5



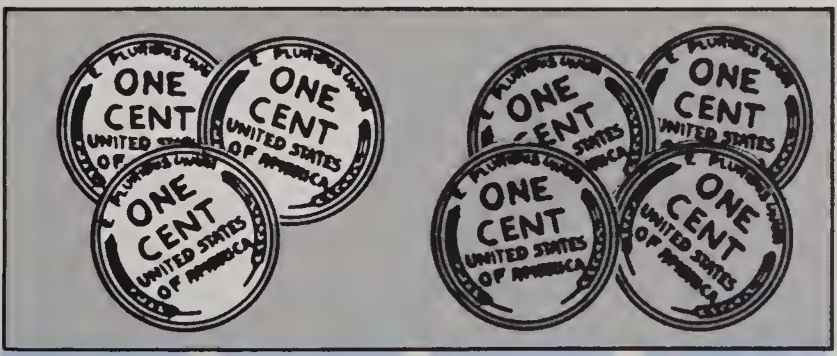
7 + 7 = 14

14 - 7 = 7

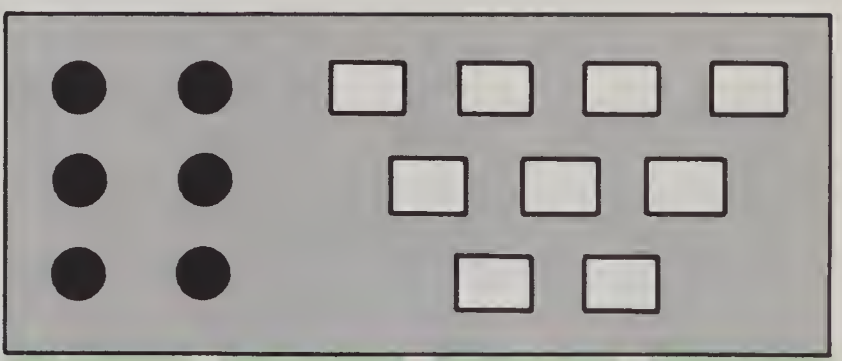


6 + 6 = 12

12 - 6 = 6


ONE CENT
UNITED STATES OF AMERICA
ONE CENT
UNITED STATES OF AMERICA
ONE CENT
UNITED STATES OF AMERICA
ONE CENT
UNITED STATES OF AMERICA
ONE CENT
UNITED STATES OF AMERICA
ONE CENT
UNITED STATES OF AMERICA
ONE CENT
UNITED STATES OF AMERICA

3	4	7	7
+4	+3	-4	-3
7	7	3	4



6	9	15	15
+9	+6	-9	-6
15	15	6	9

Notes on Pupil Page 85

Tasks: *In each section, pupils discover the rule governing the number line jumps and number pattern formation. They use this information to make the appropriate entries in the blue and green boxes of the number patterns below each number line. For the breakout at the bottom right of the page, pupils write in the blue box the time suggested by the clock face pictured.*

Purpose: (a) To provide further experiences with the operations of addition and subtraction via number line jumps and number patterns; (b) to use numbers in situations involving measurement (time).

Unifying Ideas: Structure; addition and subtraction; functions and relations; measurement.

Materials for Teacher: A demonstration clockface with minute and hour hands.

Materials for Pupils: Cuisenaire® Rods.

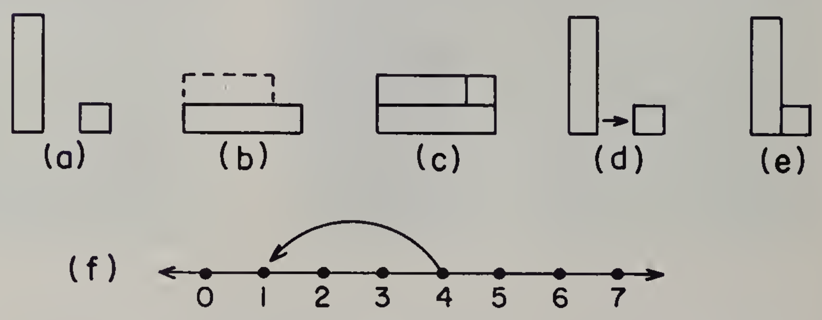
The Lesson:

Write a list of the numbers 0 through 20 on the board. Have the class count as you point to each number. Place a check mark above the 0, 2, and 4 and invite someone to place check marks above the other even numbers. Have the class count by two's, starting with 0.

Place a check mark below the 1, 3, and 5 and invite someone to continue your pattern. Have the class count by two's, starting with 1.

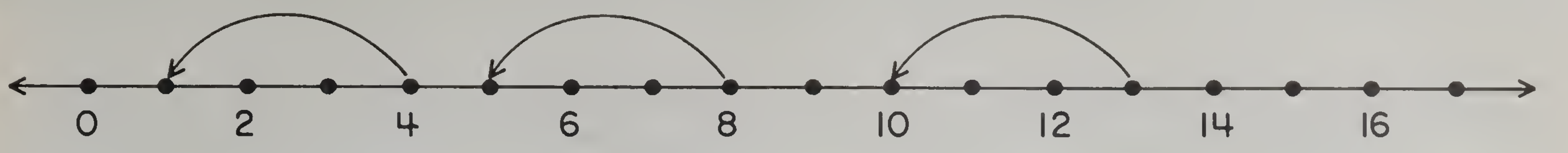
The class can now turn to page 85. Have them note the following: In section A, the arrowhead of the first jump points to an unlabeled point. Ask which number pair below the line tells them the number the arrow points to. Have them describe the connection between the second jump and the second number pair. In section B, have them describe the connection between the first two jumps and the first two number pairs. Most pupils can now rapidly complete page 85 on an individual basis. For those who seem to need more sensory data, proceed somewhat as follows.

Have the group place a purple rod and a white rod on their desks. Ask what rod should cover the purple rod so that the white rod fills the remaining space. Have them find a light green rod and cover the purple rod to verify that the white rod fills the space. Have them return the light green rod to their pile and move the purple toward the white saying, "4 goes to 1." Describe their actions on a number line. Here is a sketch of the activities just described:



Headline Story: It's 7 o'clock. What time will it be in 3 hours? . . . in 5 hours? . . . etc.

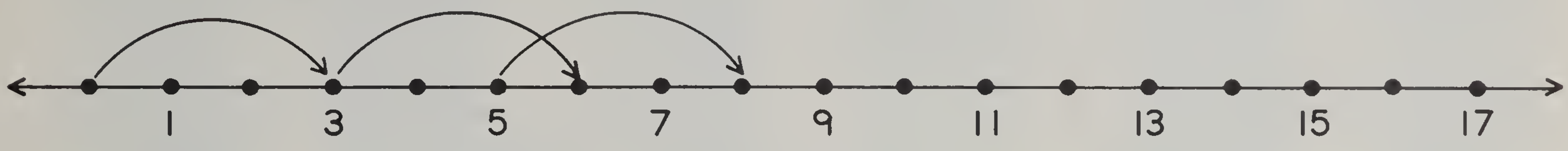
A.


0
2
4
6
8
10
12
14
16

4 → 1 8 → 5 13 → 10 3 → 0 11 → 8

15 → 12 12 → 9 10 → 7 17 → 14 7 → 4

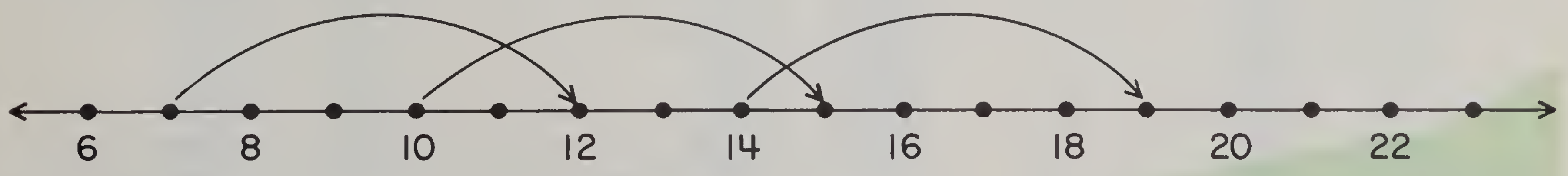
B.


1
3
5
7
9
11
13
15
17

3 → 6 0 → 3 5 → 8 8 → 11 4 → 7

10 → 13 13 → 16 9 → 12 14 → 17 7 → 10

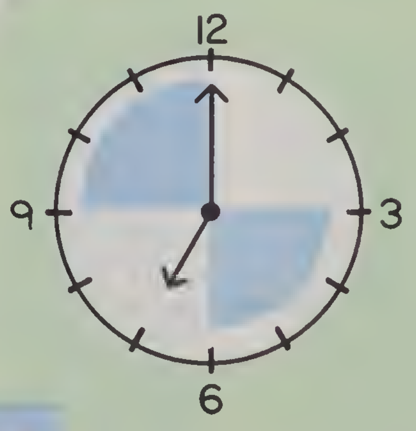
C.


6
8
10
12
14
16
18
20
22

10 → 15 7 → 12 14 → 19

17 → 22 15 → 20 9 → 14

6 → 11 13 → 18 16 → 21



7 o'clock

Notes on Pupil Page 86

Tasks: *Pupils demonstrate their discovery of each pattern by writing in the correct responses as required.*

Purpose: To help pupils notice a pattern in given data which they can then apply by continuing the pattern.

Unifying Ideas: Functions and relations.

Materials for Teacher: Colored chalk.

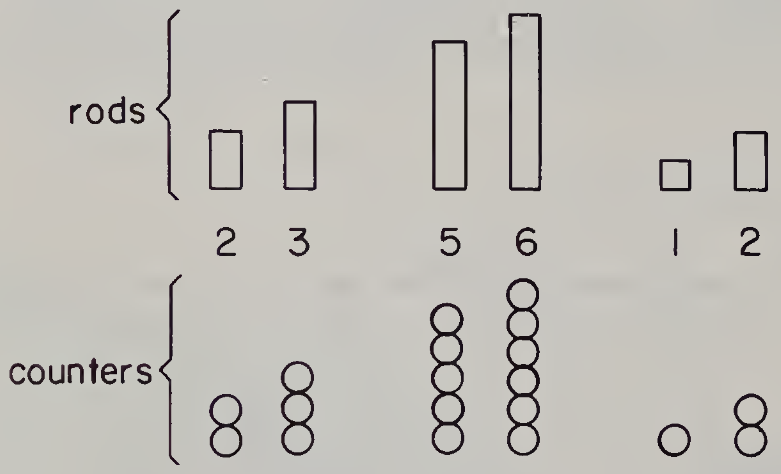
Materials for Pupils: Desk counters and Cuisenaire® Rods.

The Lesson:

Ask volunteers to call out their "favorite number." As each is called out, you reply according to some secret yet simple rule. After a few rounds, record the results on the chalkboard. The record might look like the following:

Called out	Reply
2	3
5	6
1	2

Pupils can record these results visually at their desks with rods or counters—representing the number called out on the left and the reply on the right; e.g.:



As some pupils show signs of having guessed your rule, ask them if they can tell what your reply will be to some number they call out. "You said your favorite number is 4. What do you think I will reply?" In this way a pupil can demonstrate his discovery of the rule—without giving away the secret; each example is another clue to those who are still looking for the rule.

Such rules can be further explored as real-life situations; e.g., all we know is that Bill has one more pencil than Jim has. Now the class can complete such statements as:

"If Jim has 5 pencils, then Bill has ___ pencils."
"If Jim has no pencils, then Bill has ___ pencils."
"If Bill has 4 pencils, then Jim has ___ pencils."

A record of the questions and appropriate answers might be:

Number of pencils Jim has	Number of pencils Bill has
5	6
0	1
3	4

Change the rule several times before inviting the class to turn to page 86 in their MATH WORKSHOP books for independent work at their desks.

Many classmates will want to continue playing "What's My Rule?" as a game, making up their own rules for the others to discover. Also, you may want to put several difficult examples on the board and leave them for several days. The first pupil to reveal the "secret" becomes the "expert" for that example.

Be sure to review the accompanying booklet, *Games and Enrichment Activities*, for suggestions you may wish to incorporate in your daily lesson plan.

Headline Story: Tell a story that fits:

$$5 = \square + \triangle$$

0 1 2 3 4 5 6 7 8 9

A

2 → 3
5 → 6
1 → 2

4 → 5
0 → 1
3 → 4

B

5 → 4
9 → 8
1 → 0

4 → 3
7 → 6
2 → 1

C

0 → 2
1 → 3
2 → 4

5 → 7
3 → 5
4 → 6

D

5 → 3
6 → 4
7 → 5

8 → 6
9 → 7
13 → 11

E

1 → 4
5 → 8
0 → 3

4 → 7
3 → 6
6 → 9

F

3 → 0
4 → 1
7 → 4

5 → 2
8 → 5
6 → 3

Notes on Pupil Page 87

Tasks: *Pupils note that a vertical jump from a point on one number line chunk to a point on the chunk directly below it represents the addition of 10, that a vertical jump to a point on the line directly above it represents the subtraction of 10. They use this information to make the appropriate entries in the blue and green boxes of the number sentences at the bottom of the page. For the breakout at the lower right of the page, pupils write in the green boxes the number of dimes and the value in cents of the dimes pictured.*

Purpose: To provide further experiences (a) in constructing the base ten system of numeration, (b) in establishing order on the number line, (c) in adding and subtracting with addends of 10, and (d) in using numbers in situations involving measurement.

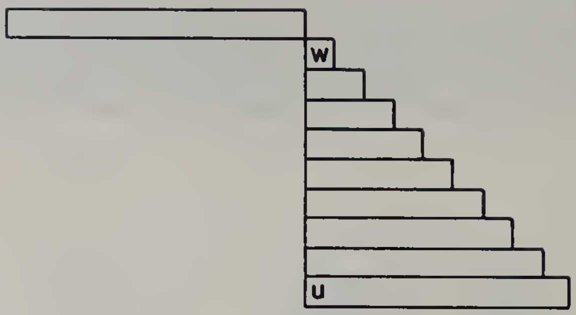
Unifying Ideas: Numeration; addition and subtraction; functions and relations; measurement; structure.

Materials for Teacher: Number line chunks as displayed on page 87; three dimes; and thirty pennies.

Materials for Pupils: Cuisenaire® Rods—ten orange and one of each of the other colors; a ditto sheet for pupils with one line drawn on it dividing the sheet into two parts.

The Lesson:

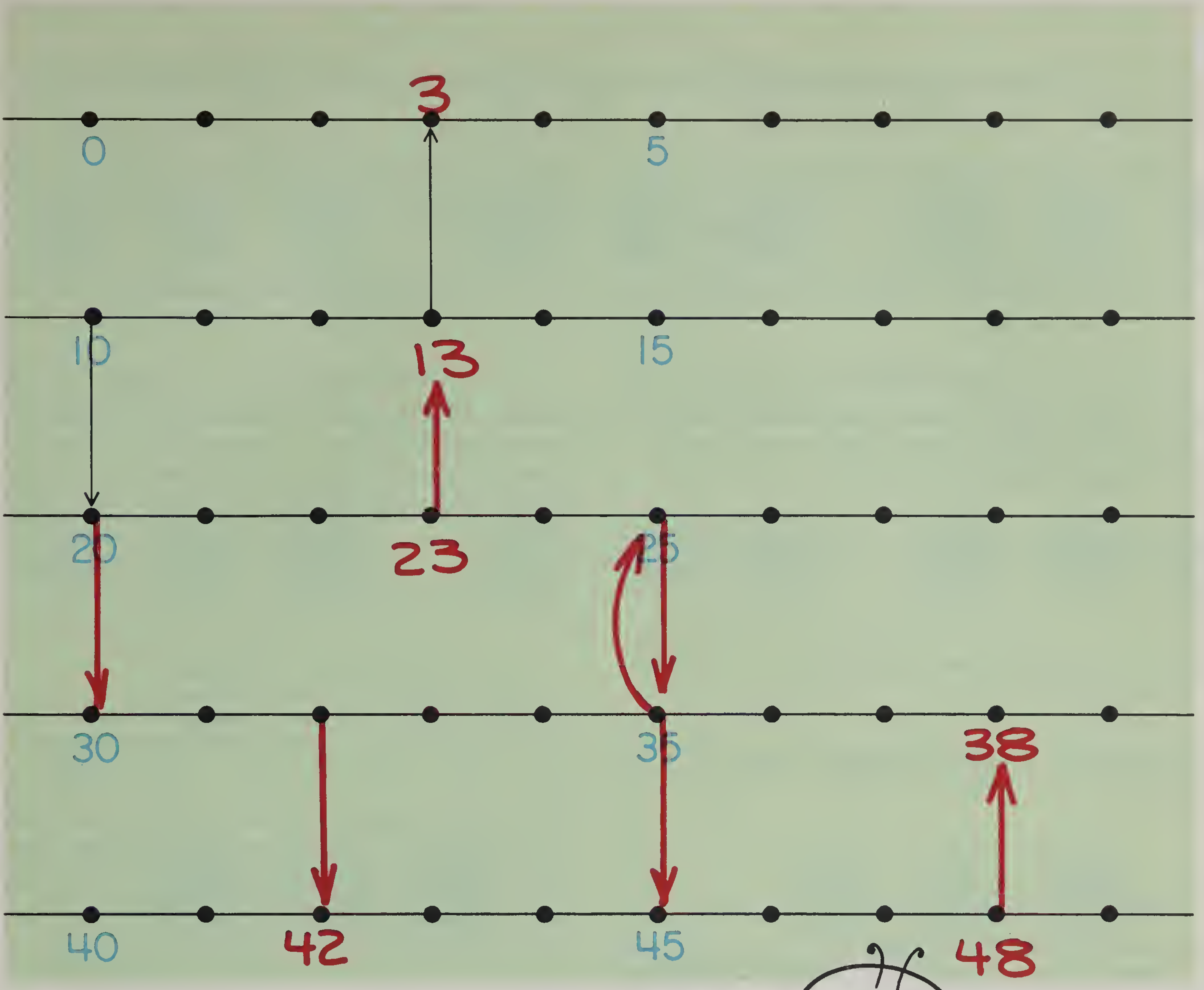
Have the class form a staircase from the white rod to the blue rod, with the orange rod starting the next row as shown below.

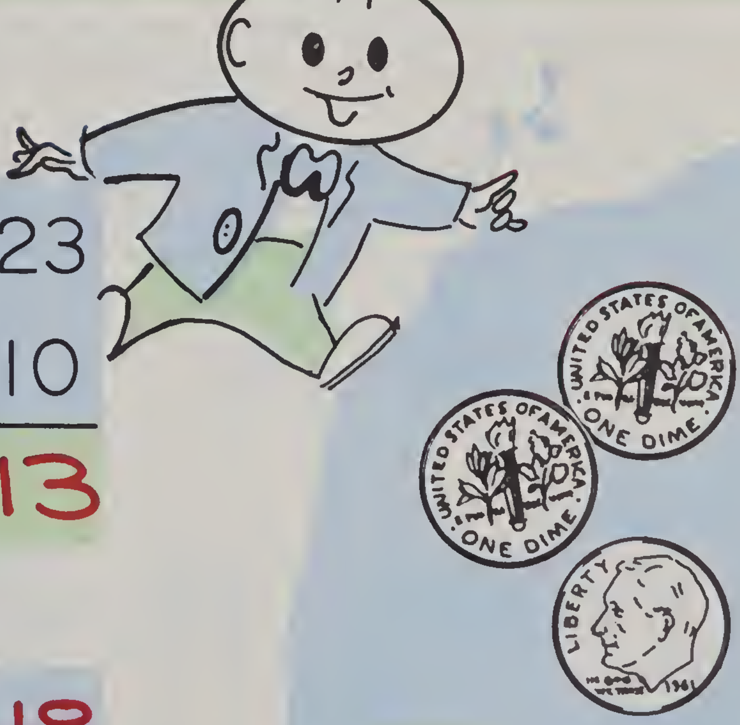


Have them count down the staircase, starting with 0 and ending with 9. When they reach 10, have them point to their arrangement as shown above. Have them slide this orange rod along the staircase as they count out the teen numbers. When they reach 20, have them slide the rod they've been using back to its original position and place another one on top of it to represent 20. They now slide both orange rods along the staircase as they count the numbers 21 through 29. For 30, they slide their two orange rods back to the original position and add one more so that they will then slide three orange rods along the staircase. Have them proceed until they count up to 100.

With the ten orange rods, have them proceed as follows: "Place the Ditto sheet on your desk. With the ten orange rods on the right side of the line, what's on the left?" (Zero). "What's on the right?" (One hundred). "Slide one orange rod across the line to the left. You now have what on the left?" (Ten). "On the right?" (Ninety). "Ten plus ninety equals what?" (One hundred). "Slide another orange rod across the line. What's on the left? The right? How much is twenty plus eighty?" And so forth. Pupils should now complete page 87 with no difficulty.

Headline Story: Alice walked 4 blocks. Al walked 10 blocks farther. (Children extend story.)


3
0
5
10
13
15
20
23
25
30
35
38
40
42
45
48


UNITED STATES OF AMERICA
ONE DIME
UNITED STATES OF AMERICA
ONE DIME
LIBERTY
1961

10 + 10 = 20

20 + 10 = 30

35 + 10 = 45

25 + 10 = 35

32 + 10 = 42

$$\begin{array}{r} 13 \\ -10 \\ \hline 3 \end{array} \qquad \begin{array}{r} 23 \\ -10 \\ \hline 13 \end{array}$$

$$\begin{array}{r} 35 \\ -10 \\ \hline 25 \end{array} \qquad \begin{array}{r} 48 \\ -10 \\ \hline 38 \end{array}$$

3 coins

30¢

Notes on Pupil Page 88

Tasks: *Pupils note the connection between the first number line jump and the first completed number sentence. Using this information, they write the appropriate numbers in the blue and green boxes and make a sketch of the number line jumps when needed.*

Purpose: To provide further experiences with (a) order (b) the addition operation via the number line and (c) in using numbers in situations involving measurement (money).

Unifying Ideas: Structure; numeration; addition and subtraction; functions and relations; measurement.

Materials for Teacher: Ten pennies; two nickels; and one dime.

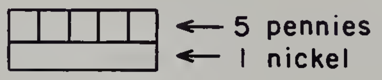
Materials for Pupils: Cuisenaire® Rods.

The Lesson:

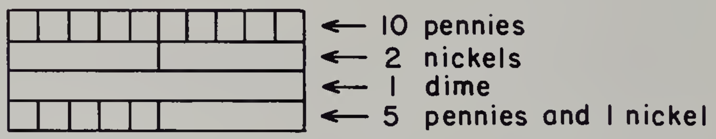
Without any help from you, have the class turn to page 88 for individual study.

After pupils finish the page, briefly discuss their work with them. Take this opportunity to give your class more experiences in dealing with money conversion situations.

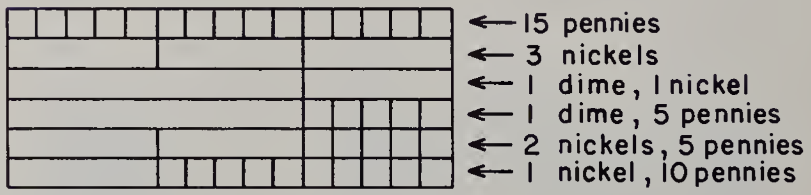
Show the class your collection of money. Tell them it consists of ten pennies, two nickels, and one dime. "If the white rod's name is one cent (or one penny), which rod represents a nickel?" Using white and yellow rods, show two different ways of representing five cents.



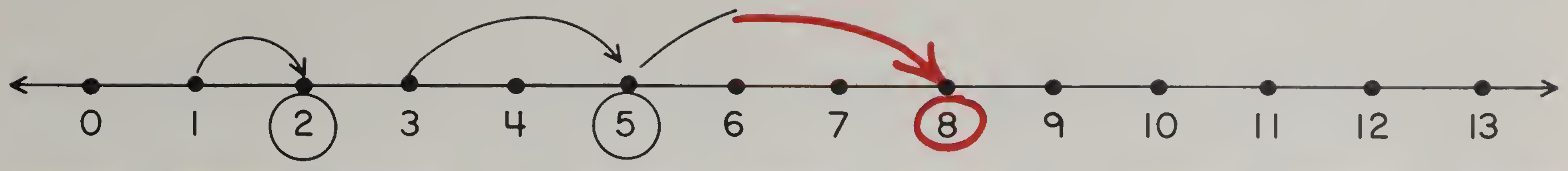
"If the white rod's name is one cent, or one penny, and the yellow rod's name is five cents, or one nickel, which rod is named ten cents or one dime?" Show ten cents using only white rods. Only yellow rods. Only orange rods. "What other ways could we show ten cents using white, yellow, or orange rods?" A summary of the pupil constructions looks like this:



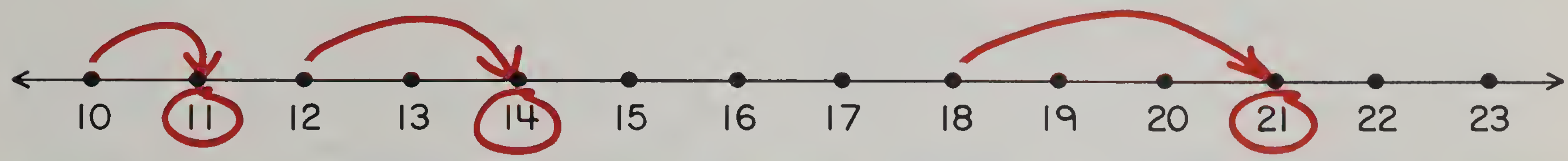
"Which way required the fewest coins? Using rods, how could we show fifteen cents? Using only pennies? Only nickels? Only dimes? If we can't use only dimes, construct all the different ways that we can represent fifteen cents."



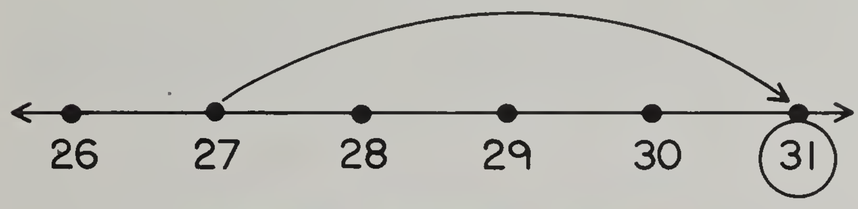
Headline Story: What is the smallest number of coins needed to make 15¢ (9¢, 13¢, 23¢, etc.)


0
1
2
3
4
5
6
7
8
9
10
11
12
13

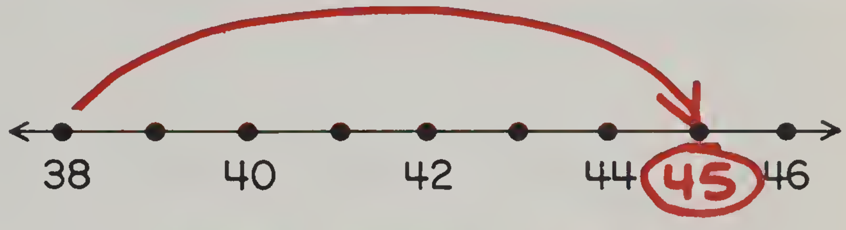
1 + 1 = 2 3 + 2 = 5 5 + 3 = 8


10
11
12
13
14
15
16
17
18
19
20
21
22
23

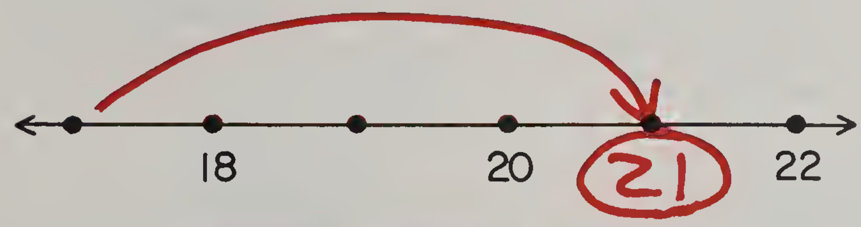
10 + 1 = 11 12 + 2 = 14 18 + 3 = 21


26
27
28
29
30
31

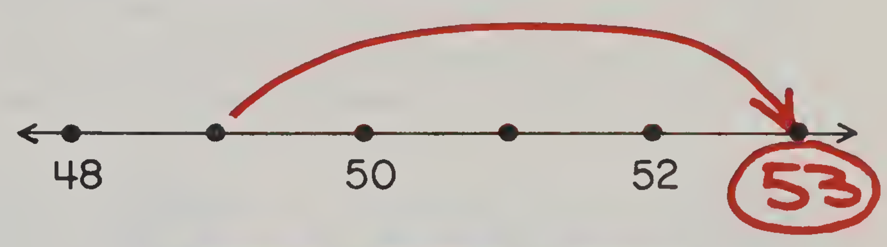
27 + 4 = 31


38
40
42
44
45
46

38 + 7 = 45



17 + 4 = 21

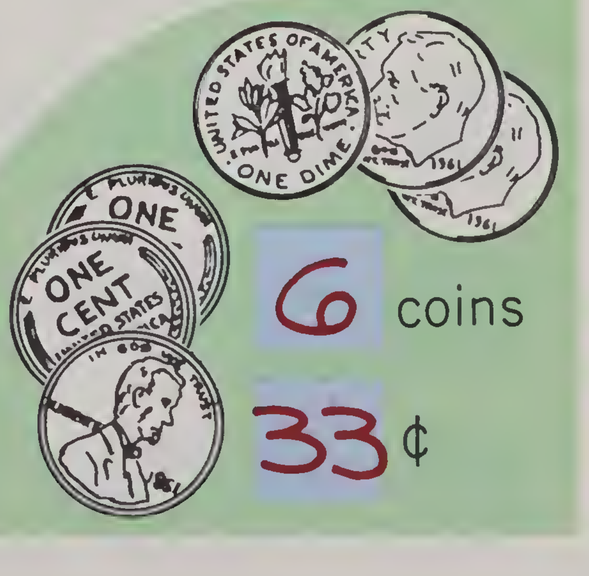


49 + 4 = 53

1 + 1 = 2 1 + 10 = 11

1 + 3 = 4 7 + 6 = 13

3 + 5 = 8 13 + 5 = 18


UNITED STATES OF AMERICA
ONE DIME
1961
ONE CENT
ONE

6 coins

33¢

Notes on Pupil Page 89

Tasks: Pupils use the clues provided by the first number line jump and the first completed number sentence to write the appropriate numbers in the blue and green boxes, complete the number sentences, and make sketches of the number line jumps when necessary.

Purpose: To extend pupil understanding of (a) order on the number line, (b) the decimal system of numeration, and (c) addition and subtraction via number patterns and number line jumps.

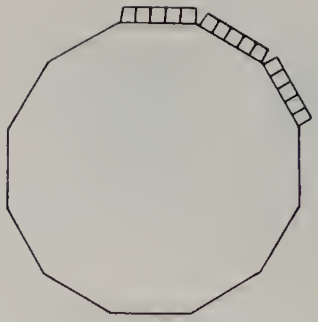
Unifying Ideas: Structure; numeration; addition and subtraction; functions and relations.

Materials for Teacher: A demonstration clock face with minute and hour hands.

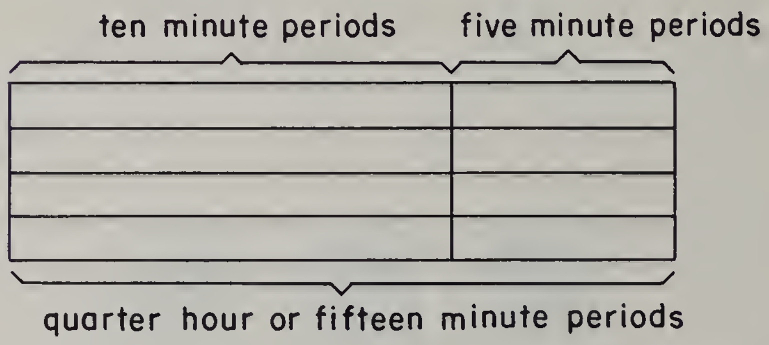
Materials for Pupils: Cuisenaire® Rods; a ditto copy of the polygon described in "The Lesson" for each pupil.

The Lesson:

After pupils complete page 89 independently, work with the dittoed polygon. "If the white rod's name is 'one minute,' place five minutes on one of the lines of the figure on your page. Five more. Five more." Here is a summary of their actions:

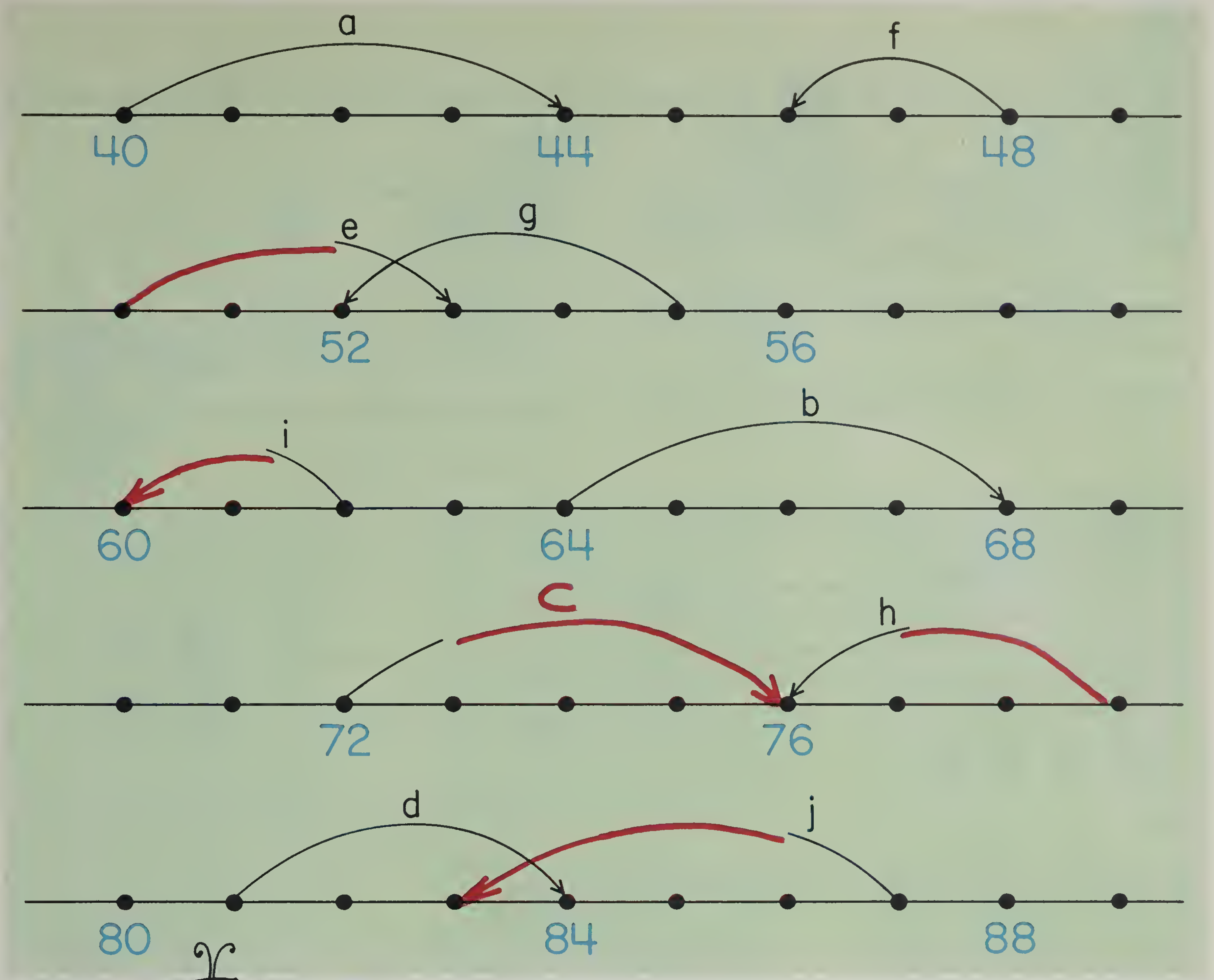


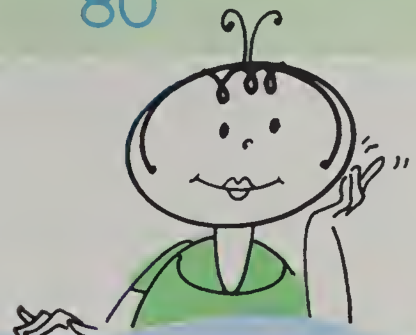
Have the pupils complete the construction of the sixty minutes on their dittoed figure. "Exchange each five minutes for a yellow rod. How many five minute periods are there? Using yellow rods, form as many fifteen-minute trains as possible. How many quarter-hour periods did you get? For each quarter-hour period, exchange as many five-minute periods as possible for ten-minute periods." They should now have eight rods arranged like this:

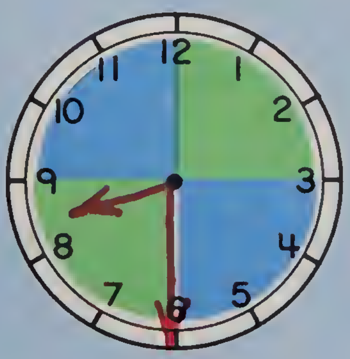


"Show two thirty-minute periods. Exchange the four five-minute periods for two ten-minute ones. How many orange rods do you now have? Show one hour."

Headline Story: It's 8:30. In a half-hour it will be ___.


a
f
40
44
48
e
g
52
56
i
b
60
64
68
c
h
72
76
d
j
80
84
88




12
1
2
3
4
5
6
7
8
9
10
11

The time is –

8:30

a 40 + 4 = 44

b 64+4 = 68

c 72 + 4 = 76

d 81+3 = 84

e 50+ 3 = 53

f 48 − 2 = 46

g 55 − 3 = 52

h 79 − 3 = 76

i 62 − 2 = 60

j 87 − 4 = 83

Notes on Pupil Page 90

Tasks: *Pupils complete each example.*

Purpose: To test pupil achievement.

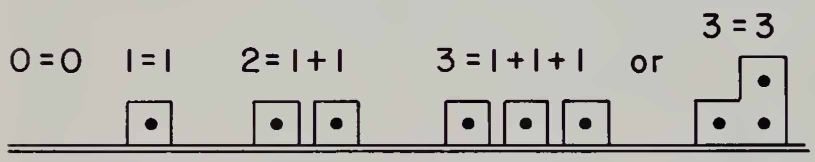
Materials for Teacher: Cards as described in "The Lesson."

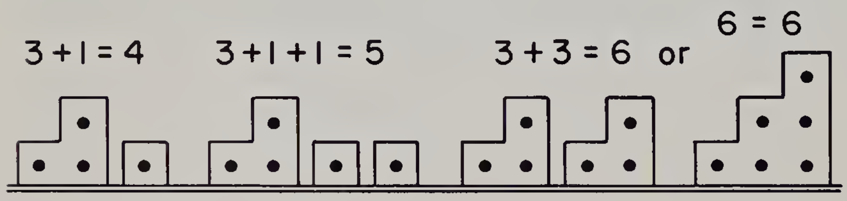
Materials for Pupils: No special materials needed.

The Lesson:

Make three sets of cards like those at the top of page 90.

Present this problem: "Using *not more than* three out of the twelve cards, can you arrange them along the chalkrail to represent each number less than 22?" (As the cards are arranged, write the combinations on the board as indicated below.)





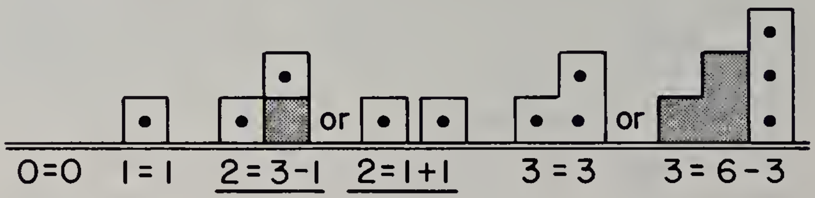
A summary of results so far:

0 = 0
1 = 1
2 = 1+1
3 = 1+1+1 = 3
4 = 3 + 1
5 = 3 + 1 + 1
6 = 3 + 3 = 6

Experiments continue and the record grows:

7 = 3+3+1 = 6+1
8 = 6+1+1
9 = 3+3+3 = 6+3
10 = 6+3+1 = 10
11 = 10+1
12 = 10+1+1 = 6+6 = 6+3+3
13 = 10+3 = 6+6+1
14 = 10+3+1
15 = 6+6+3
16 = 10+6
17 = 10+6+1
18 = 6+6+6
19 = 10+6+3
20 = 10+10
21 = 10+10+1

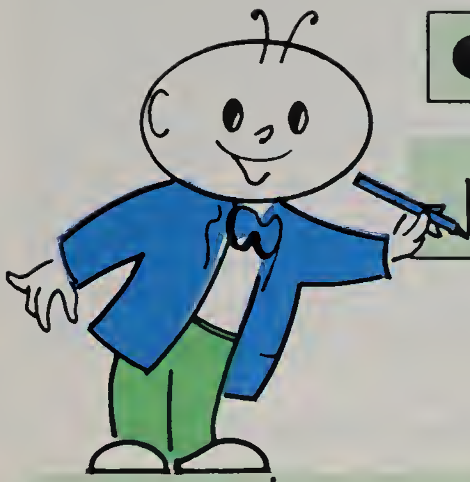
A second problem: "If one card placed face down on another suggests subtraction, can you represent all numbers through 10 with not more than two cards?"

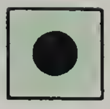


and

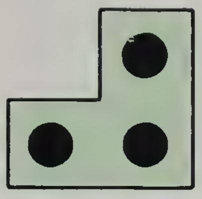
4 = 3+1 = 10-6   5 = 6-1   6 = 6 = 3+3   etc.

Headline Story: Mary spent a dime and a nickel. Make a headline to fit.

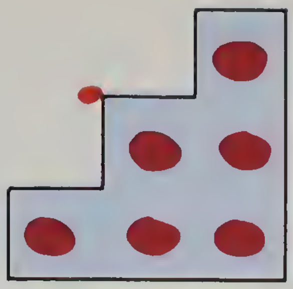




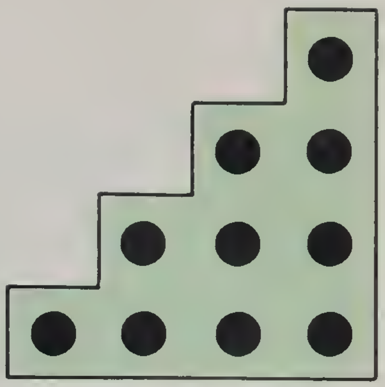
1



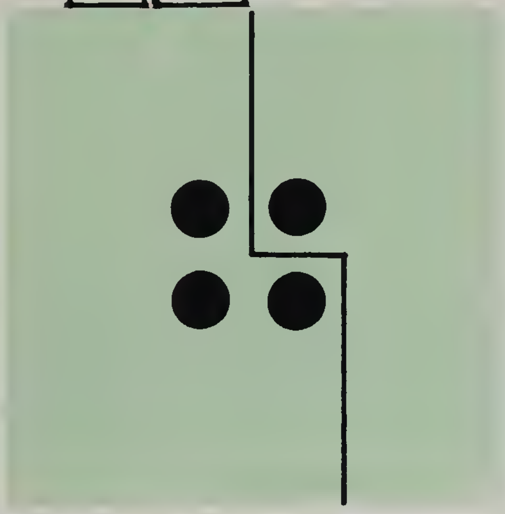
3



6



10

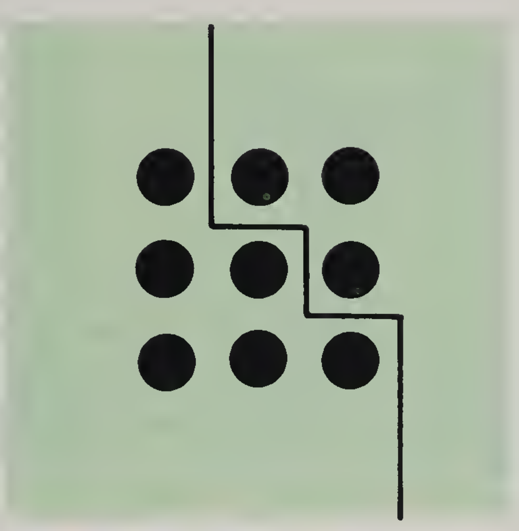


3 + 1 = 4

1 + 3 = 4

4 - 1 = 3

4 - 3 = 1

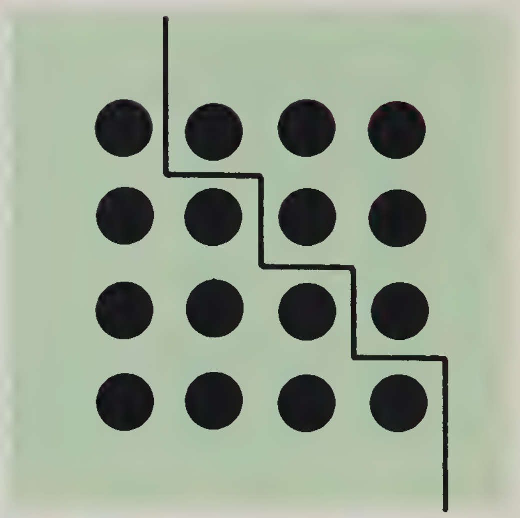


6 + 3 = 9

3 + 6 = 9

9 - 6 = 3

9 - 3 = 6

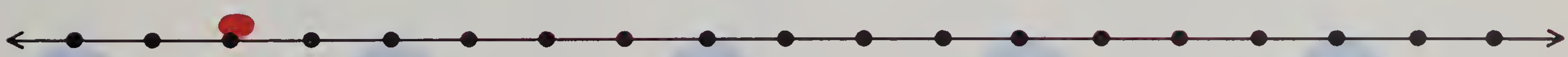


10 + 6 = 16

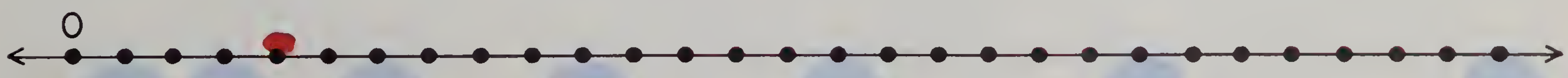
6 + 10 = 16

16 - 10 = 6

16 - 6 = 10



4 8 12 16 20



1 3 6 10 15 21 28

Notes on Pupil Page 91

Tasks: *Pupils complete each example.*

Purpose: To test pupil achievement.

Materials for Teacher: No special materials are needed.

Materials for Pupils: No special materials are needed.

The Lesson:

List addition facts in groups of three and add the numbers written in each column as shown below:

$$\begin{array}{ccccc} & & (a) & & \\ 2 & + & 1 & = & 3 \\ 1 & + & 4 & = & 5 \\ \underline{0} & + & \underline{2} & = & \underline{2} \\ 3 & & 7 & & 10 \end{array} \qquad \begin{array}{ccccc} & & (b) & & \\ 3 & + & 1 & = & 4 \\ 2 & + & 5 & = & 7 \\ \underline{5} & + & \underline{4} & = & \underline{9} \\ 10 & & 10 & & ? \end{array}$$

Notice that in example (a) the results we found by adding the columns (3, 7, 10) suggest that we write 3 + 7 = 10. If this holds in all such cases, then we have a shortcut in example (b) instead of finding the sum of 4, 7, and 9, we might find the sum of 10 and 10.

$$\begin{array}{r} 4 \\ 7 \\ \underline{9} \\ \underrightarrow{10 + 10} = 20 \end{array}$$

If we count out 4 objects, then 7 more and then 9 more—and count the whole collection—we find that we do have 20.

Try several groups of three statements. Does this method work in each case?

Consider a group of three subtraction facts:

$$\begin{array}{ccccc} 3 & - & 2 & = & 1 \\ 5 & - & 1 & = & 4 \\ \underline{4} & - & \underline{2} & = & \underline{2} \\ 12 & & 5 & & 7 \end{array} \qquad \begin{array}{ccccc} 5 & - & 4 & = & 1 \\ 8 & - & 5 & = & 3 \\ \underline{7} & - & \underline{2} & = & \underline{5} \\ 20 & & 11 & & 9 \end{array}$$

Is it the case that 12 − 5 = 7 and 20 − 11 = 9?

Here is a puzzle: With each of the numbers 1, 2, 3, 4, 5, 6, 7, 8, and 9 used only once, complete the following:

$$\begin{array}{ccccccc} \_\_ & + & \_\_ & + & \_\_ & = & \underline{15} \\ \_\_ & + & \_\_ & + & \_\_ & = & \underline{15} \\ \_\_ & + & \_\_ & + & \_\_ & = & \underline{15} \\ 15 & + & 15 & + & 15 & = & 45 \end{array} \qquad \begin{array}{cccccccc} & & (answers:) & & & & \\ 8 & + & 1 & + & 6 & = & \\ 3 & + & 5 & + & 7 & = & \\ \underline{4} & + & \underline{9} & + & \underline{2} & = & \underline{\underline{\quad}} \\ & + & & + & & = & \end{array}$$

(Rows and columns in the answer can be interchanged, but the same three numbers will always appear in the same column or the same row.)

Headline Story: It's 6:15. How long is it until 7 o'clock?

7 + 3 = 10
2 = 6 − 4
19 + 1 = 20
10 = 16 − 6

9 − 4 = 5
0 = 15 − 15
17 − 9 = 8
0 − 0 = 0

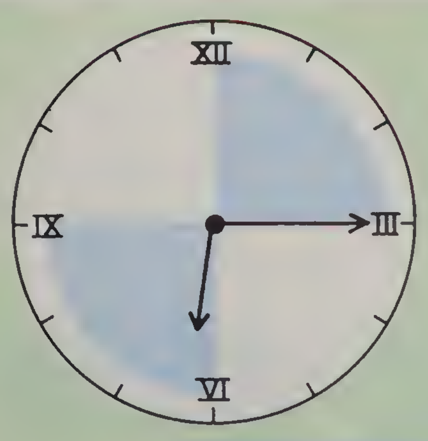
8 = 16 − 8
15 = 25 − 10
18 + 3 = 21
0 + 0 = 0

2	3	1	4
3	3	8	6
5	4	1	0
10	10	10	10

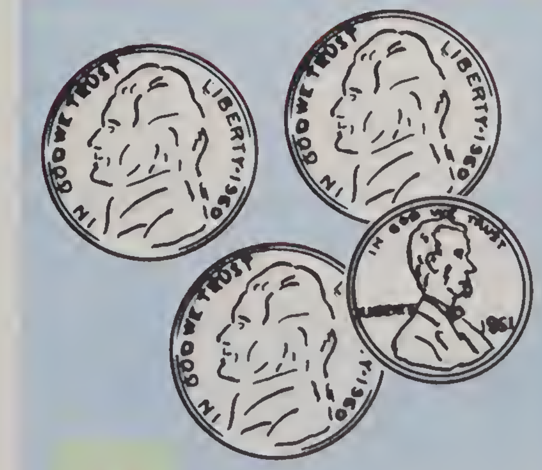
8	5	9	0	1
1	5	3	8	12
6	5	3	7	2
15	15	15	15	15

2	9	5	3
4	1	7	8
6	2	0	1
12	12	12	12

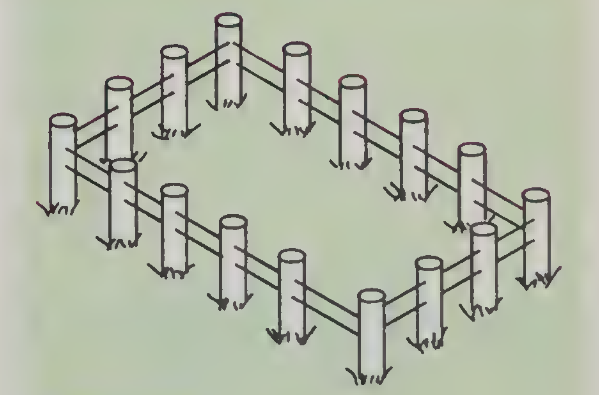
5	3	2	3	6
5	2	9	11	7
10	15	9	6	7
20	20	20	20	20


XII
III
VI
IX

The time is 6:15


IN GOD WE TRUST
LIBERTY 1960

16 ¢



16 posts

Notes on Pupil Page 92

Tasks: *Pupils complete all number sentences. In the six upper sections the sentences should represent the actions in the pictures.*

Purpose: To explore the number 12, and regrouping.

Unifying Ideas: Addition and subtraction; functions and relations; measurement.

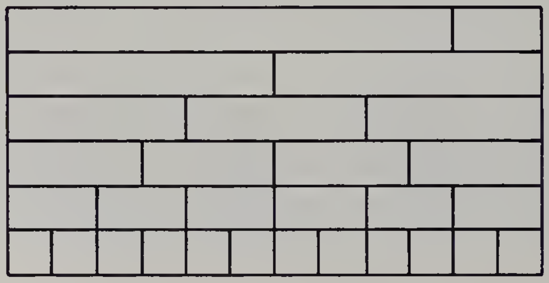
Materials for Teacher: An empty one-dozen egg carton.

Materials for Pupils: Cuisenaire® Rods.

The Lesson:

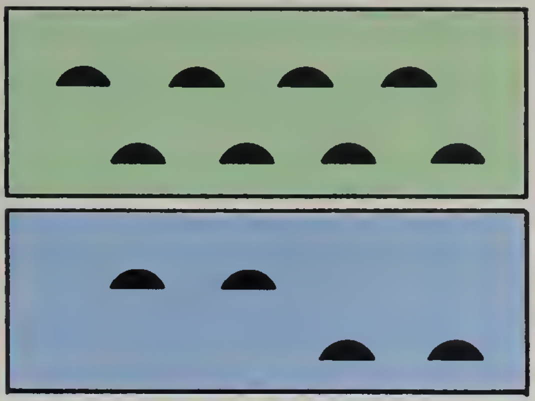
Have pupils turn to page 92. Ask if anyone can tell the class what's going on in the first picture. If no one volunteers, proceed somewhat as follows: "What is the connection between the 8 on the green shaded box and the set of counters in the green section of the picture? How many counters are in the blue section of the picture? What number should we put in the blue box of the number sentence? How many counters in all are there in the blue and green sections of the picture? How should we complete the number sentence at the bottom of the first section?"

"What number should we use to make the number sentence at the bottom of the second section true?" Most, if not all, of the class can complete page 92 without any further guidance. Discuss the completed page with the class. "What number is page 92 a study of?" Tell the class that another name for a set of twelve objects is one dozen. "Who knows how many eggs in one dozen? In a half dozen? In a third of a dozen? In a quarter of a dozen? Who would have more eggs—Billy who has a third of a dozen or Tilly who has a quarter of a dozen?" Have the class construct a train of an orange and red rod. Call the train 12, or one dozen. Have them construct as many trains as possible which look different, but are made of rods of one color and are as long as the 12-train. Here is a sketch of their construction:



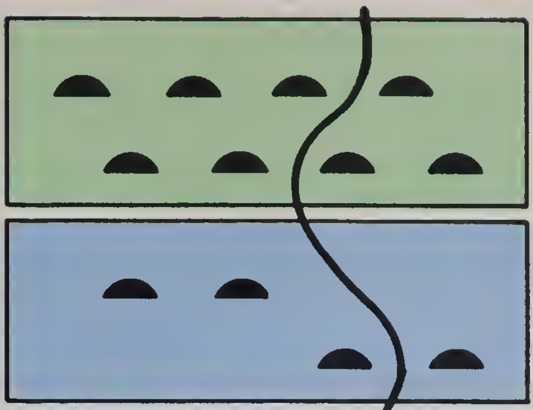
Hold up one half dozen, using rods. (A dark green rod, two light green rods, three red rods, or six white rods are all possibilities.) Hold up one third of a dozen. (One purple, two reds, four whites, or a red and two whites are all possibilities.) Hold up one quarter of a dozen. (One light green rod, a red and white rod, or three white rods are all possibilities.)

Headline Story: Mrs. Jones used 2 out of a dozen of eggs. (Children extend story.)



8
+ 4
12

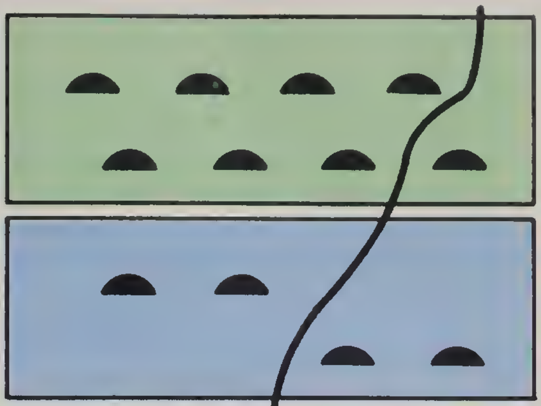
8 + 4 = 12



5
+ 3
8

3
+ 1
4

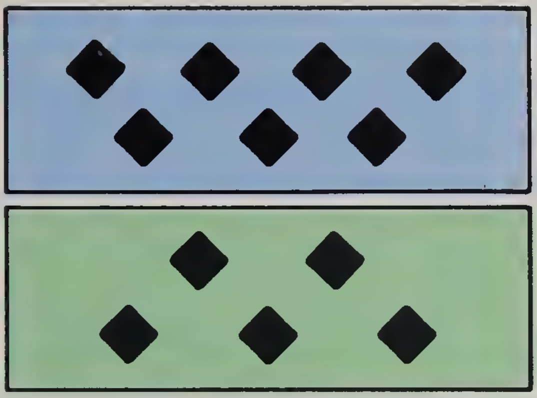
8 + 4 = 12



7
+ 2
9

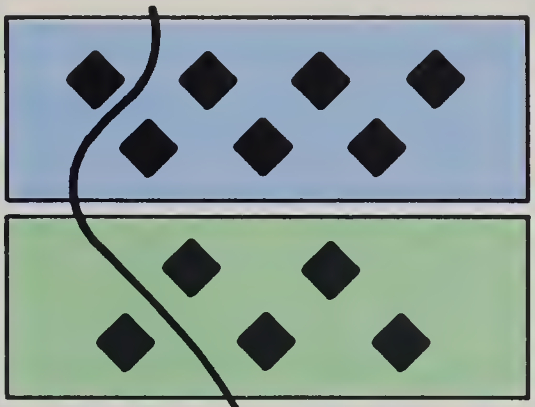
1
+ 2
3

9 + 3 = 12



7
+ 5
12

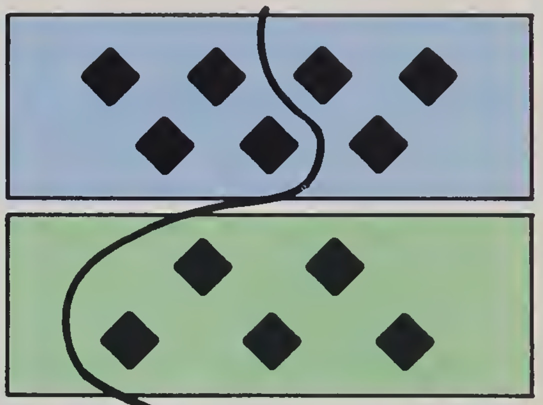
7 + 5 = 12



1
+ 1
2

6
+ 4
10

2 + 10 = 12



4
+ 0
4

3
+ 5
8

4 + 8 = 12

---

0 + 12 , 11 + 1

1 + 11 = 12

12 + 0 = 12

6 + 6 = 12

12 = 3 + 9

12 = 5 + 7

10 + 2 = 12

Notes on Pupil Page 93

Tasks: Pupils complete all number sentences.

Purpose: To explore the number 15 and regrouping; to provide pupils with measurement experiences dealing with ounces and pounds.

Unifying Ideas: Addition and subtraction; functions and relations; measurement.

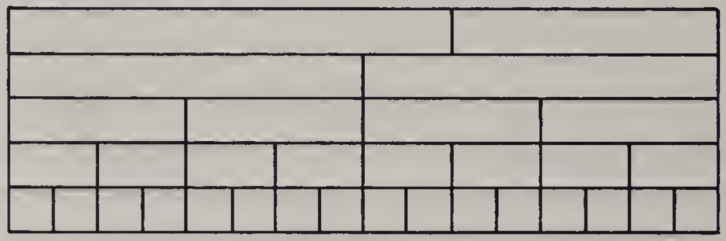
Materials for Teacher: A balance scale; ounce and pound weights.

Materials for Pupils: Cuisenaire® Rods.

The Lesson:

Have the class turn to page 93. Ask how this page is like page 92. How is it different? Assign the page for individual study. Briefly discuss and summarize the class findings.

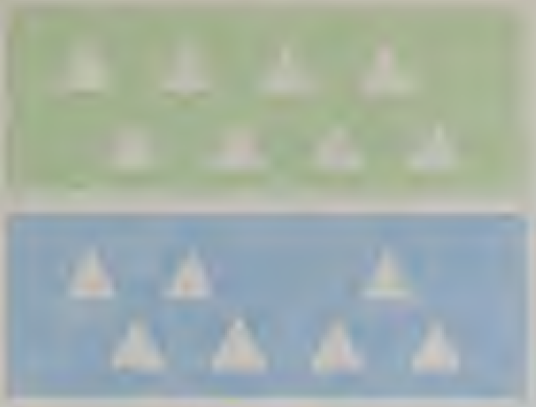
Pass out some ounce and pound weights so that each pupil has a chance to compare the different weights. Place an ounce weight on each pan of the balance scales to show the class the balance. Remove one of the weights to show the imbalance. Place the pound weight on the empty pan. Have different pupils come up and place ounce weights on the other pan while the rest of the class keeps a tally with their white rods. When the scales balance again, have the class make a train of their white rods. It should be as long as a train of 1 orange and 1 dark green rod. "If the white rods represent ounces, how many ounces in one pound? Then the two new names for the train could be what? (Sixteen ounces and one pound). Hold up an 'ounce train.' Hold up a 'pound train.' Using rods of one color, build as many different looking trains as possible which are as long as the 'pound train.' " Here is a summary of their constructions:



"Show an 'eight ounce' train. What is another name for an 'eight ounce' train? Show the two 'half-pound' trains. Hold up a 'four ounce' train. How many 'four ounce' trains make a train as long as a 'one pound' train? Hold up a 'quarter-pound' train. How many 'two ounce' trains make a 'pound' train? Hold up a 'one-eighth pound' train. How many ounces equal one pound? Show a 'one-sixteenth of a pound' train."

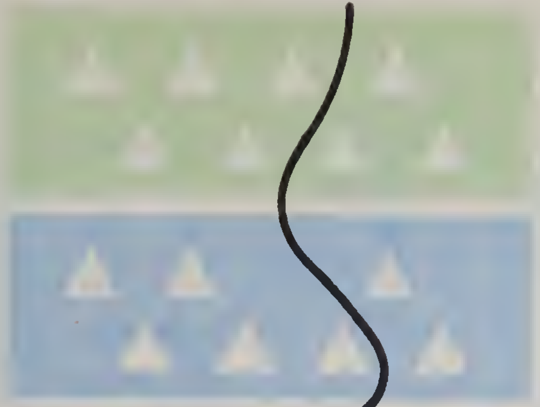
Headline Story: Complete the headline and tell a story that fits:

○ + △ = 16



8
+ 7
15

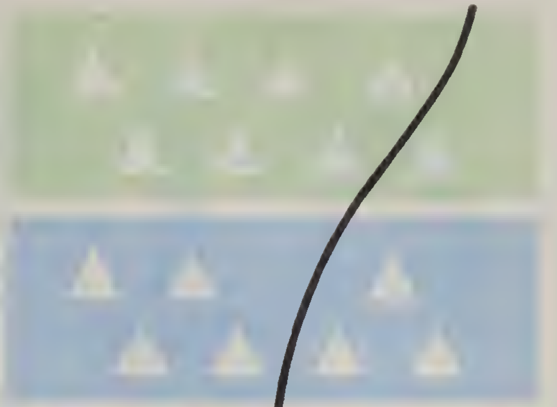
8 + 7 = 15



5
+ 5
10

3
+ 2
5

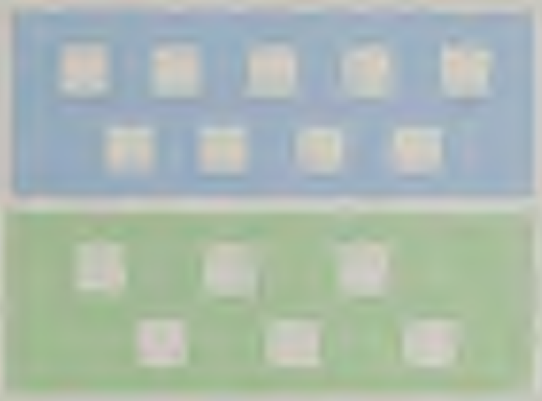
10 + 5 = 15



7
+ 4
11

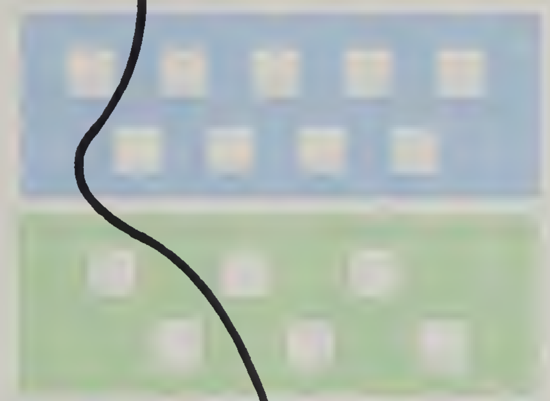
1
+ 3
4

11 + 4 = 15



9
+ 6
15

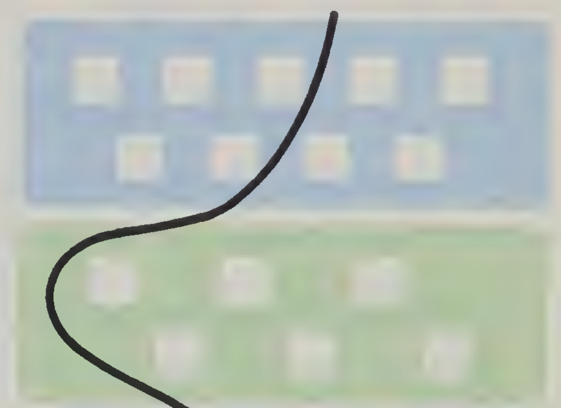
9 + 6 = 15



1
+ 2
3

8
+ 4
12

3 + 12 = 15



5
+ 0
5

4
+ 6
10

5 + 10 = 15

0+15; 2+13; 12+3; 14+1

1 + 14 = 15

13 + 2 = 15

7 + 8 = 15

15 = 15 + 0

15 = 4 + 11

6 + 9 = 15

Notes on Pupil Page 94

Tasks: Pupils connect points according to the instructions below each diagram.

Purpose: To familiarize pupils with ideas such as point, straight line, triangle, and square through the use of a straight-edge.

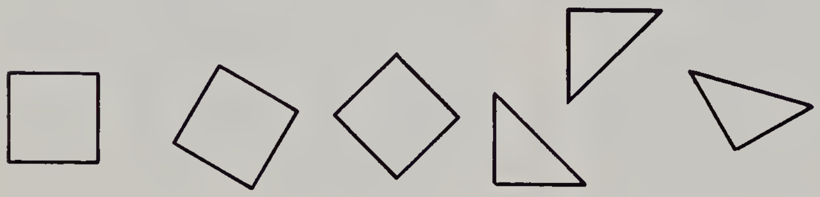
Unifying Ideas: Geometry.

Materials for Teacher: A yard stick; a piece of string (about a yard long); colored chalk; demonstration triangles and squares.

Materials for Pupils: A straight-edge and sharpened pencil for each pupil.

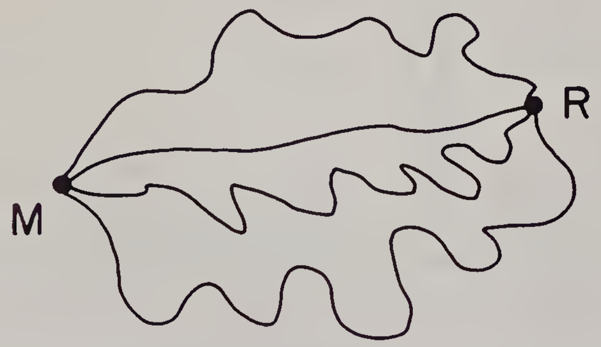
The Lesson:

Display several different size triangles and squares of wood, cardboard, or metal. Place and hold some at different angles. Is the shape still that of a square? . . . a triangle?



The shape remains the same. The name we use refers to an attribute of an object that does not change as it is moved around.

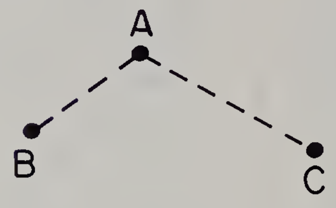
Put points on the board and label them with letters. Have children put points at random on a piece of paper at their desks. Connect a pair of points on the board with several kinds of lines drawn freehand—none being a very good representation of a straight line.



If four ants raced from M to R along the routes indicated, which ant would probably win the race? Assuming all four ants can cover the same distance in the same time, which would probably reach point R last? Could we draw a line between M and R that would be a shorter route than any shown? Where would it go?

Erase all lines, leaving points labeled M and R. Would someone draw a line to indicate the shortest route? What could we use to help draw a line that suggests that shortest route? (Stretch a string, any kind of straight-edge, sighting from one to the other, etc.) Discuss and demonstrate the relative advantages of each. The straight-edge as something rigid to follow with the chalk seems most helpful. Have each child use a straight-edge to connect the points he has indicated on his paper.

Introduce children to the method used on page 94 to give directions for joining pairs of points.

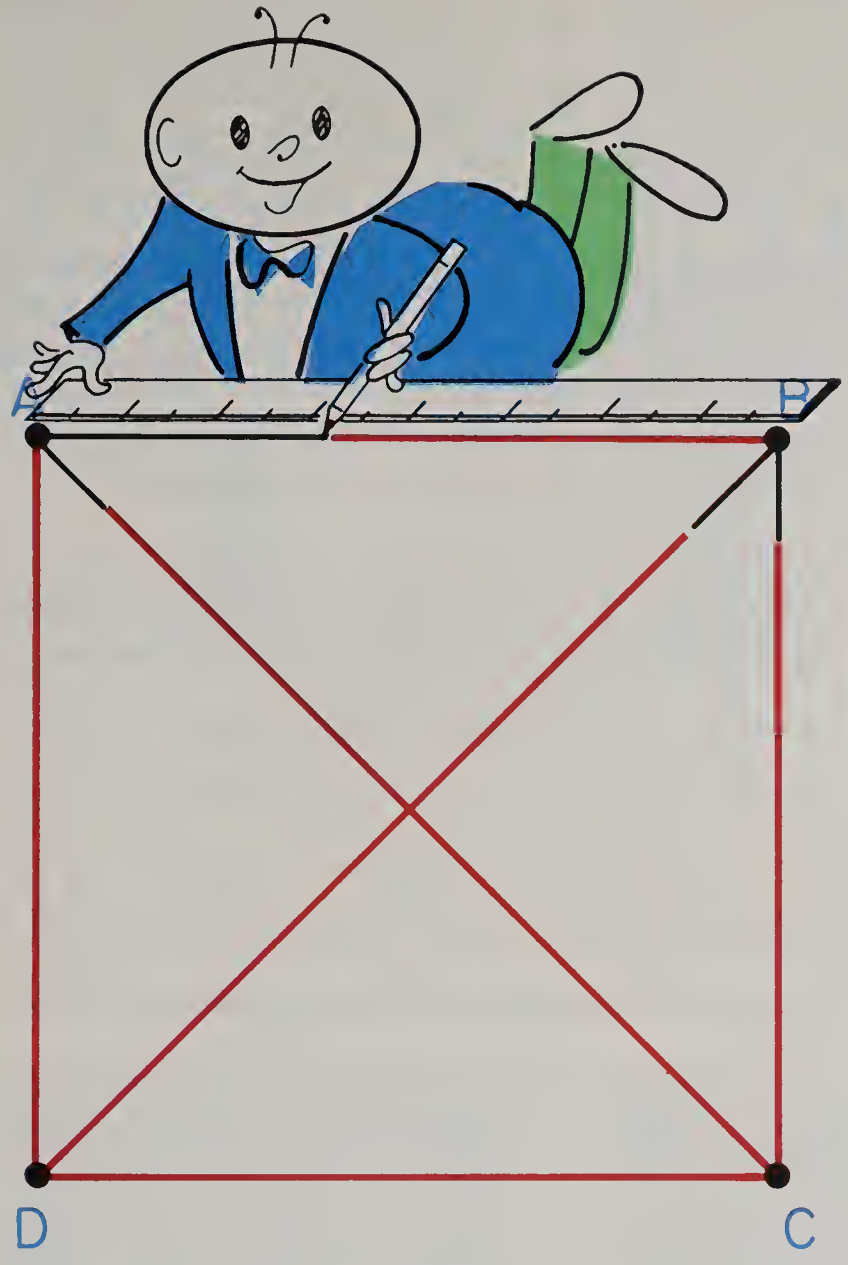


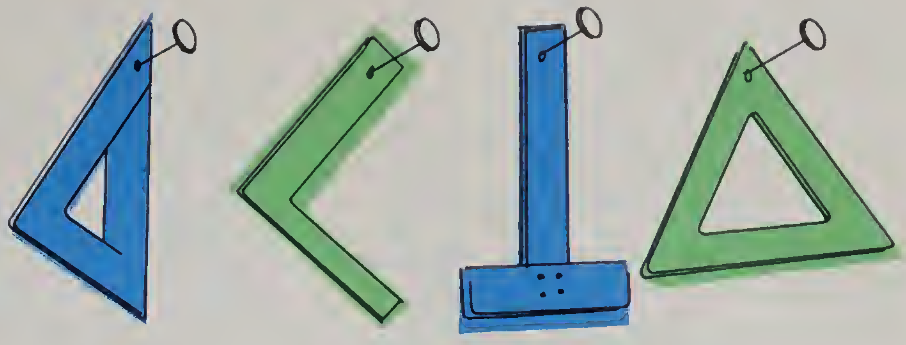
$\overline{AC}$ and $\overline{AB}$

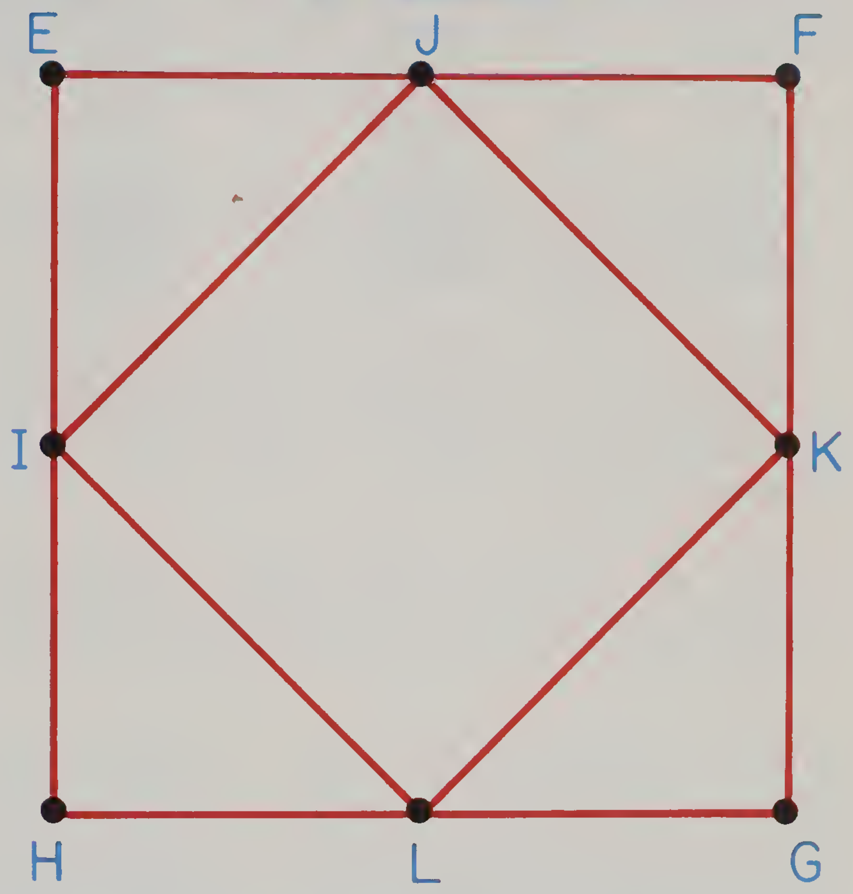
Can any other straight lines be drawn connecting a pair of points? . . . How would the directions to draw that line be given? . . . $\overline{BC}$ or $\overline{CB}$. Would a line drawn from B to C look the same as one drawn from C to B after the line was finished? Of course, so we need not worry about the order in which a pair of points is listed. The entire page 94 can be reproduced and easily completed on the chalkboard.

Be sure to review the accompanying booklet, *Games and Enrichment Activities*, for suggestions you may wish to incorporate in your daily lesson plan.

Headline Story: Find objects in the room that have square surfaces.

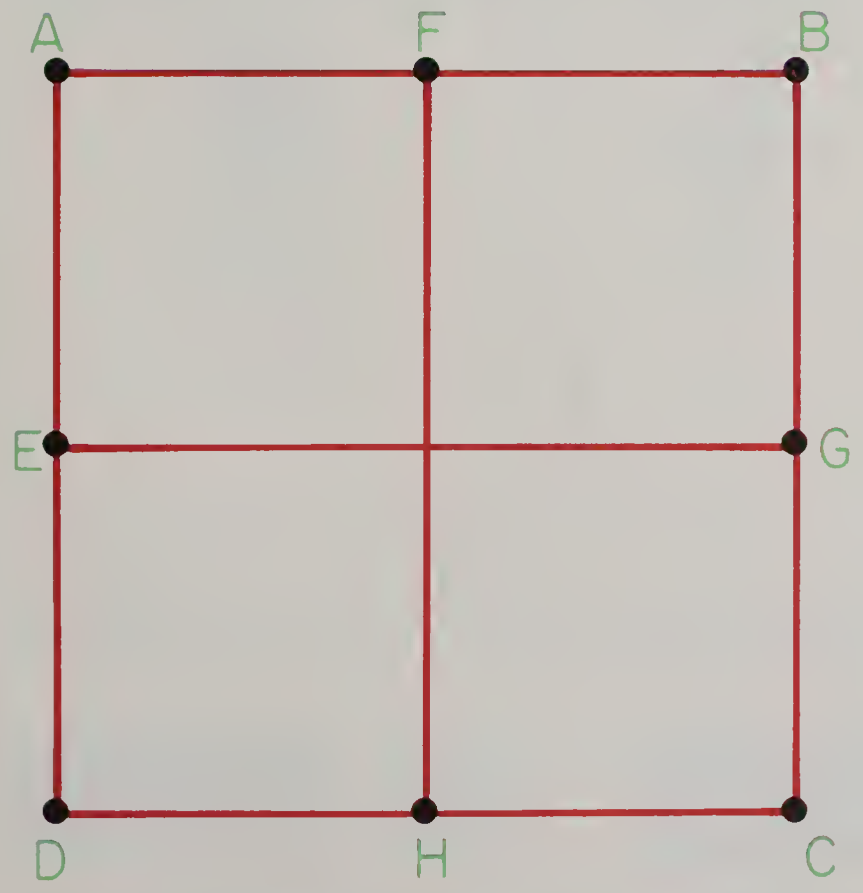


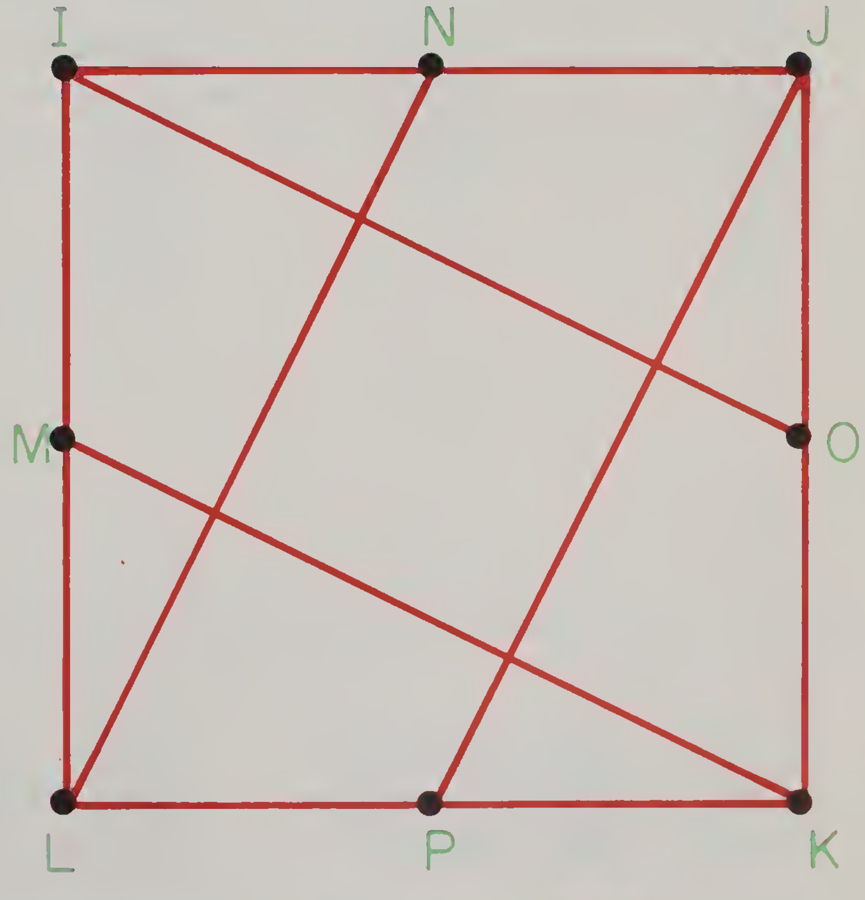




$\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DA}$, $\overline{AC}$, $\overline{BD}$

$\overline{EF}$, $\overline{FG}$, $\overline{GH}$, $\overline{HE}$, $\overline{IJ}$, $\overline{JK}$, $\overline{KL}$, $\overline{LI}$





$\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DA}$, $\overline{EG}$, $\overline{FH}$

$\overline{IJ}$, $\overline{JK}$, $\overline{KL}$, $\overline{LI}$, $\overline{MK}$, $\overline{NL}$, $\overline{OI}$, $\overline{PJ}$

Notes on Pupil Page 95

Tasks: Pupils complete all number sentences.

Purpose: To explore the number 18 (a) via regrouping, (b) via the operations of addition and subtraction, and (c) via jumps along the number line.

Unifying Ideas: Structure; addition and subtraction; functions and relations.

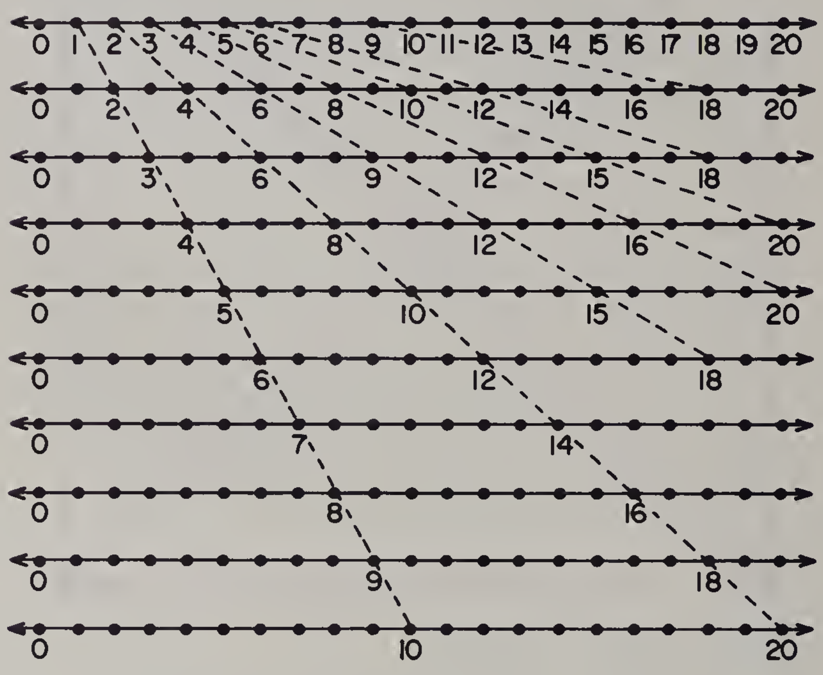
Materials for Teacher: Number line chunks with twenty-one points prominent on each chunk (use masking tape); a yardstick; colored chalk for each jumping pattern of the lesson.

Materials for Pupils: Ditto copies of number line chunks similar to the teacher's demonstration set; a foot rule.

The Lesson:

Have the class turn to page 95 for individual study. Pupils who finish quickly should check with you before making themselves available as consultants to those requesting assistance.

Place one of your number line chunks on the board and label a point as 0. Have someone make jumps of 1 from 0 and then label each point in succession while pupils at their desks label their first number line accordingly. "Did we land on the point 18?" Place a second number line chunk on the board and label a starting point 0. Have someone come up and take jumps of 2, labeling each successive landing point while the class does likewise at their desks. "Did we land on 18?" Proceed in a similar manner for jumps of 3, 4, 5, 6, 7, 8, 9, and 10. A summary of their results look like this:



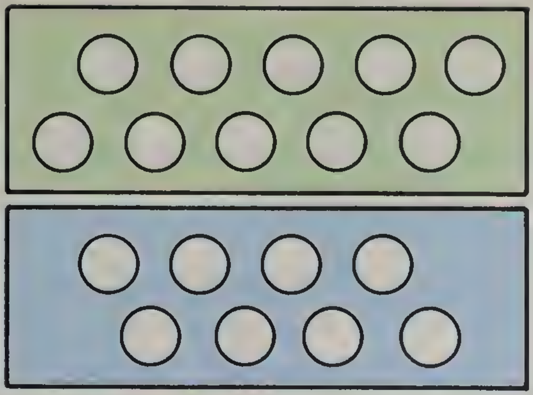
Starting at 0, which size jumps allowed us to land on 18? (Answer: 1, 2, 3, 6, 9.)

Ask if anyone notices any pattern as they scan their labeled number lines on the chalkboard model.

Note the multiple patterns suggested by the dotted lines.

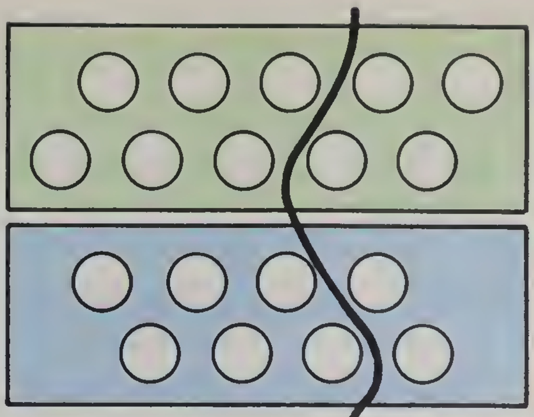
Headline Story: Complete the headline and tell a story that fits:

$18 = \square + \triangle$



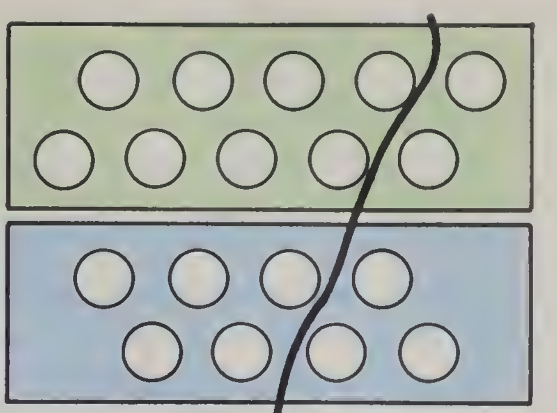
10
+ 8
18

10 + 8 = 18



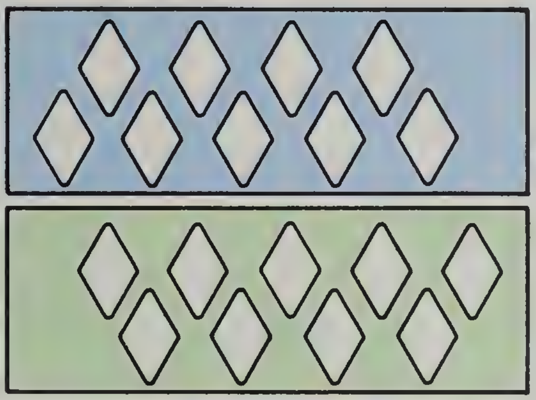
6	4
+ 6	+ 2
12	6

12 + 6 = 18



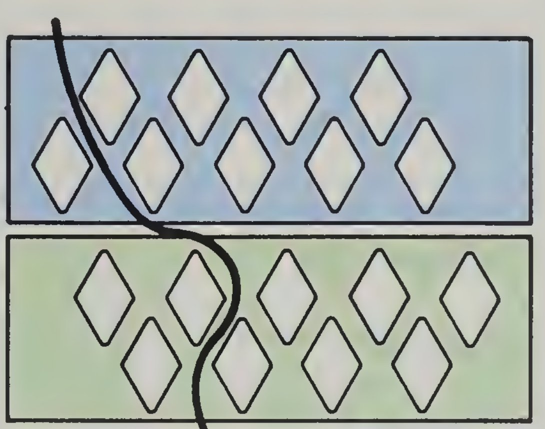
8	2
+ 5	+ 3
13	5

13 + 5 = 18



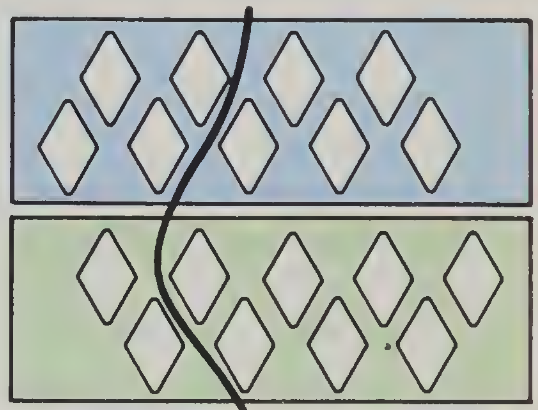
9
+ 9
18

9 + 9 = 18



1	8
+ 3	+ 6
4	14

4 + 14 = 18



4	5
+ 2	+ 7
6	12

6 + 12 = 18

7 other possible answers

11 + 7 = 18

15 + 3 = 18

16 + 2 = 18

18 = 1 + 17

18 = 18 + 0

8 + 10 = 18

Notes on Pupil Page 96

Tasks: *Pupils color the interior of like rectangles the same color, establish and write the pattern of multiples of 10 based on the clue for rectangle B in section (a); finally, they write the sums of various multiples of 10 found in sections (c) to (g).*

Purpose: (a) To provide pupils with further experiences with the base ten system of numeration; (b) to anticipate multiplication via the addition and subtraction of multiples of 10; and (c) to use numbers in situations involving measurement (area).

Unifying Ideas: Numeration; addition and subtraction; multiplication and division; measurement.

Materials for Teacher: No special materials are

Materials for Pupils: Cuisenaire® Rods.

The Lesson:

Have pupils turn to page 96, and assign it as independent study. Discuss and summarize the completed page with the class.

Have pupils complete the following sentences:

50 + □ = 50
40 + □ = 50
30 + □ = 50
20 + 30 = □
□ + 40 = 50

30 + □ + □ = 50
□ + 30 + 10 = 50
10 + 10 + □ = 50
20 + 20 + 10 = □
□ + 20 + 10 = 50
□ + □ + 10 = 50

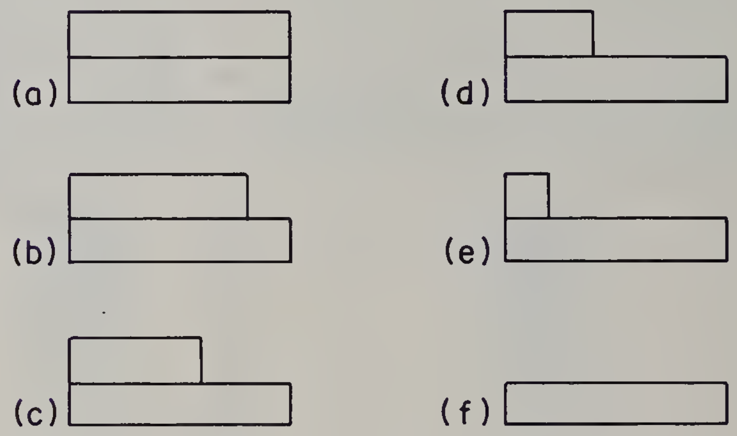
20 + □ + □ + □ = 50
50 = 10 + □ + 10 + 10
50 = □ + □ + 20 + 10
□ = 10 + 10 + 10 + 20
□ + □ + □ + □ + □ = 50

Remember, in a sentence with frames, you must write the same number in all frames of the same shape. For example, the first sentence in the second column should be completed as 30 + 10 + 10 = 50 and not as 30 + 5 + 15 = 50. (This rule of "same number for frames of the same shape" does NOT mean that differently shaped frames must be filled with different numbers.)

Establish agreement that the white rod represents 10. Then have pupils complete the following number sentences and construct models based on this agreement. The diagram below shows models for sentences (a) through (f).

(a) 50 − 50 = △
(b) 50 − 40 = △
(c) 50 − 30 = △

(d) 50 − 20 = △
(e) 50 − △ = 40
(f) 50 − △ = 50



Headline Story: Complete the headline and make a story to fit:

10 + 20 + 30 = △

a

Color
10 A
20 B
30 C
40 D
50 E

b

30 C
40 D
Color
10 A
50 E
20 B

c

20 10 10
20 20

d

40 20 20
40 40

e

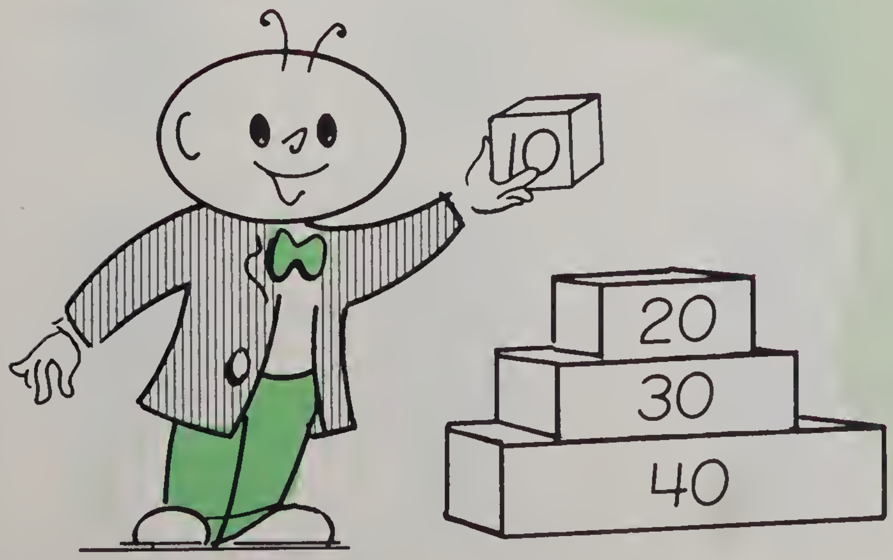
30 20 10
30 30

f

50 50
50 40 10

g

50 30 20
50 50



Notes on Pupil Page 97

Tasks: *Pupils use the information indicated by the number line jumps to complete the number sentences in the top half. For the bottom half of the page, pupils are to write the appropriate numbers in the blue and green boxes, using the number line provided to assist them when necessary.*

Purpose: To provide pupils with further experience (a) with second decade numbers primarily via addition and subtraction on the number line; and (b) in using numbers in situations involving measurement (money, weight, time, length).

Unifying Ideas: Structure; addition and subtraction; functions and relations; measurement.

Materials for Teacher: 16 pennies; 16 one-ounce weights; a clockface with minutes shown on it; 2 meter sticks; 2 yardsticks.

Materials for Pupils: No special materials needed.

The Lesson:

Assign page 97 for individual study. Discuss and summarize the completed page with the class.

Divide the class into five measuring teams, assigning the following tasks to them:

Measuring-Teams A and B. Team A prepares to report to the class on how it can use pennies to demonstrate each of the first three number sentences. Team B checks Team A's work.

Measuring-Teams C and D. Team C prepares to report on how it can use one-ounce weights to demonstrate the next two number sentences. Team D checks Team C's work.

Measuring-Teams E and F. Team E prepares to report on how they can use the clockface to demonstrate the next four number sentences. Team F checks Team E's work.

Measuring-Teams G and H. Team G prepares to report on how it can use meter sticks to draw lines on the board to demonstrate the next three number sentences. Team H does likewise for the next three number sentences.

Measuring-Teams I and J. Team I prepares to report on how it can use a yardstick to draw lines on the board to demonstrate the next three number sentences. Team J does likewise for the last three number sentences.

After all measuring teams have completed their tasks, have each team give its report to the class. Discuss the significance of the lesson with the class; e.g.,

Could Teams A and B have used one-ounce weights instead of pennies?

Could Teams C and D have used minutes instead of ounces?

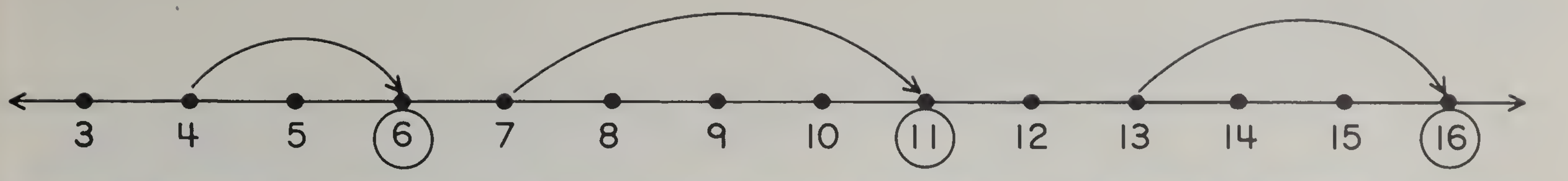
Could Teams E and F have used centimeters instead of minutes?

Could Teams G and H have used inches instead of centimeters?

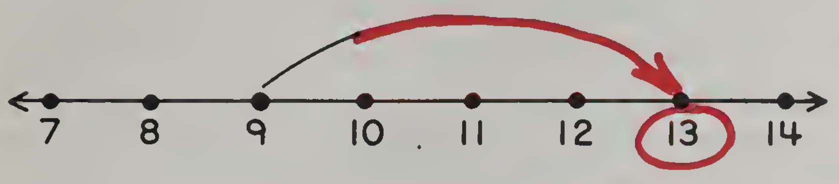
Could any of the measuring units have been used to demonstrate any of the number sentences?

Headline Story: Make up a story to fit this headline:

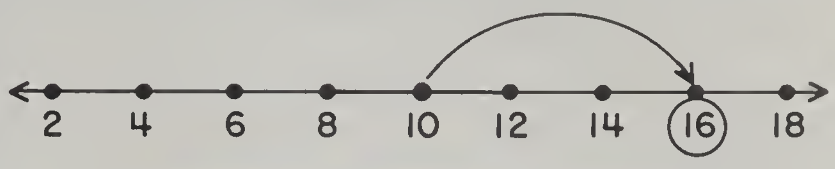
$$7 + \triangle = \triangle + 7$$


3 4 5 6 7 8 9 10 11 12 13 14 15 16

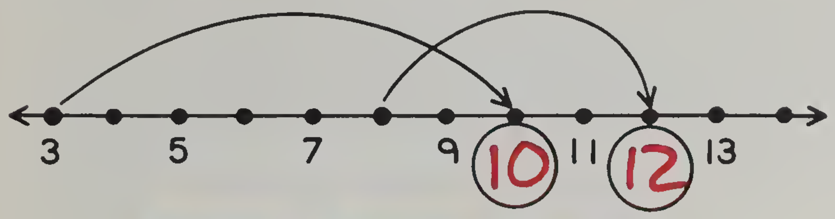
4 + 2 = 6   7 + 4 = 11   13 + 3 = 16


7 8 9 10 11 12 13 14

9 + 4 = 13

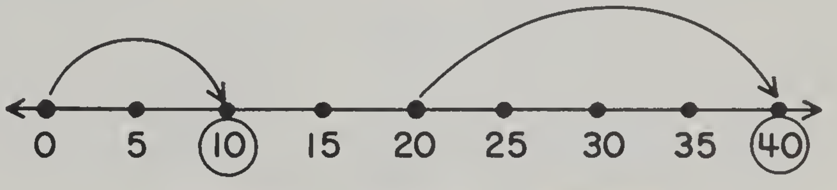

2 4 6 8 10 12 14 16 18

10 + 6 = 16


3 5 7 9 10 11 12 13

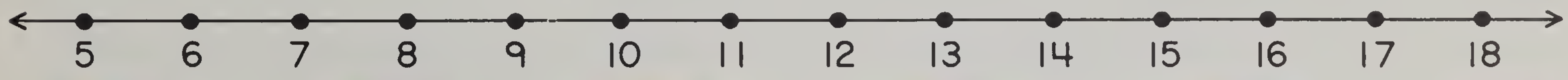
3 + 7 = 10

8 + 4 = 12


0 5 10 15 20 25 30 35 40

0 + 10 = 10

20 + 20 = 40


5 6 7 8 9 10 11 12 13 14 15 16 17 18

5	6	8	5	9	6
+ 6	+ 5	+ 5	+ 8	+ 6	+ 9
11	11	13	13	15	15

7	10	5	13	14	6
+ 10	+ 7	+ 13	+ 5	+ 6	+ 14
17	17	18	18	20	20

Notes on Pupil Page 98

Tasks: *Pupils use the clues provided on each number line chunk to help them label unnamed points.*

Purpose: (a) To provide pupils with further opportunities to establish order on the number line and (b) to extend their knowledge of the base ten system of numeration.

Unifying Ideas: Structure; number and counting; numeration.

Materials for Teacher: No special materials needed.

Materials for Pupils: No special materials needed.

The Lesson:

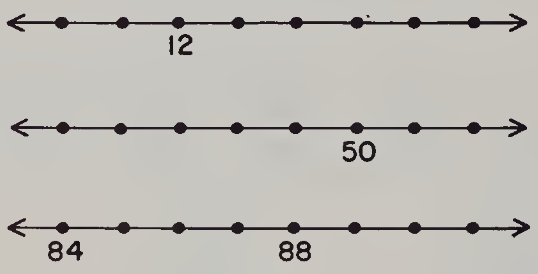
Play some counting forward and counting backward games similar to the following:

"From the number 10, count backward two numbers and count forward three numbers."

"From 60, count backward five numbers and forward five numbers."

"What are the numbers between 30 and 40?"

Draw some number line chunks on the board similar to the following and discuss naming the points.



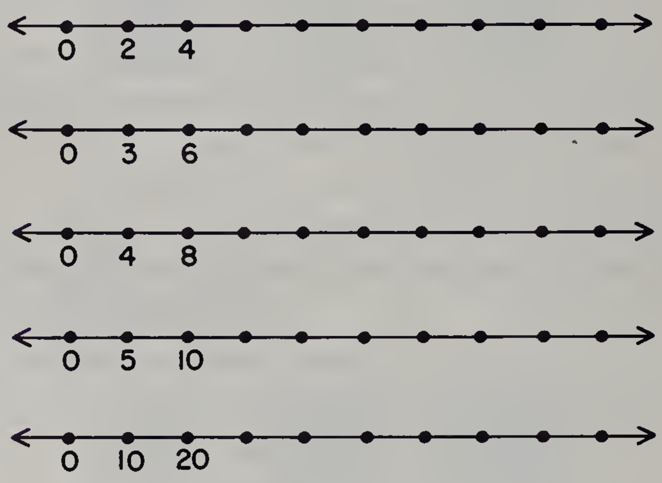
etc.

Have pupils turn to page 98 and complete it. Discuss and summarize their findings with the class. Note that the answers given on the pupil's page for the last number line chunks are the ones pupils are most likely to give. There are many more correct possibilities. Here are some for the next to the last chunk:

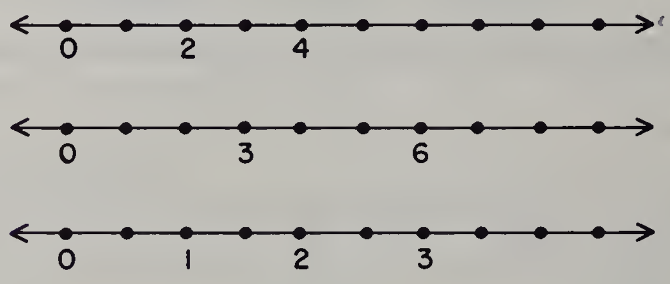
898, 899, 900, 901
8998, 8999, 9000, 9001

Show the first of these possibilities to your brightest student and let him suggest others for the last two chunks.

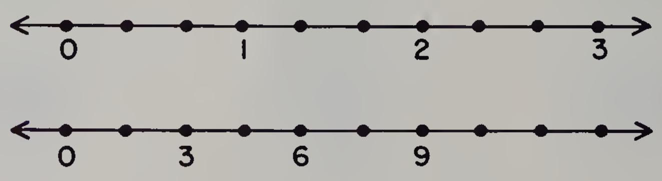
Sketch each of the following on the board and have pupils label the dots.



Alter the situation as follows so that pupils supply the missing numbers:

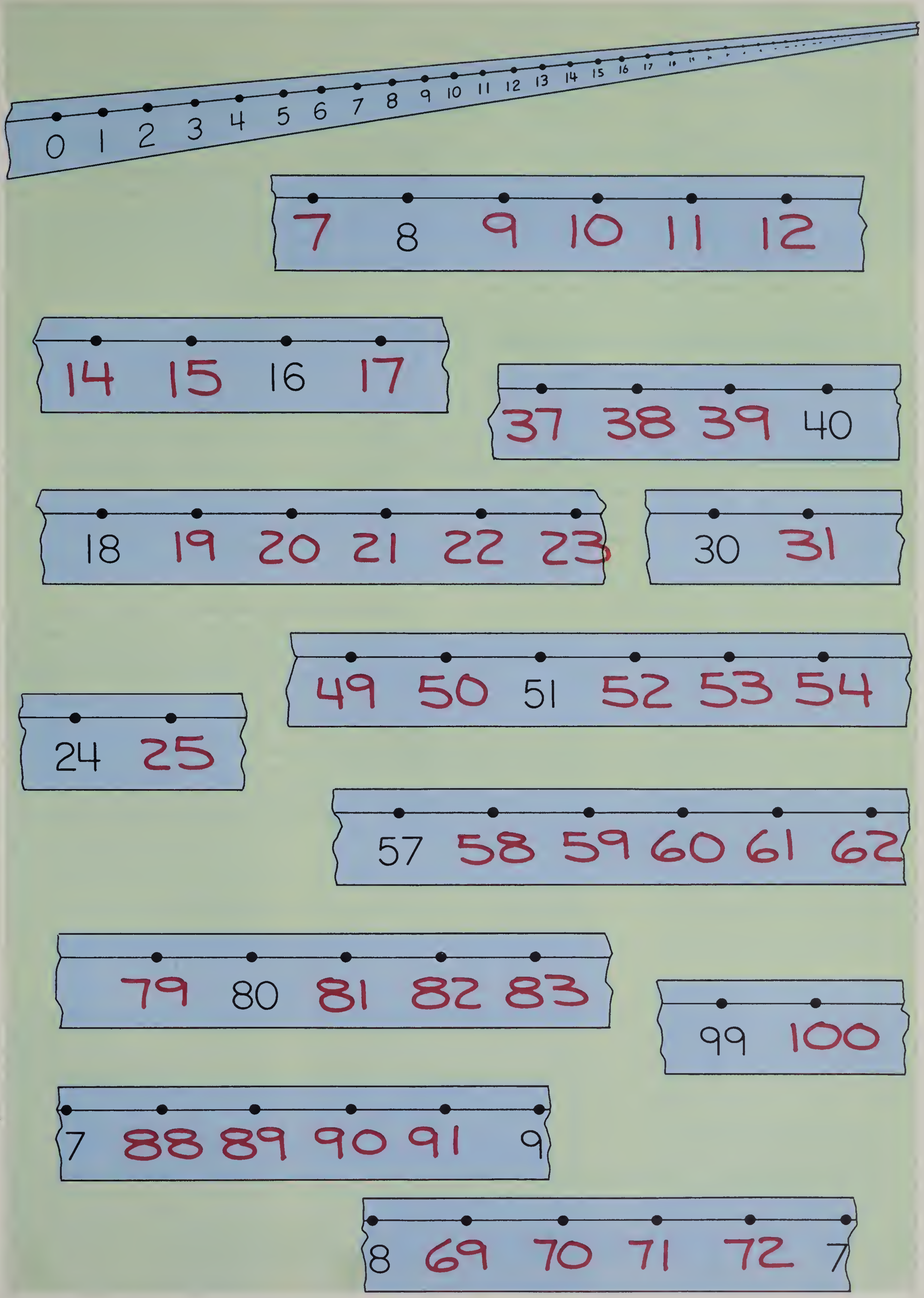


The last situation forces an important issue. If it goes well, try some more like the following:



Headline Story: Extend the series indicated:

37 __ 33 31 __ __ __ 23


0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17
7 8 9 10 11 12
14 15 16 17
37 38 39 40
18 19 20 21 22 23
30 31
49 50 51 52 53 54
24 25
57 58 59 60 61 62
79 80 81 82 83
99 100
7 88 89 90 91 9
8 69 70 71 72 7

Notes on Pupil Page 99

Tasks: *Pupils record in each blue box the value in cents of the coins displayed in that box.*

Purpose: To provide pupils with further experiences in using numbers in situations involving measurement (money).

Unifying Ideas: Measurement; addition and subtraction.

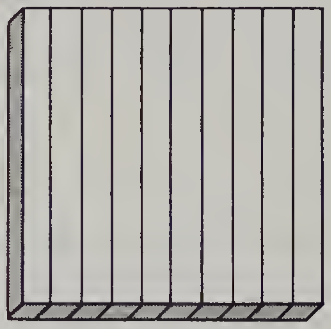
Materials for Teacher: Some pennies and dimes.

Materials for Pupils: Cuisenaire® Rods.

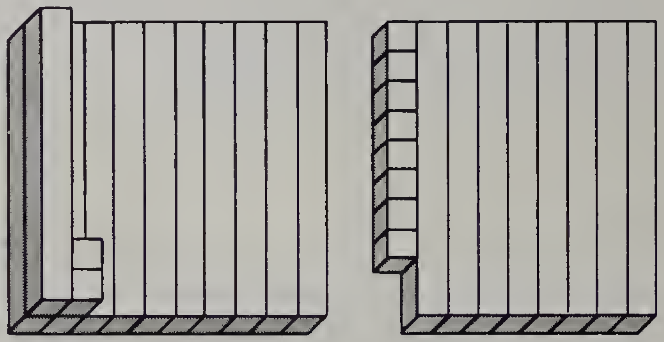
The Lesson:

Distribute your pennies and dimes. Have the class examine them carefully so that they know what is stamped on the front and back of each coin. Ask several pupils to describe the front and back of each coin.

Have the class turn to page 99. Discuss the first picture with the class. Assign the page for individual study. Discuss the completed page with the class. Have the class form a square of orange rods as shown below:

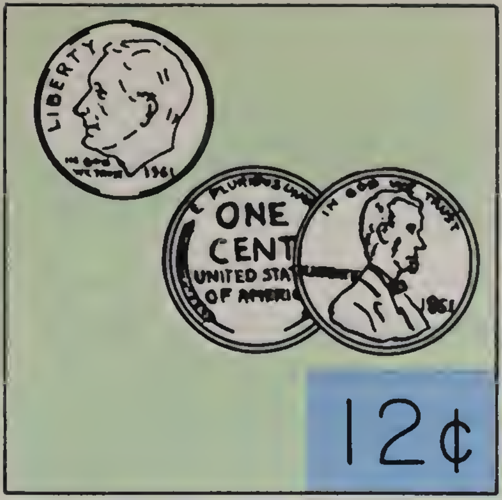


Tell the class that for the rest of the lesson, white rods will be called pennies, orange rods will be called dimes, and the square of orange rods, one dollar. Have them represent the first picture, using their rods. "Cover the orange square with a dime and two pennies. How much of the dollar is left over?" A sketch of their actions looks like this:

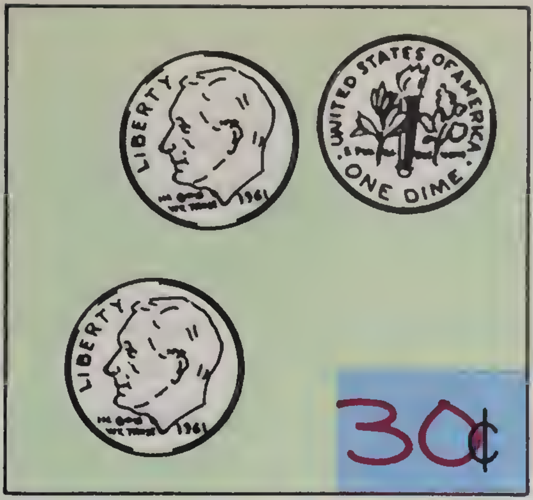


Form several two-member play teams. Each team member takes turns at being a customer and a clerk. The customer pretends he has purchased something for an amount equal to that represented in one of the twelve boxes on the pupil page. He points to the box and says, "I am going to buy a _____ which costs this much. If I give you a dollar bill, how much change will you give me?" The clerk computes the change and tells the customer the amount. The customer then checks the clerk.

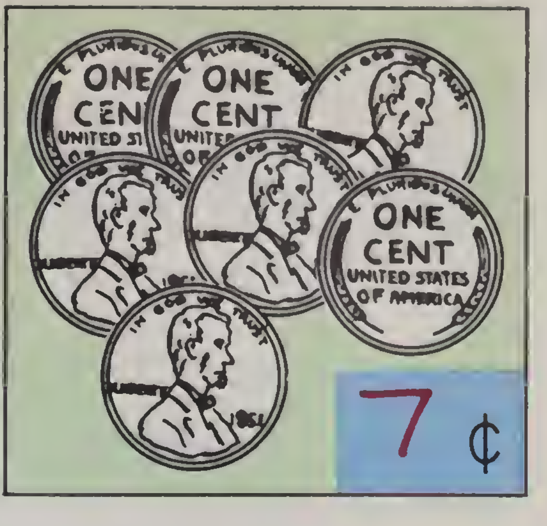
Headline Story: How many ways can you make 25¢ with an even number of coins?



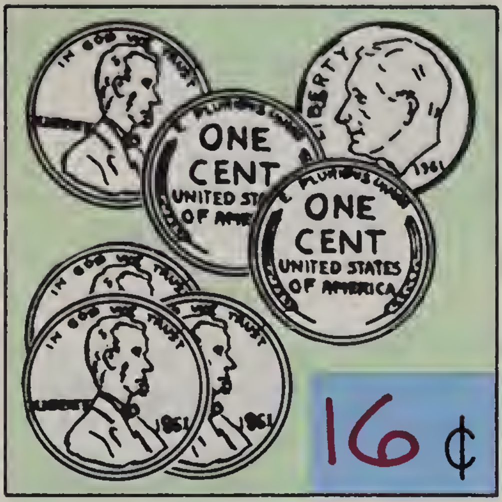
12¢



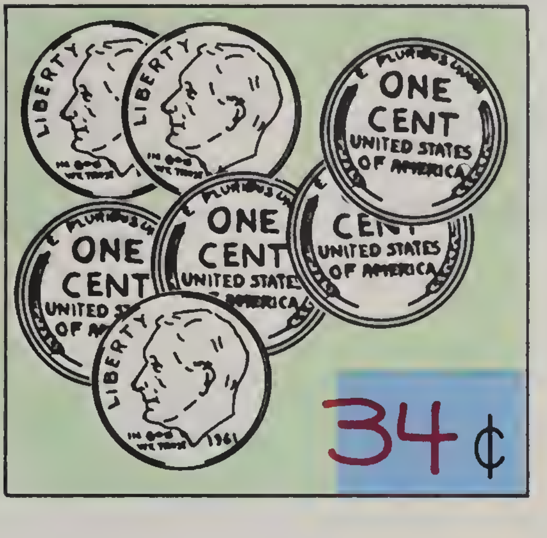
30¢



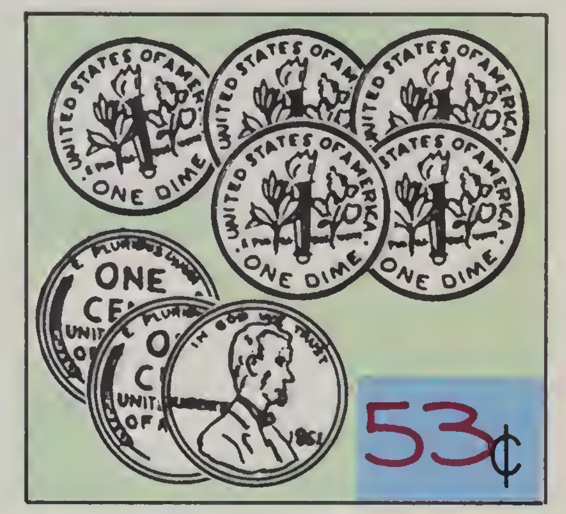
7 ¢



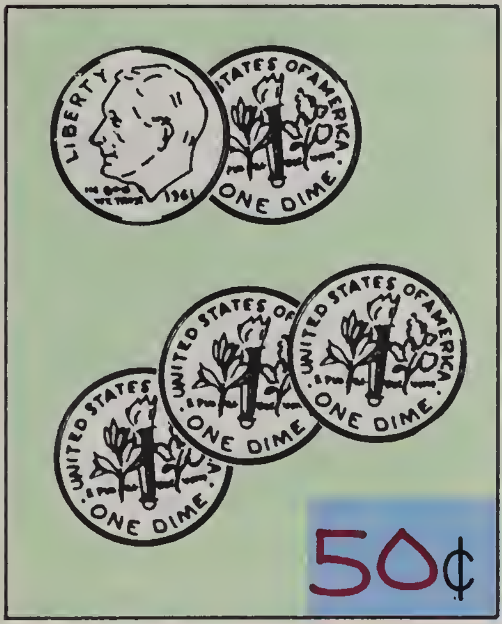
16 ¢



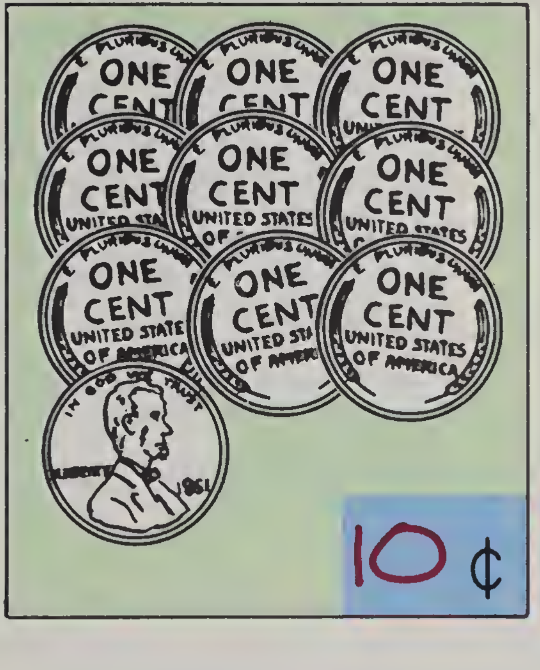
34 ¢



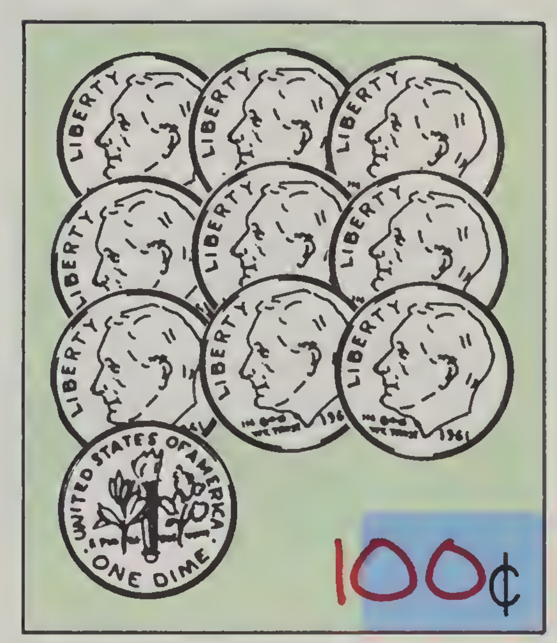
53¢



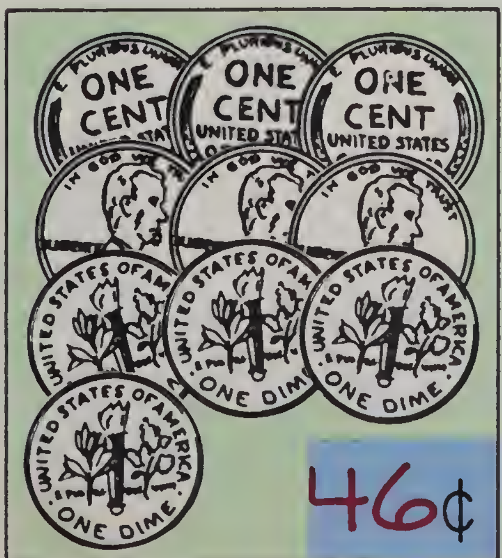
50¢



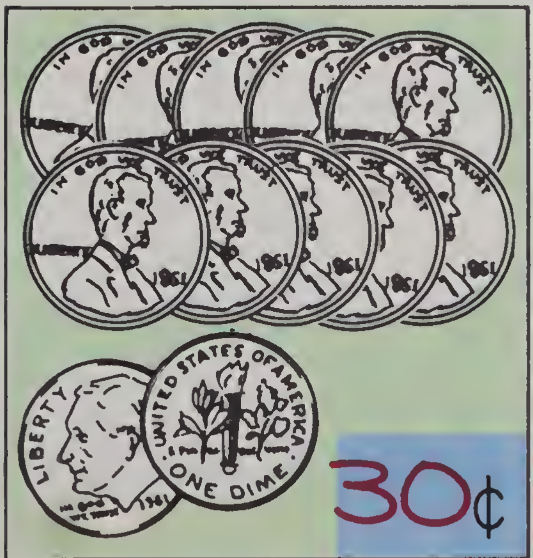
10 ¢



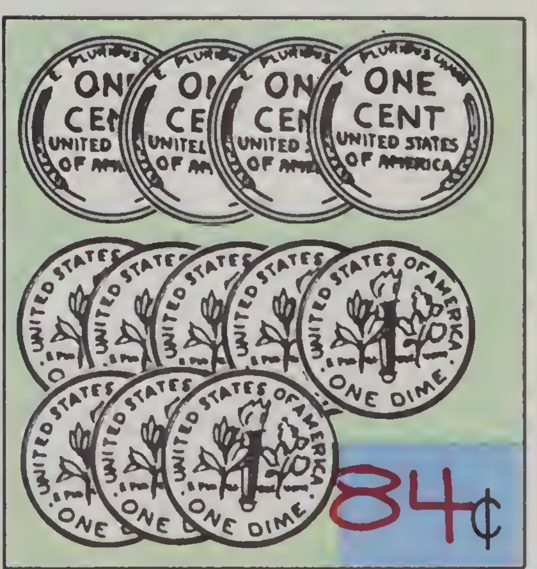
100¢



46¢



30¢



84¢

Notes on Pupil Page 100

Tasks: *Pupils complete each example.*

Purpose: To test pupil achievement.

Materials for Teacher: No special materials are needed.

Materials for Pupils: No special materials needed.

The Lesson:

Think of a number greater than 4 and less than 16. Think of another . . . and another . . . until five have been given and written on the board, such as:

5		13
	10	
8		15

Combine these (any number may be repeated) to make true sentences in the following forms:

____ + ____ = 20

20 − ____ = ____

such as:

5 + 15 = 20 15 + 5 = 20 20 − 15 = 5 20 − 5 = 15	fact team
10 + 10 = 20 20 − 10 = 10	fact team

Divide the class into two teams—A and B—and let each select its own five numbers greater than 4 and less than 16. The team wins that makes the most sentences of these numbers in the forms:

____ + ____ = 20 and 20 − ____ = ____

A	B
5 10 13 8 15	7 , 10 , 13 9 , 11
(all listed above)	(three fact teams)

The selection by team B will afford ten true statements in the form specified—two 4-member fact teams and one 2-member fact team:

7 + 13 = 20 (4 members)

11 + 9 = 20 (4 members)

10 + 10 = 20 (2 members)

The game can be made more exciting and challenging by changing the forms permitted, such as:

____ + ____ + ____ = 20 ____ + ____ − ____ = 20

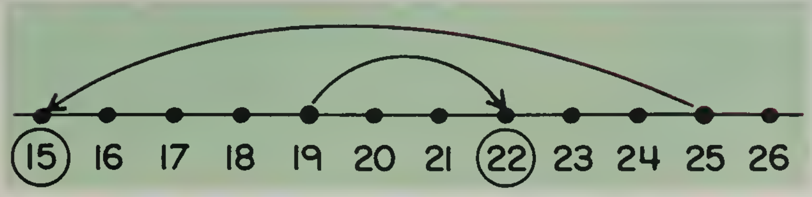
Now team A, with the above selections, could write such expressions as:

5 + 5 + 10 = 20 15 + 13 − 8 = 20 etc.

Be sure to review the accompanying booklet, *Games and Enrichment Activities*, for suggestions you may wish to incorporate in your daily lesson plan.

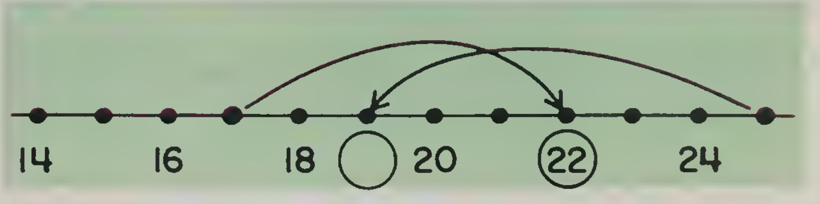
Headline Story: Tina spent 37¢ and gave the clerk a half-dollar. (Pupils extend.)

25 26 27 28 29 30 31 32 33 34 35


⑮ 16 17 18 19 20 21 ㉒ 23 24 25 26

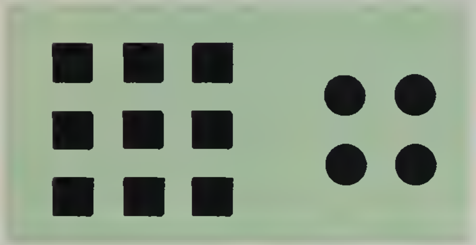
19 + 3 = 22

25 − 10 = 15


14 16 18 ○ 20 ㉒ 24

17 + 5 = 22

25 − 6 = 19



9 + 4 = 13

4 + 9 = 13

13 − 9 = 4

13 − 4 = 9

10 + 5 = 15

10 + 10 = 20

12 + 10 = 22

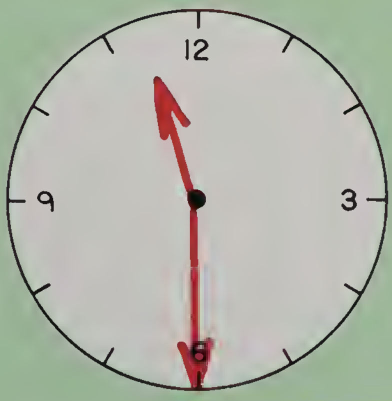
19 + 2 = 21

3 + 17 = 20

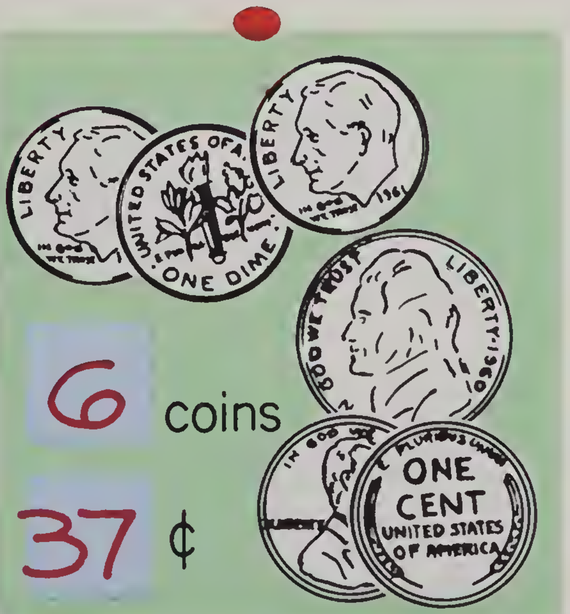
11 + 11 = 22

$$\begin{array}{r}18\\+\ 2\\\hline 20\end{array}\qquad\begin{array}{r}18\\+10\\\hline 28\end{array}\qquad\begin{array}{r}18\\+12\\\hline 30\end{array}$$

$$\begin{array}{r}25\\-\ 5\\\hline 20\end{array}\qquad\begin{array}{r}25\\-\ 7\\\hline 18\end{array}\qquad\begin{array}{r}25\\-10\\\hline 15\end{array}$$

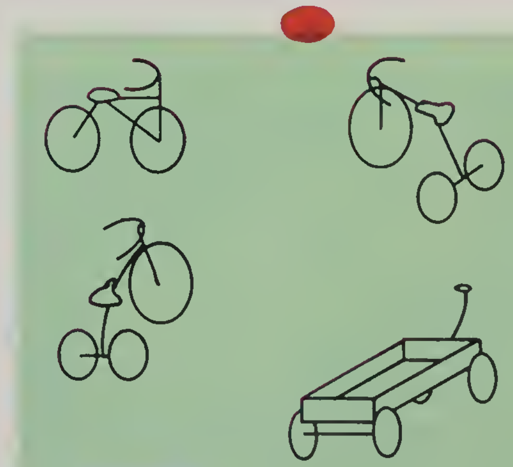


The time is 11:30


LIBERTY
IN GOD WE TRUST
UNITED STATES OF AMERICA
ONE DIME
1961
ONE CENT

6 coins

37 ¢



12 wheels

Notes on Pupil Page 101

Tasks: *Pupils complete each example.*

Purpose: To test pupil achievement.

Materials for Teacher: No special materials are needed.

Materials for Pupils: No special materials are needed.

The Lesson:

Consider three groupings of indicated addition facts. In each case, add the numbers in each column.

(a)
3 + 4 = 7
8 + 1 = 9
11 5 16

(b)
2 + 3 + 1 = 6
5 + 0 + 4 = 9
7 3 5 15

(c)
2 + 1 + 4 = 7
3 + 3 + 3 = 9
5 + 1 + 3 = 9
10 5 10 25

We notice that 11 + 5 = 16, 7 + 3 + 5 = 15, and 10 + 5 + 10 = 25.

After several examples of each kind are worked out, erase the expressions and leave the results found; or, in the cases given above, consider the following:

(a)
___ + ___ = 7
___ + ___ = 9
11 + 5 = 16

(b)
___ + ___ + ___ = 6
___ + ___ + ___ = 9
7 + 3 + 5 = 15

(c)
___ + ___ + ___ = 7
___ + ___ + ___ = 9
___ + ___ + ___ = 9
10 + 5 + 10 = 25

Can you find at least three different ways to reconstruct each of the three examples? There are many more, but here are three ways to reconstruct the first:

7 + 0 = 7
4 + 5 = 9
11 + 5 = 16

2 + 5 = 7
9 + 0 = 9
11 + 5 = 16

6 + 1 = 7
5 + 4 = 9
11 + 5 = 16

If you want to add a degree of complexity, limit numbers to be used to all odd numbers, and 0 only if needed; or all even numbers, and 1 only if needed. (Without 0 and 1, some would be impossible.) Who can complete each with the fewest uses of 0 and 1?

Headline Story: Can you make 45¢, using exactly 14 coins?

8	8	16
3	6	9
11	14	25

7	9	16
7	4	11
14	13	27

5	8	13
9	9	18
14	17	31

1	2	3	6
6	5	4	15
7	7	7	21

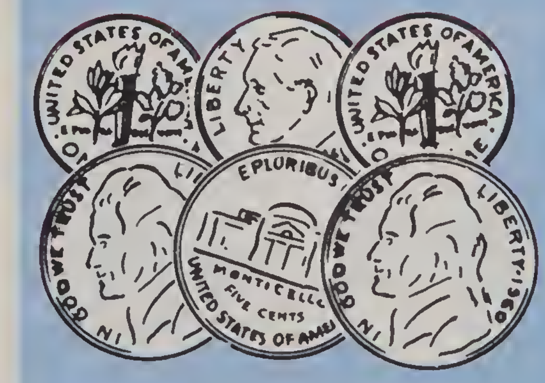
9	6	3	18
1	3	5	9
10	9	8	27

8	1	6	15
3	5	7	15
4	9	2	15
15	15	15	45

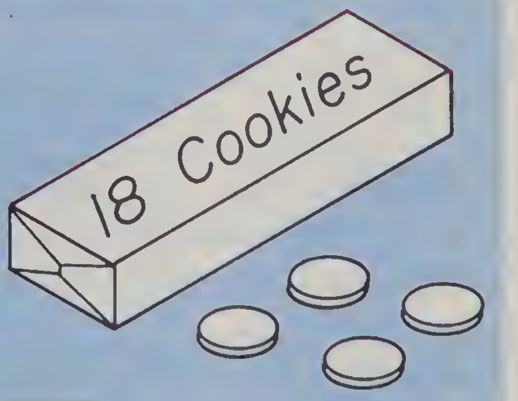
5	1	4	10
8	6	4	18
2	14	4	20
15	21	12	48

Bill Jones has two brothers and three sisters. Bill is three years old.

13 words



45¢



22 cookies

Notes on Pupil Page 102

Tasks: Pupils complete the number line and look for patterns in the given data, filling in the blanks to maintain the patterns.

Purpose: To provide pupils with further opportunities to explore number patterns and their related rules of formation.

Unifying Ideas: Functions and relations.

Materials for Teacher: A chalkboard pointer.

Materials for Pupils: No special materials are needed.

The Lesson:

Divide the class into two groups to work on page 102. One group works on patterns A and C; the other on B and D. After the examples on page 102 are completed, pupils describe their discoveries. Put more difficult examples of each pattern on the chalkboard for the pupils to consider.

A	B
8 , 9 → 10	15 , 14 → 13
9 , ___ → ___	11 , ___ → ___
___ ___ → 14	___ , 29 → ___
19 , ___ → ___	___ , ___ → 39

C	D
9 , 11 → 10	6 , 8 → 10
18 , ___ → 19	7 , ___ → ___
25 , ___ → ___	___ , 16 → ___
___ , 30 → ___	___ , ___ → 12

Encourage pupils to discuss what number is appropriate for each blank. Then, with a pointer, indicate a blank and call for a response from a particular pupil. Select blanks at random, speeding up the tempo, allowing a very brief time for an answer.

At any point, the level of difficulty can be increased by (1) extending the examples of each pattern, using larger numbers, and (2) initiating other examples such as:

E	F
2 , 5 → 8	10 , 21 → 32
0 , 3 → 6	8 , 19 → 30
4 , 7 → ___	3 , 14 → ___
___ , 9 → ___	___ , 37 → ___
___ , ___ → 24	___ , ___ → 55

or (3) using any rule that is appropriate to challenge the group of pupils involved.

Now consider having the class form work teams of two members each. "A" writes a number on his paper. "B" takes the number and, using his secret rule, writes his answer to "A":

A	1
B	3

"A" writes another number. "B" uses his rule to answer "A":

A	1	3
B	3	5

"A" writes still another number. "B" answers:

A	1	3	2
B	3	5	4

"A" must now determine "B"'s rule.

Headline Story: Make up a headline that fits: Walt broke all but 3 of a dozen eggs.

WHAT'S MY RULE?

0 1 2 3 4 5 6 7 8 9

A

1, 2 → 3
6, 7 → 8
3, 4 → 5

7, 8 → 9
5, 6 → 7
2, 3 → 4

B

5, 4 → 3
2, 1 → 0
7, 6 → 5

4, 3 → 2
9, 8 → 7
6, 5 → 4

C

1, 3 → 2
4, 6 → 5
2, 4 → 3

7, 9 → 8
3, 5 → 4
0, 2 → 1

D

0, 2 → 4
3, 5 → 7
4, 6 → 8

2, 4 → 6
5, 7 → 9
1, 3 → 5

Notes on Pupil Page 103

Tasks: *Pupils discover the specific pattern in each of the four lists at the bottom of the page and use the number line at the top of the page to help them complete each list in accordance with the jump rule they have discovered.*

Purpose: (a) To provide pupils with further experiences in addition and subtraction via the number line; (b) to help them discover patterns.

Unifying Ideas: Addition and subtraction; functions and relations.

Materials for Teacher: Demonstration models of the number lines shown in "The Lesson"; flannel board or cork board; colored chalk.

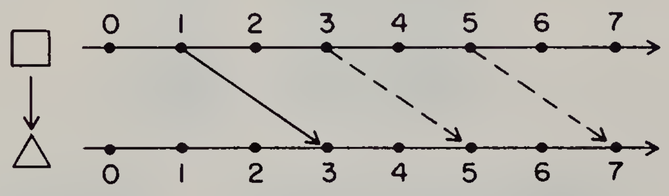
Materials for Pupils: Copies of your number line models for work at their desks.

The Lesson:

By correctly matching the first two entries in each list at the bottom of the page with the appropriate jumps on the number line at the top of the page, pupils should be able to complete the rest of the blanks. Let pupils put their findings on the chalkboard and discuss how they discovered the pattern.

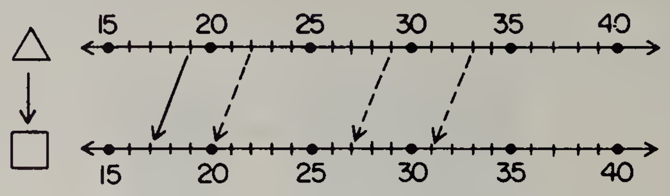
On the lower left quarter of the page, the two sentences with frames show that each of the jump rules is the inverse of the other. Similarly, the jump rules exemplified by the Tables in the lower right quarter are inverses of each other.

Consider following up page 103 with an alternative diagram appropriate to this activity: two parallel number lines.

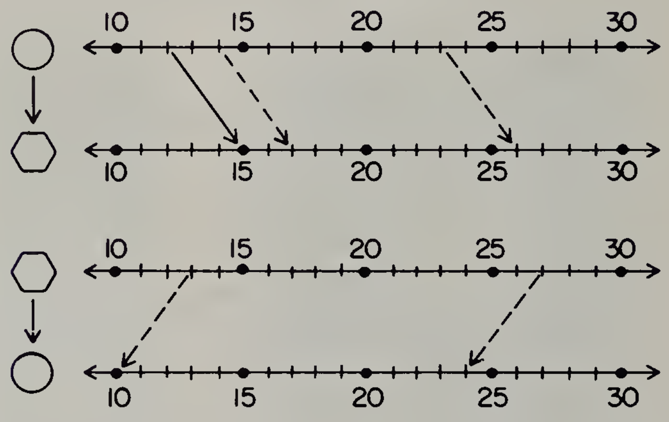


Invite different pupils to suggest a starting number. After the class locates the landing point on their number lines, invite others to use your demonstration model for all the class to see and check against their findings.

The second pattern emerges from another pair of number lines.

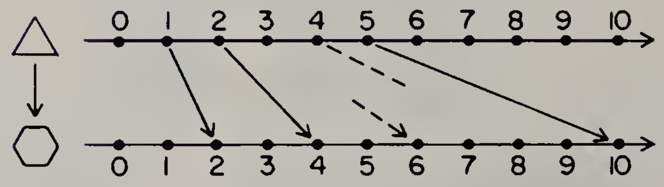


The third and fourth patterns, when shown together in this fashion, suggest pictorially that *adding three* can be reversed by *subtracting three!*



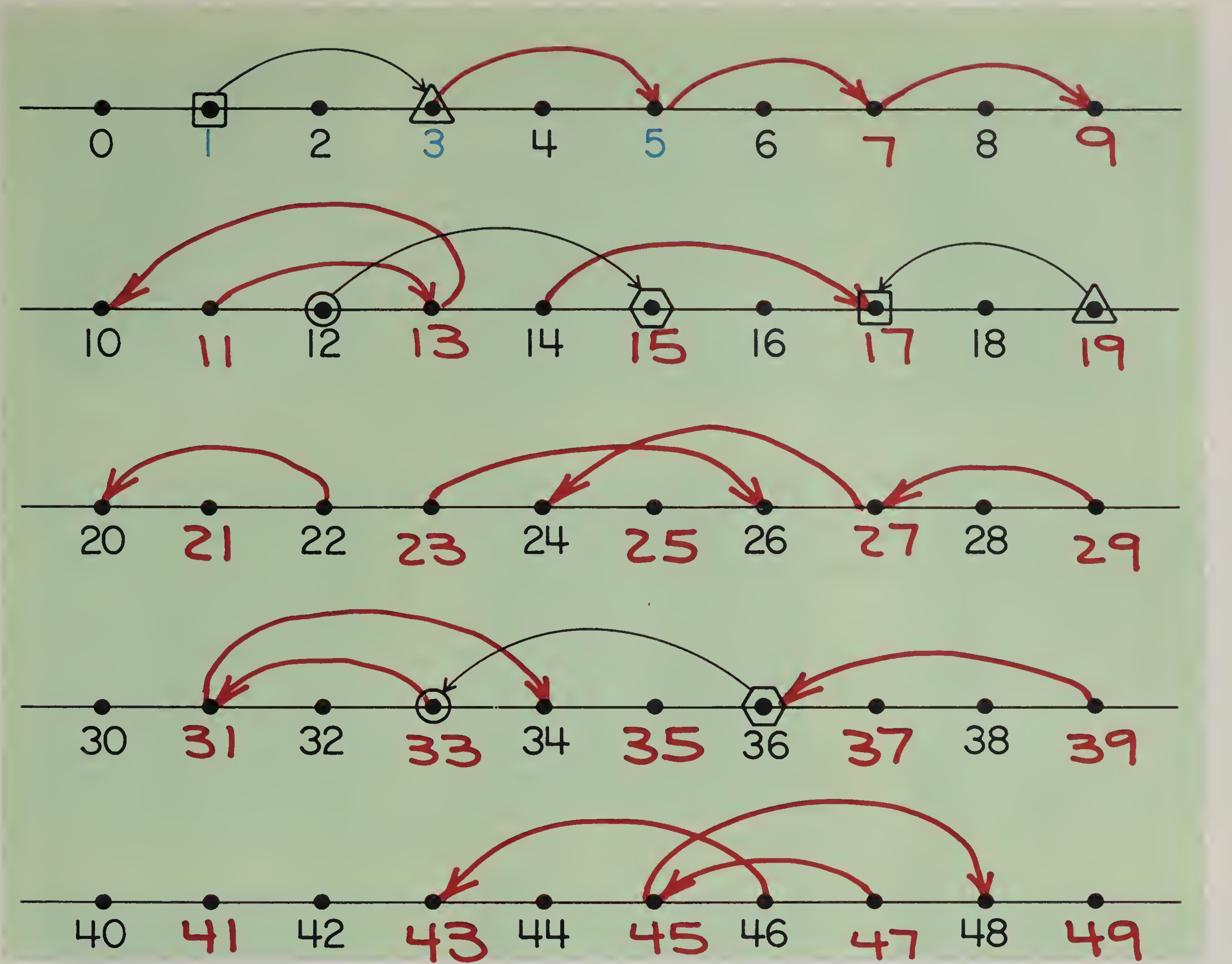
Now invite the class to use their number lines to explore a "doubling" rule:

△ → ⬡
2 → 4
5 → 10
1 → 2
4 → __
__ → 6



Pupils like to invent their own "What's My Rule?" games, giving a few clues and challenging the others to discover the pattern.

Headline Story: How many ways can you make 25¢ with an odd number of coins?


0
1
2
3
4
5
6
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47
48
49

□ → △	△ → □	○ → ⬡	⬡ → ○
1 → 3	19 → 17	12 → 15	36 → 33
3 → 5	22 → 20	45 → 48	27 → 24
5 → 7	47 → 45	14 → 17	39 → 36
7 → 9	29 → 27	31 → 34	13 → 10
11 → 13	33 → 31	23 → 26	46 → 43

Notes on Pupil Page 104

Tasks: *Pupils complete the sketches for examples II through IX, guided by the clues given in the chart at the bottom of the page. They also complete that chart by using the information acquired through completing the sketches.*

Purpose: To help pupils further develop the notion of rows and columns—the array representation of the idea underlying multiplication.

Unifying Ideas: Multiplication and division.

Materials for Teacher: Colored chalk, thin rectangular strips, and demonstration counters for your flannel or cork board.

Materials for Pupils: Sharpened pencil, straightedge, paper and counters for work at their desks.

The Lesson:

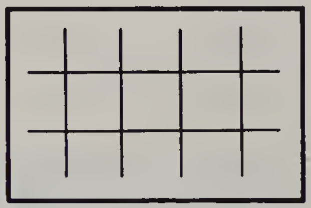
Invite pupils to examine diagram I and column I in the table at the bottom. "In what way is the first column of figures a written record of the first diagram?" (The 4 indicates the number of horizontal lines, 3 the number of vertical lines, while the 12 indicates the number of intersection points.)

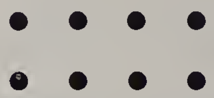
Diagram II poses a problem. Ask what information they get from the diagram. (2 horizontal lines.) What information do they get from column II? (5 vertical lines.) Have them use the straightedge and a pencil to construct a model on their papers based upon this information. "Place a counter on each point where one line cuts across another. Count your counters and decide which number goes in the blank space at the bottom of column II." Have several pupils assist you in constructing a flannel board model of the completed task.

For sketch number III have pupils pair off. "A" dictates the information given by diagram III and column III of the table. "B" completes the sketch. "A" then places counters at each intersection point. "B" counts and records the number of counters placed on the diagram.

Examples IV, VII, VIII and IX ask a somewhat different question. Consider IV. "Puzzle out the connection between the blank diagram IV and the clues given in column IV. What does the number 3 tell us?" (The number of horizontal lines to be drawn.) "A" draws them on his paper. "What is the number 9 a record of?" (The number of intersecting points they should have after drawing in the correct number of vertical lines.) "A" now draws in the vertical lines. "B" counts out 9 counters. After each vertical line is drawn, "B" places counters on his desk — one for each intersection. When his supply of 9 counters is exhausted, both pupils are in a position to verify and record the number of vertical lines required to complete the sketch.

Pupils can now do the following: "A" constructs a diagram using straight-edge and pencil. "B" duplicates the sketch, using only his counters.

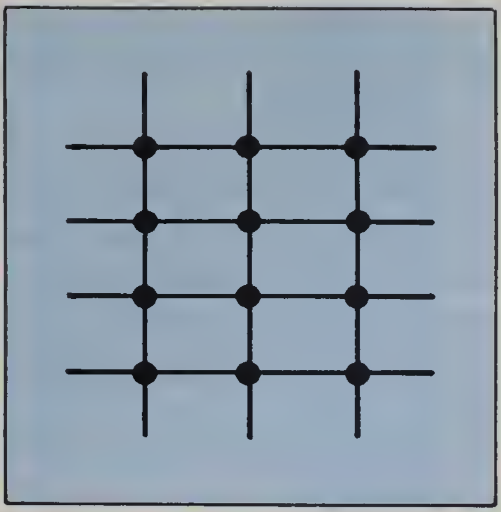




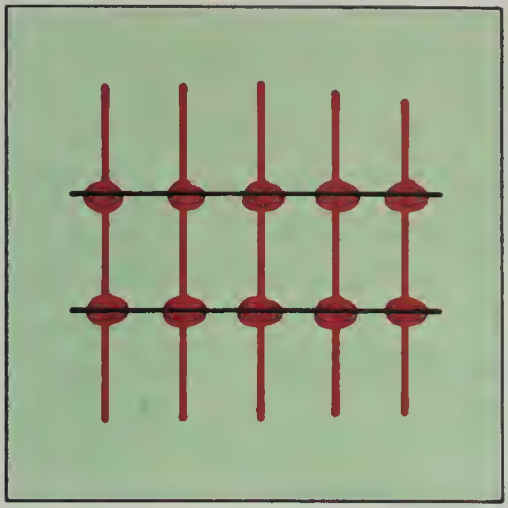
Each team member writes his own record of his actions. They then compare their records to be sure there is agreement. Now complete page 104.

Headline Story: Make up a headline that fits: March has 1 more day than April.

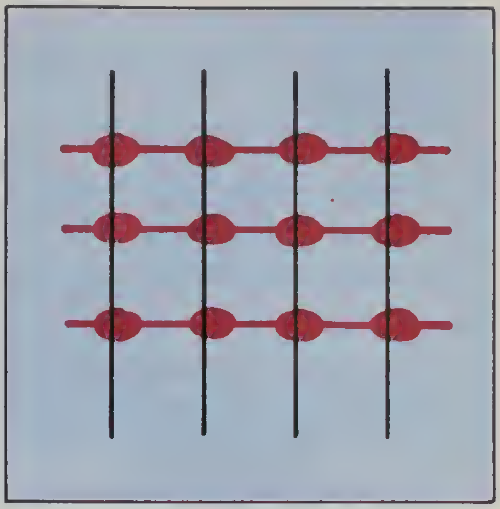
I



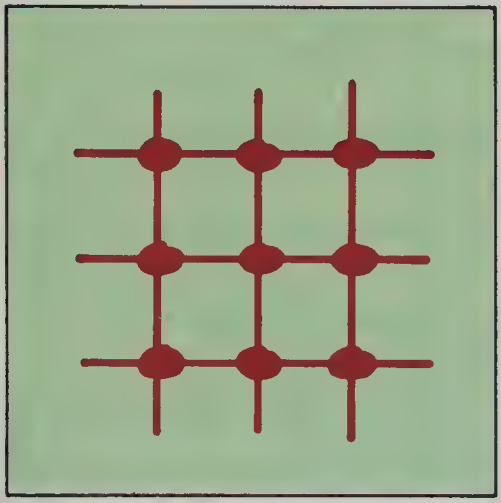
II



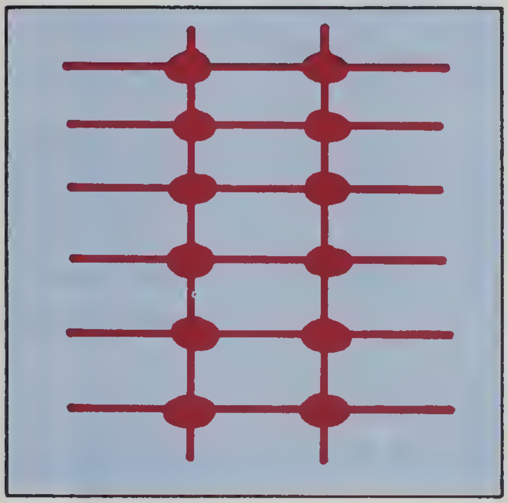
III



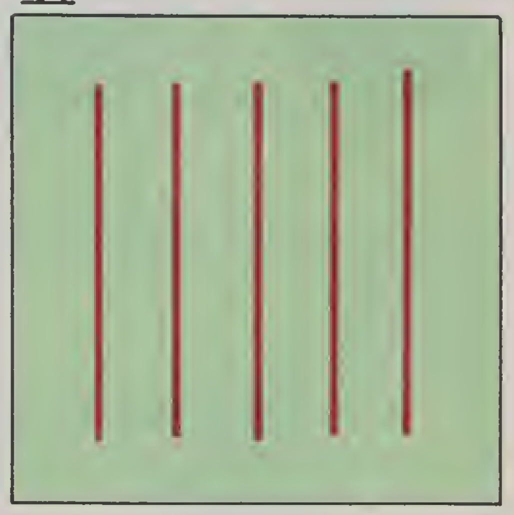
IV



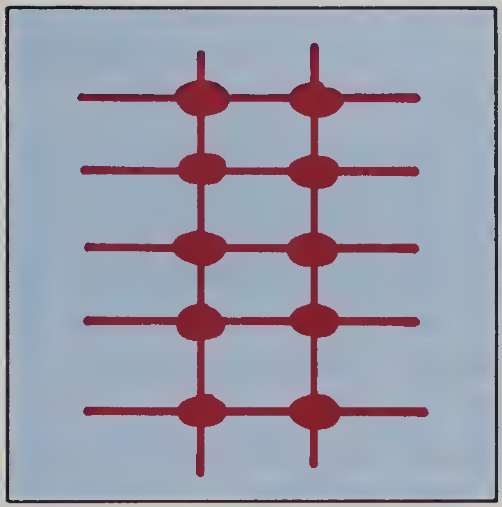
V



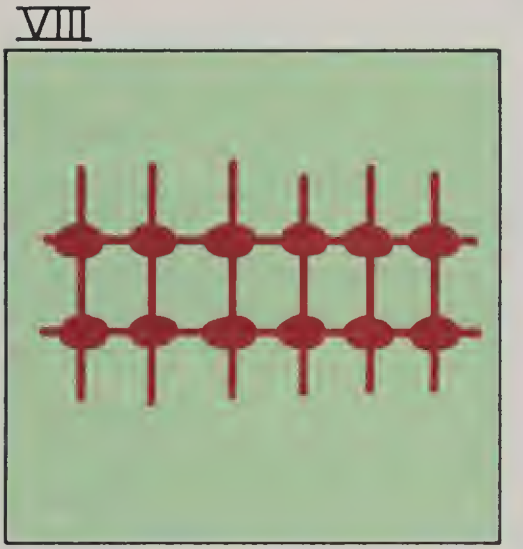
VI



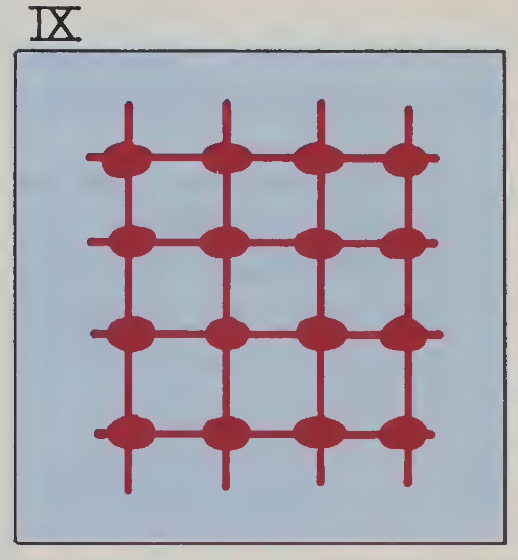
VII



VIII



IX



	I	II	III	IV	V	VI	VII	VIII	IX
— Lines	4	2	3	3	6	0	5	2	4
| Lines	3	5	4	3	2	5	2	6	4
┼	12	10	12	9	12	0	10	12	16

Notes on Pupil Page 105

Tasks: *Pupils write numbers in the tinted boxes only, according to the patterns suggested by the given clues for each situation.*

Purpose: To encourage pupils to search for patterns that lead to short cuts to counting.

Unifying Ideas: Numeration; multiplication and division.

Materials for Teacher: A few sets of demonstration size number cards labeled 1-30; a flannel board or cork board; colored chalk.

Materials for Pupils: A set of number cards labeled 1-30 for work at their desks.

The Lesson:

Use number cards to begin building the following pattern on your flannel board or cork board:

1	2	3
4		

1	2	3
4	5	6
7	8	9
10	11	12

Ask pupils to build the same pattern at their desks while others come up to the board to help you complete the demonstration model. Make a written record of the final version on the chalkboard. Now begin a new pattern:

1	5	
2		
3		
4		

1	5	9
2	6	10
3	7	11
4	8	12

Have the class discuss the various number sequences they see. Notice in particular that the right hand column of the first record shows multiples of 3; i.e., 3, 6, 9, 12.

Arrange the class into pairs or buddy teams. One buddy reconstructs the first record at his desk while the other reconstructs the second record. Each buddy turns over some of his number cards and asks his buddy to tell him which number cards were turned over.

one buddy

1	2	3
	5	
7		
10		

the other buddy

1		
2	6	10
	8	

Encourage pupils to develop speed and accuracy in playing the game.

Use grids (patterns) with larger numbers of rows and columns. Each pattern will have a row or column anticipating a multiplication table.

1	2	3
4	5	6
7	8	9
10	11	12

1	2	3	4
5			8
			12
			16

1	2	3	4	5
6	7			10
				15
				20
				25
				30

1	2	3	4	5	6
					12
					18
					24

Shift from desk work and flannel board displays to the chalkboard as the patterns get larger.

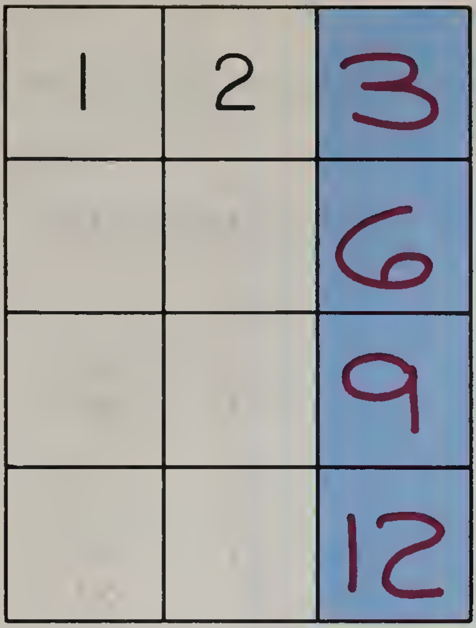
Sometimes ask for the numbers in the rows and columns in sequential order. Ask what numbers would appear in the shaded column if the grid were extended. Check each answer by constructing one or two rows. Erase some of the numbers in the extended pattern, leaving others in as hints for pupils to reconstruct the record.

Pupils may now turn to page 105 in their MATH WORKSHOP book for individual study.

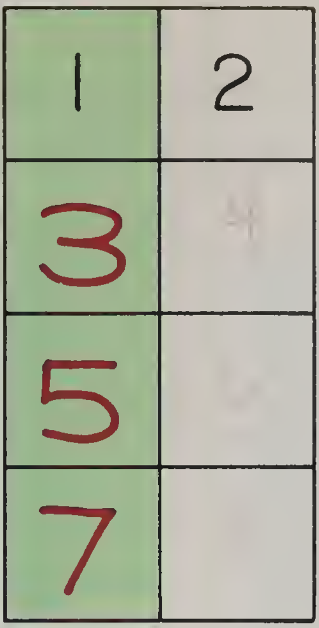
Be sure to review the accompanying booklet, *Games and Enrichment Activities*, for suggestions you may wish to incorporate in your daily lesson plan.

Headline Story: Complete the pattern suggested:

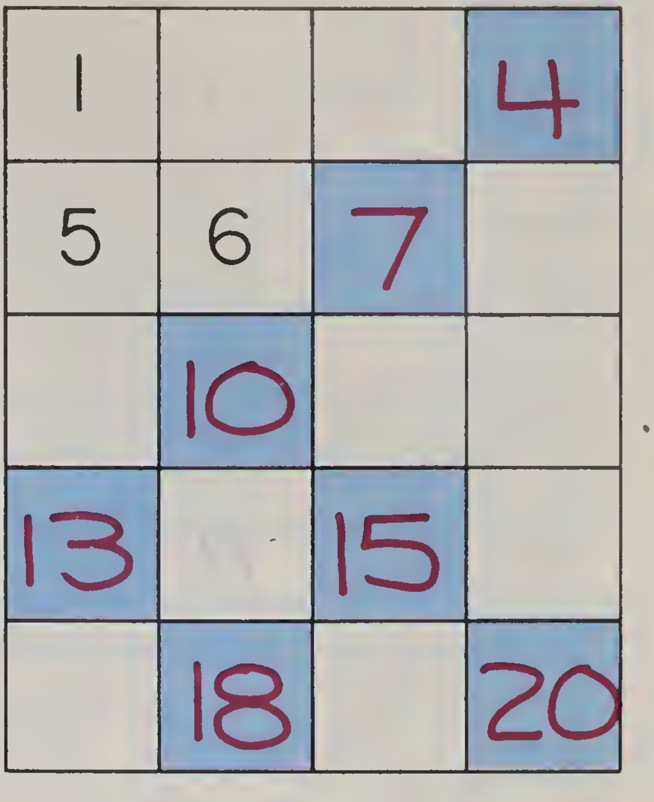
__ 3 __ 9 12 __ 18 __



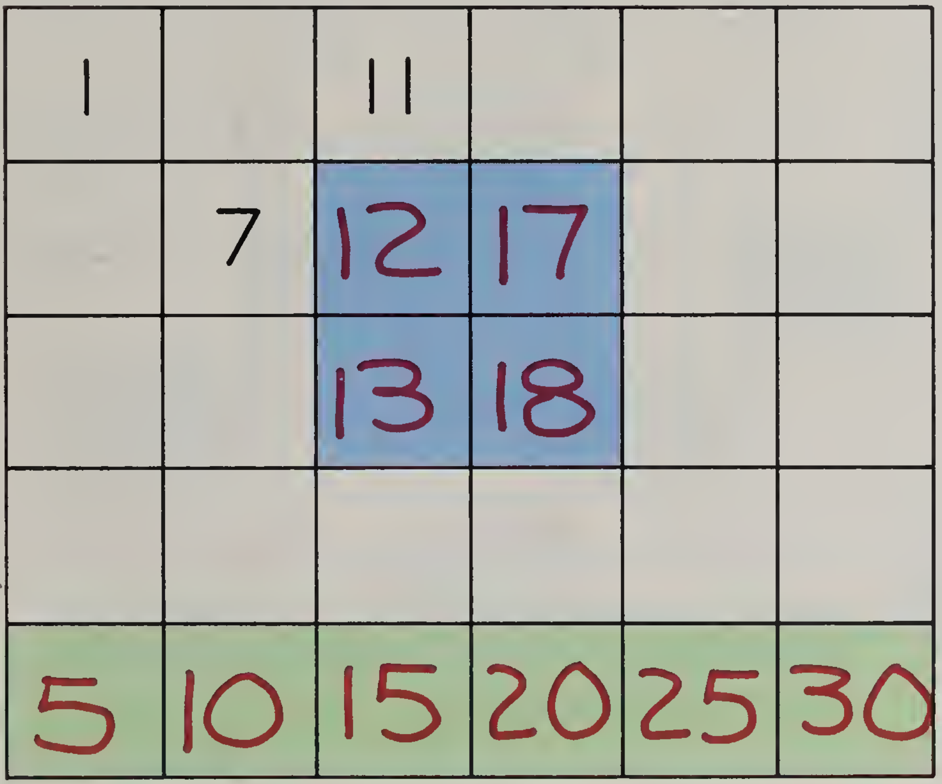
1	2	3
		6
		9
		12



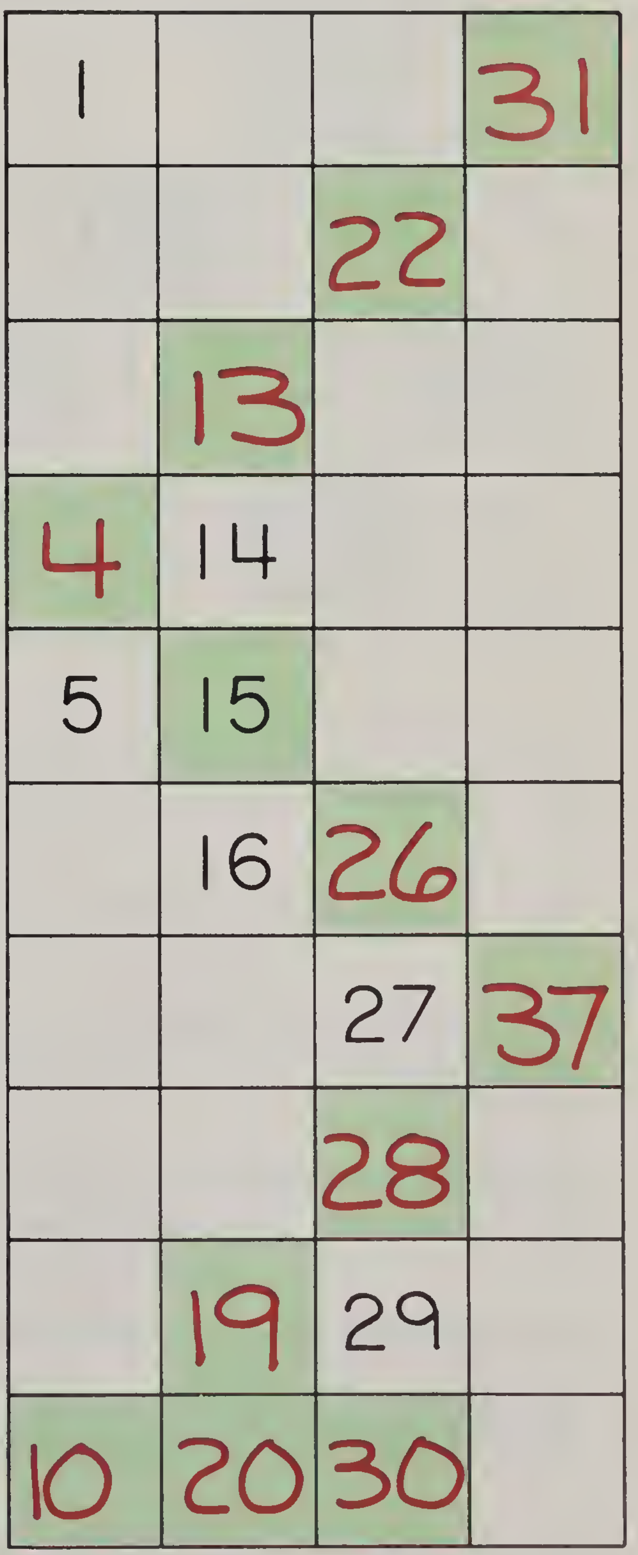
1	2
3	
5	
7	



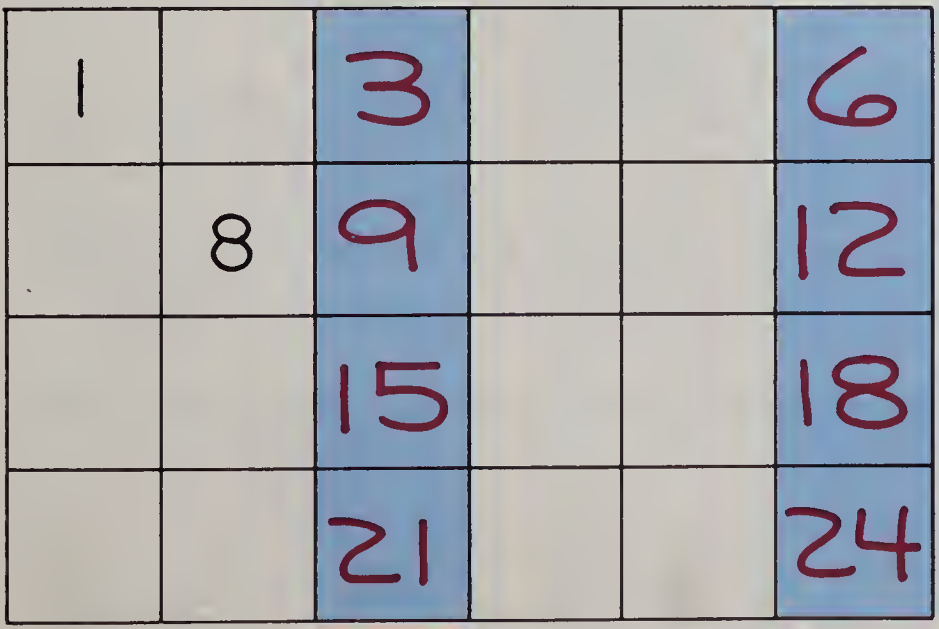
1			4
5	6	7	
	10		
13		15	
	18		20



1		11			
	7	12	17		
		13	18		
5	10	15	20	25	30



1			31
		22	
	13		
4	14		
5	15		
	16	26	
		27	37
		28	
	19	29	
10	20	30	



1		3			6
	8	9			12
		15			18
		21			24

Notes on Pupil Page 106

Tasks: *Pupils complete the pattern of jumps along each number line and fill in only those numbers that would occur in the tinted boxes.*

Purpose: To provide pupils with experiences in working with multiples of 2, 3, and 4.

Unifying Ideas: Multiplication and division.

Materials for Teacher: A demonstration size number line (labeled); four empty half-pint milk containers and five empty quart containers; a supply of sand or water sufficient to carry out experiments; colored chalk.

Materials for Pupils: Desk number line (labeled); counters, pencil and paper.

The Lesson:

How many shoes in a pair? . . . in 2 pair? . . . in 3 pair? Keep a record of pupils' statements.

No. of pairs	1	2	3	4	etc.
No. of shoes	2	4	6	8	

Have your pupils act out each of the following situations: How many pupils are there if there are 3 sets of 2 partners? . . . 1 set? . . . 4 sets? This time be sure the list of results is a random one.

No. of sets of partners	3	1	4	2	etc.
No. of pupils	6	2	8		

Erase one of the rows from the random record.

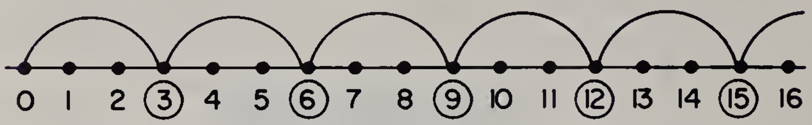
No. of sets of partners					etc.
No. of pupils	6	2	8	4	

If there are 6 pupils, how many pairs of partners? If there are 2 pupils, how many pairs of partners? Etc. Now erase the completed list and replace it with:

No. of sets of partners	3	4	1	2	etc.
No. of pupils					

If there are 3 sets of partners, how many pupils in all? Etc.

Display your model number line. Have your pupils use their desk number lines. Starting at 0 on a number line and taking jumps of 3, what would our landing points be? Have pupils help you complete the picture on your demonstration number line.



Have the class decide on the method of keeping a record of landing points.

Landing points	3	6	9	12	etc.
No. of jumps	1	2	3	4	

Erase the completed record, leaving just the number line record on display. Rearrange in random order the record just erased so that one number of each pair is missing.

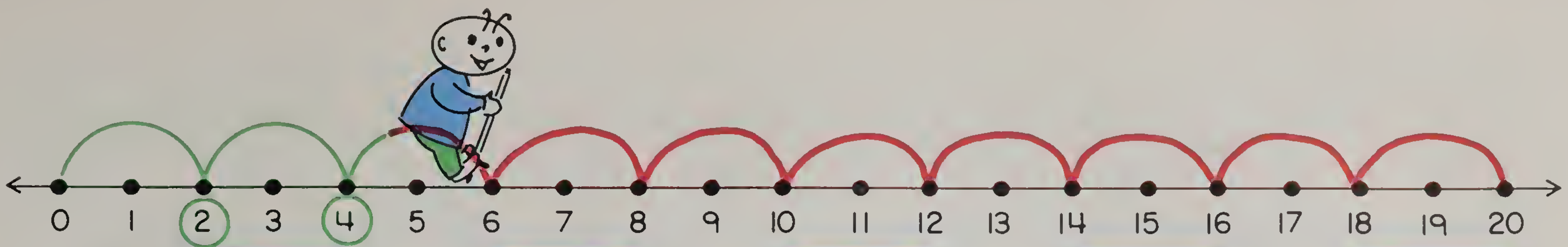
Landing points	6			9	etc.
No. of jumps		1	4		

Invite pupils to complete each pair and to verify their results on the number line.

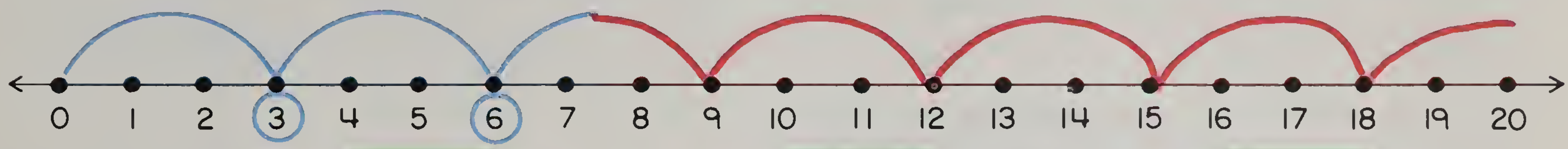
You might involve your pupils in the following simple experiment: How many glasses (half-pint containers) of water or sand do we need to fill a 1 quart container? (8) . . . a 2 quart container? . . . a 3 quart container? Etc. Have the pupils decide on a method of recording their findings.

Headline Story: Complete the pattern:

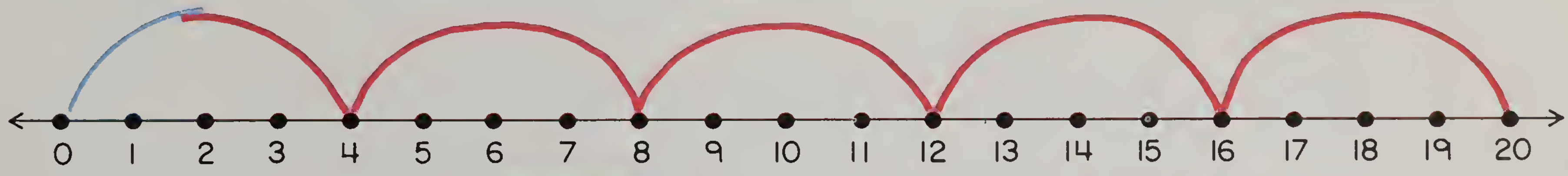
___ 5 9 13 ___ 21 ___ ___


0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20

	2		4		6		8		10
	12		14		16		18		20
	22		24		26		28		30


0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20

		3			6			9	
	12			15			18		
21			24			27			30
		33			36			39	


0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20

			4				8		
	12				16				20
			24				28		
	32				36				40
			44				48		

Notes on Pupil Page 107

Tasks: *Pupils record the number of small square tiles required to cover each block.*

Purpose: (a) To provide pupils with further experiences in addition involving numbers 1 to 15; (b) to anticipate systems of numeration (Base 2 or binary pattern: 1, 2, 4, 8, 16, 32 . . .).

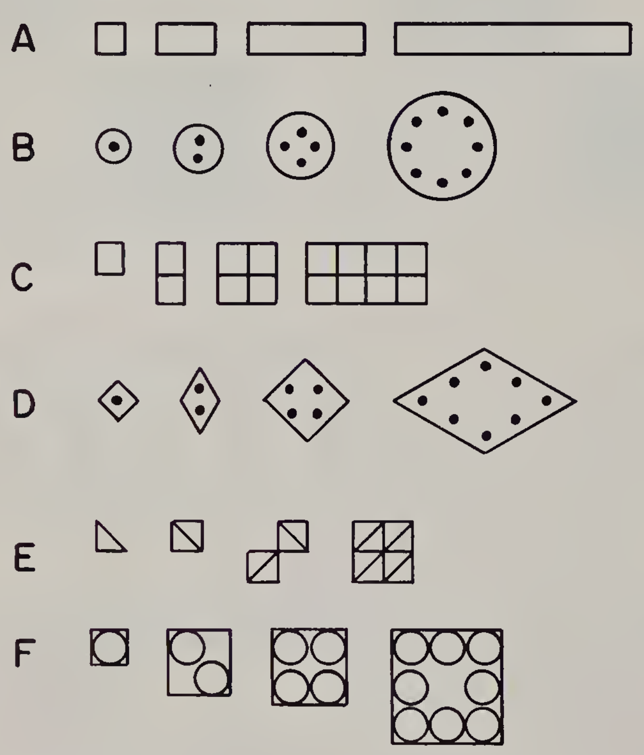
Unifying Ideas: Addition and subtraction; numeration.

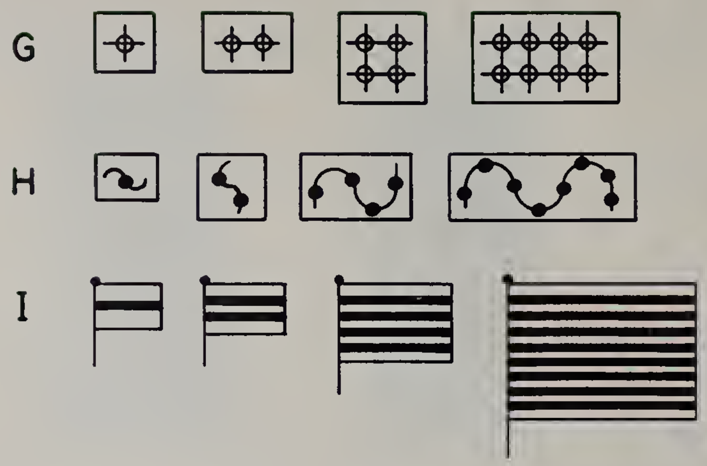
Materials for Teacher: Demonstration size tag board models of the diagrams described in "The Lesson."

Materials for Pupils: Nine envelopes, each containing a set of cards with diagrams as shown in "The Lesson."

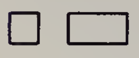
The Lesson:

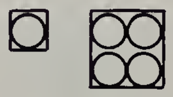
Divide the class into 9 research teams. Each team is given an envelope containing one of the sets of 4 cards shown below:

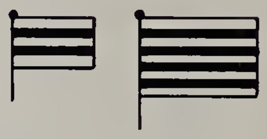


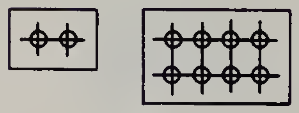


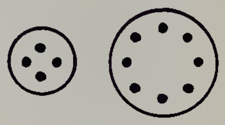
Each team's task is to make models of as many numbers as possible. What numbers do *single* cards suggest? (1, 2, 4, 8) *Pairs* of cards? (3, 5, 6, 9, 10, 12)


$1 + 2 = 3$


$1 + 4 = 5$


$2 + 4 = 6$


$2 + 8 = 10$


$4 + 8 = 12$

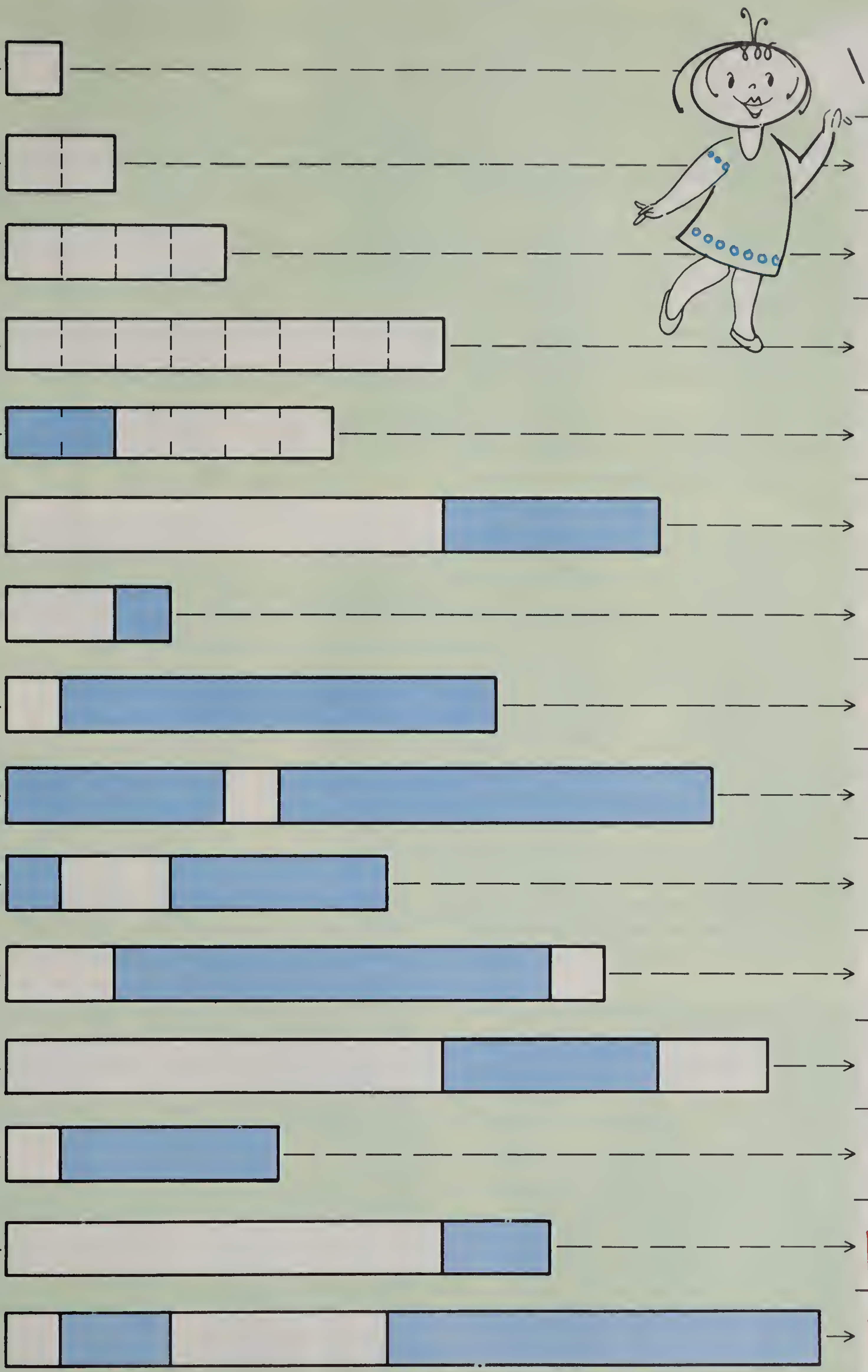
Three cards? (7, 11, 13, 14)

$1 + 2 + 4 = 7$ $\quad$ $1 + 4 + 8 = 13$

$1 + 2 + 8 = 11$ $\quad$ $2 + 4 + 8 = 14$

Have each team use demonstration size models to report its findings to the class. Keep a record on the chalkboard. "What is the same about each team's report?" Problem: "Can any team construct a model of any number other than 1 to 15 using one set of cards?" (No—but someone may see that if the pattern in any set is extended to include a model for 16, they can continue the search.)

Headline Story: Extend the pattern: 1 2 4 8 ___ ___ ___ ___



a 1

b 2

c 4

d 8

e 6

f 12

g 3

h 9

i 13

j 7

k 11

l 14

m 5

n 10

o 15

Notes on Pupil Page 108

Tasks: Pupils list the fact team suggested by each diagram or sketch.

Purpose: To provide pupils with further experience with teen numbers and related fact teams.

Unifying Ideas: Addition and subtraction.

Materials for Teacher: Demonstration size counters for use on a flannel board or cork board; yardstick or pointer; colored chalk.

Materials for Pupils: Counters; pencil and paper.

The Lesson:

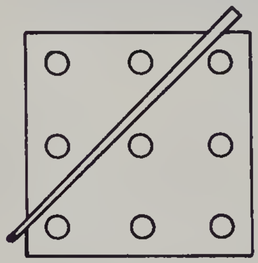
Arrange the diagram below on your flannel board or cork board alongside the list of open number sentences as shown:

3 + 5 = ___

1 + ___ = 12

7 = ___ − 4

9 = 3 + ___

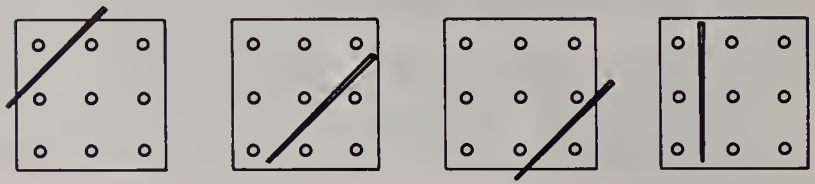


Invite pupils to turn each open number sentence into a true one. Ask the pupils to use their desk counters to duplicate your flannel board display. Which of the complete number sentences is an appropriate title for the arrangement on display? Most pupils would agree that 9 = 3 + 6 would fit their arrangement well. (However, some pupil may present a plausible reason for another selection. If he does, he should not be discouraged, but complimented.)

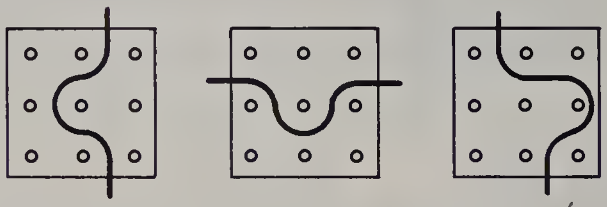
While 9 — 3 = 6 is an appropriate title, note that all members of this fact team are equally fitting:

9 = 3 + 6	6 + 3 = 9	9 − 3 = 6	6 = 9 − 3
9 = 6 + 3	3 + 6 = 9	9 − 6 = 3	3 = 9 − 6

Develop several situations similar to the one above by altering the location of your pointer. Pupils duplicate your arrangement with their pencils and may suggest their own. Some possibilities:



A sketch on the chalkboard would allow:



Each situation offered should be accompanied by several open number sentences, one of which when completed is an appropriate title.

Now present the following situation for pupils to work on at their desks:

3 + 7 = ___

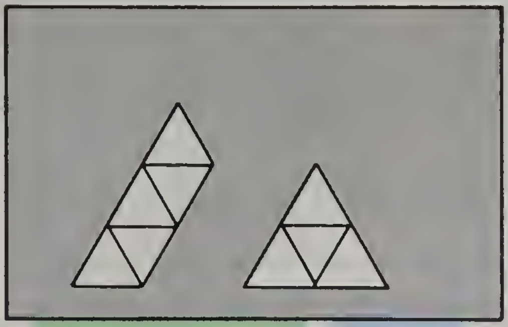
___ + 3 = 10

10 − 3 = ___

___ − 7 = 3

They must (a) complete each number sentence and then (b) construct an appropriate model for the entire fact team. Invite different pupils to draw sketches of their arrangements on the chalkboard. Encourage pupils to provide episodes from real life that contain a similar situation. Encourage pupils to include unusual twists: 10 — 7 = 3 . . . I've got a nickel and two pennies; when I save 3 more pennies, I can exchange all the coins for a dime.

Headline Story: Write the fact team suggested by: Eight cents change from a quarter.

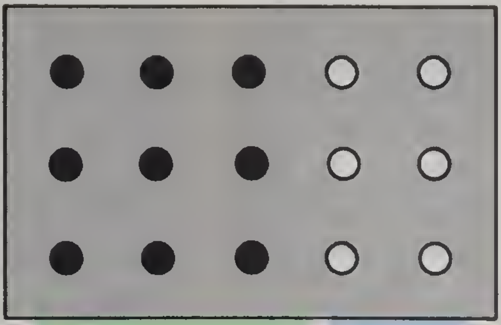


$5 + 4 = 9$

$4 + 5 = 9$

$9 - 5 = 4$

$9 - 4 = 5$

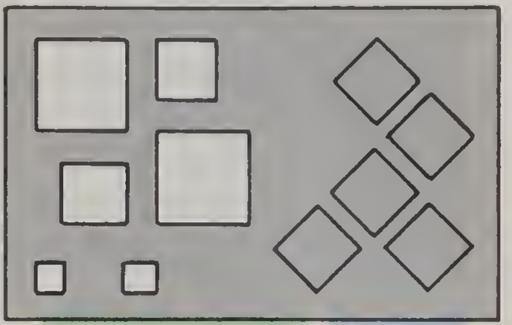


$9 + 6 = 15$

$6 + 9 = 15$

$15 - 6 = 9$

$15 - 9 = 6$

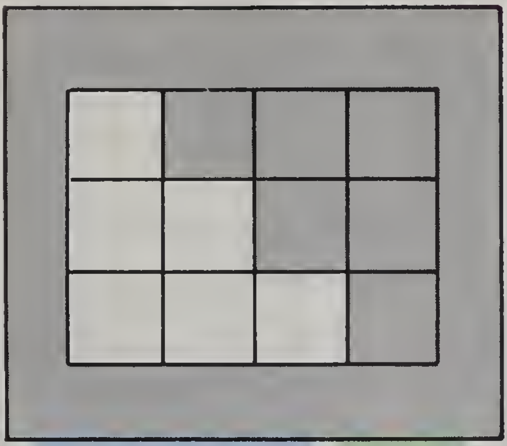


$6 + 5 = 11$

$5 + 6 = 11$

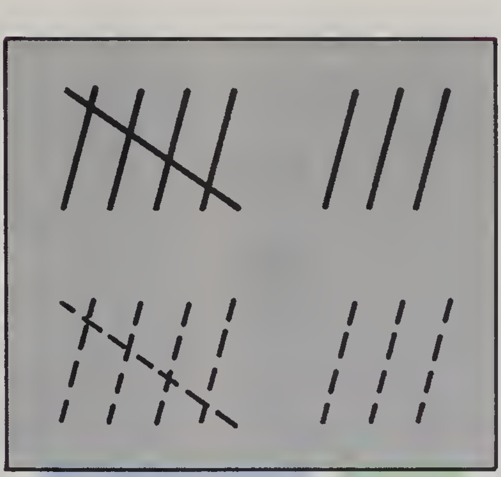
$11 - 6 = 5$

$11 - 5 = 6$



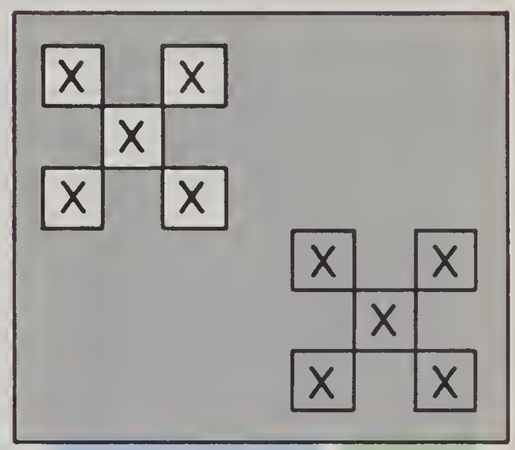
$6 + 6 = 12$

$12 - 6 = 6$



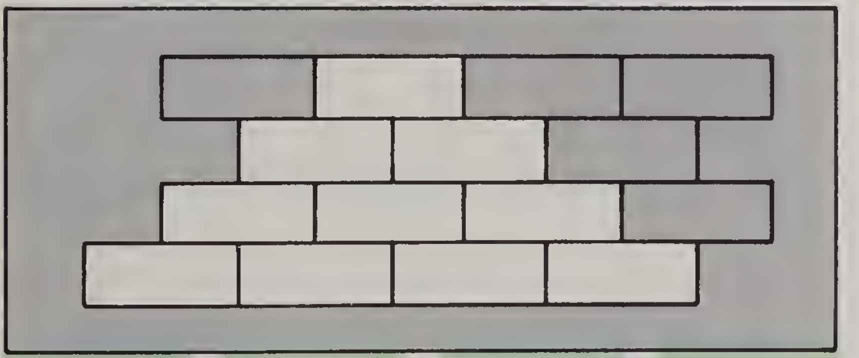
$8 + 8 = 16$

$16 - 8 = 8$

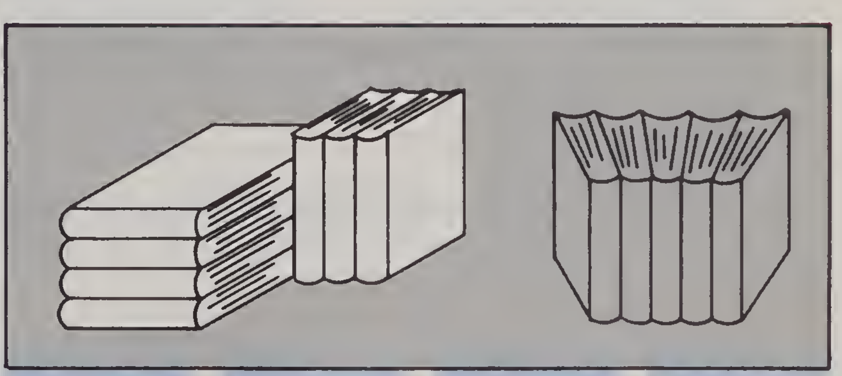


$5 + 5 = 10$

$10 - 5 = 5$



5	10	15	15
+ 10	+ 5	− 10	− 5
15	15	5	10



7	5	12	12
+ 5	+ 7	− 5	− 7
12	12	7	5

Notes on Pupil Page 109

Tasks: Pupils fill in the blanks to indicate the various methods of regrouping the items displayed in each sketch.

Purpose: To involve pupils in studying basic regrouping actions that they can use to evolve the usual addition algorism.

Unifying Ideas: Addition and subtraction.

Materials for Teacher: Four containers; 17 demonstration size counters or, if possible, 8 blocks and 9 balls; colored chalk.

Materials for Pupils: A similar supply of desk counters for each group of 5 pupils.

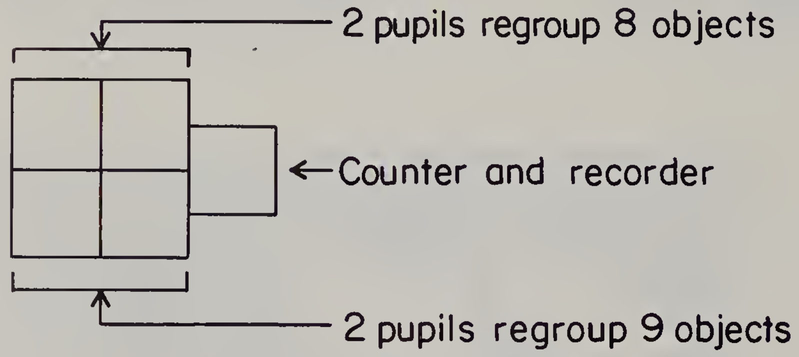
The Lesson:

Arrange the class into groups as described below. Each group receives a supply of 17 counters and paper and pencil for record keeping purposes.

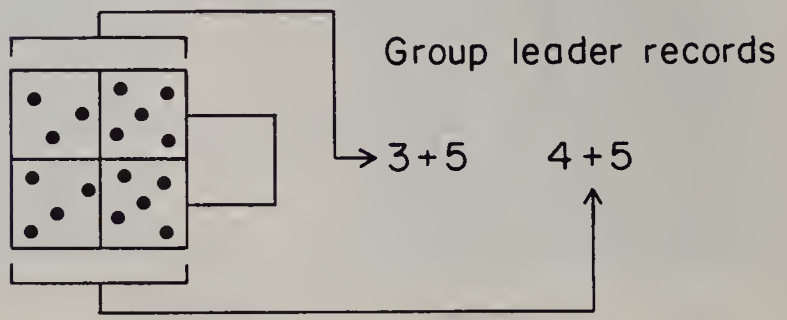
How many objects in all would we have if we combine 8 blocks with 9 balls? One way to find out is to count; another is to recall that 8 + 9 = 17; another is to discover efficient regrouping procedures for processing the given information.

Consider the following two situations. In how many ways can 8 objects be distributed between two boxes? In how many ways can 9 objects be distributed between two boxes?

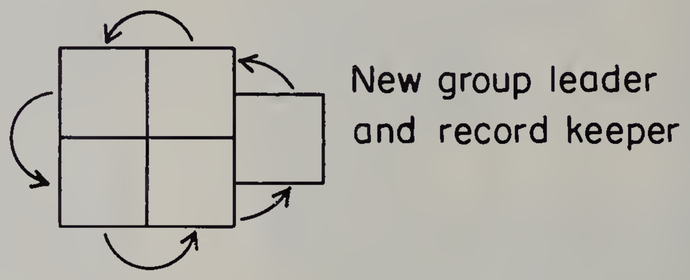
Each group of 5 pupils is organized into teams with the following arrangement of desks:



Each group leader assigns one pair of group members 8 counters and the other pair 9 counters. When the group leader gives the signal, each group constructs an arrangement of their counters on their desks. Each group then describes its arrangement to the group leader, who records the information dictated.



You might want to have each member of a group shift to the desk on his right before a new arrangement and record are made.

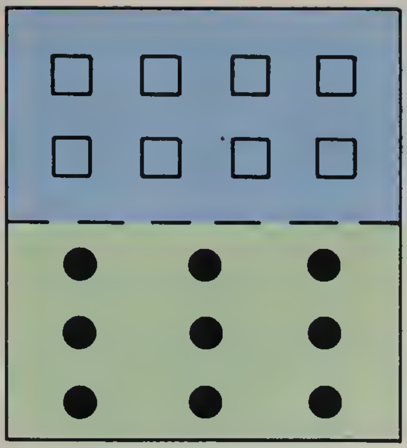
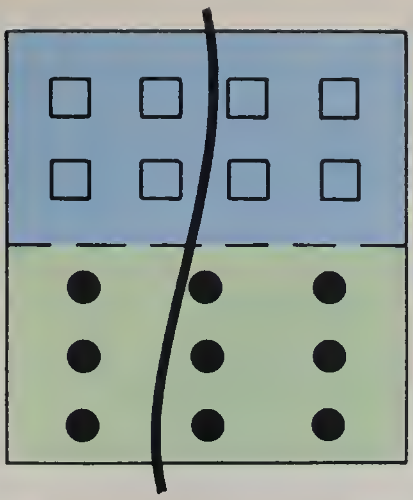
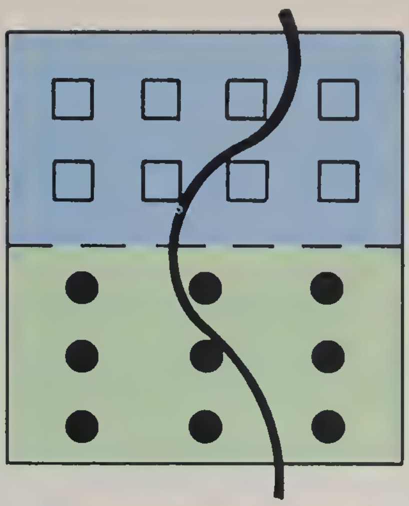
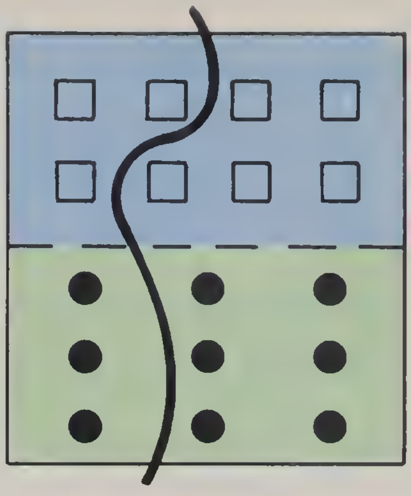


In this way every pupil will have had a chance to be a group leader and a member of the 8 team and the 9 team.

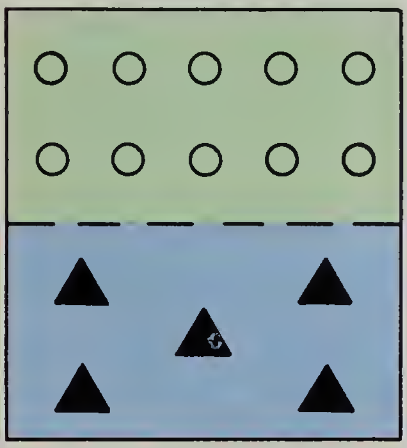
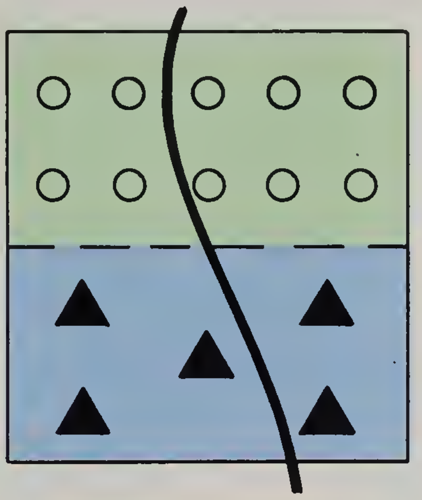
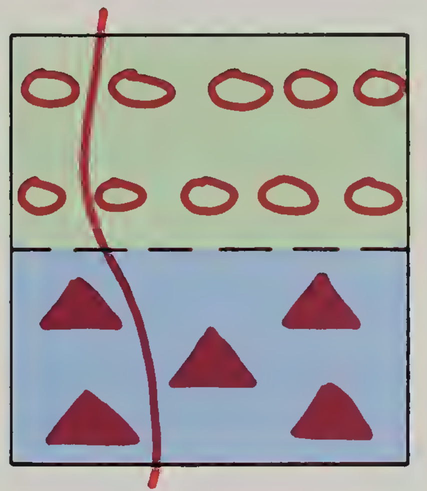
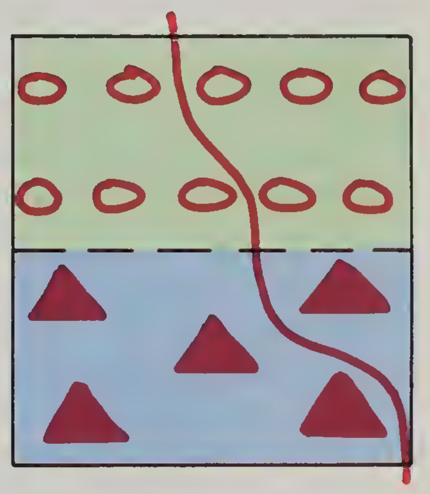
Can we combine the 8 blocks and 9 balls in two boxes so there would be 10 objects in one box? We might record some suggestions as follows:

	Box A	Box B	A	B	A	B
Blocks	4	4	7	1	7	1
Balls	3	6	3	6	0	9
	7	10	10	7	7	10

Headline Story: Tell a story that fits: □ + △ = 15 8 + ○ = ⬡

8	4 + 4	5 + 3	3 + 5
+ 9	+ 3 + 6	+ 5 + 4	+ 3 + 6
17	7 + 10	10 + 7	6 + 11

10	4 + 6	2 + 8	5 + 5
+ 5	+ 3 + 2	+ 2 + 3	+ 4 + 1
15	7 + 8	4 + 11	9 + 6

7	4 + 3	2 + 5	7 + 0
+ 6	+ 4 + 2	+ 1 + 5	+ 3 + 3
13	8 + 5	3 + 10	10 + 3

Tasks: *Pupils examine each Cross Number Puzzle carefully and fill in the missing numbers for each puzzle. They use each completed puzzle to complete the number sentence below it.*

Purpose: (a) To provide pupils with further experiences with sums and differences of the numbers 0-8; (b) to present subtraction of a number as the inverse of addition of that number; (c) to provide pupils with opportunities to discover the commutative property of addition.

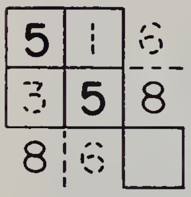
Unifying Ideas: Addition and subtraction; structure.

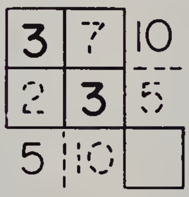
Materials for Teacher: Several packs of demonstration size number tags (0-20) for use with a demonstration size Cross Number Puzzle; colored chalk.

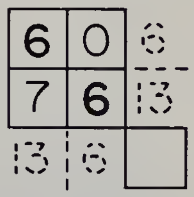
Materials for Pupils: For those who need them—counters and a blank copy of a Cross Number Puzzle suitable for desk work.

The Lesson:

Display several examples of Cross Number Puzzles in which the same number is repeated on the down-to-the-right diagonals:







Ask pupils to reconstruct each example.

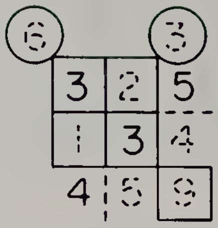
In each case we find instances of the fact that it doesn't make any difference in which order we add:

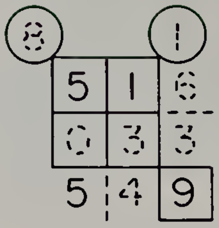
$$8 + 6 = 6 + 8 = 14$$

$$5 + 10 = 10 + 5 = 15$$

$$13 + 6 = 6 + 13 = 19$$

Display the following reconstruction situations:

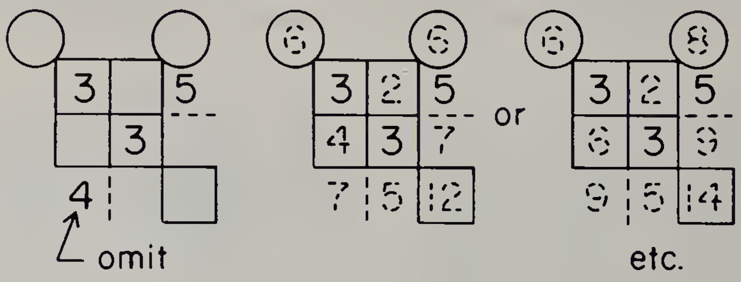




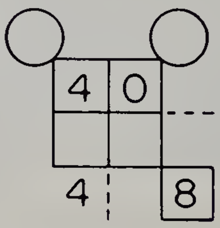
Ask pupils to reconstruct the original Cross Number Puzzle for which these are incomplete records.

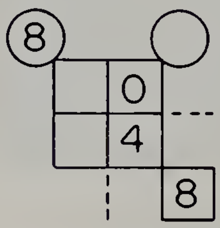
Given the same clues, can they construct a different Cross Number Puzzle? No, there is only one way to reconstruct each.

Are all the clues required in each of the examples? If 4 were omitted in the first situation below, several reconstructions are possible:



Now display the following situations:





Note the new feature in the Cross Number Puzzles on the pupil's page. The "ears" on each puzzle are used to record the sums of the numbers in the diagonals. Let pupils discover that the sum of the diagonal sums is the same as the sum of the row sums (or the sum of the column sums). This is another example of the *any order—any grouping* principle of addition.

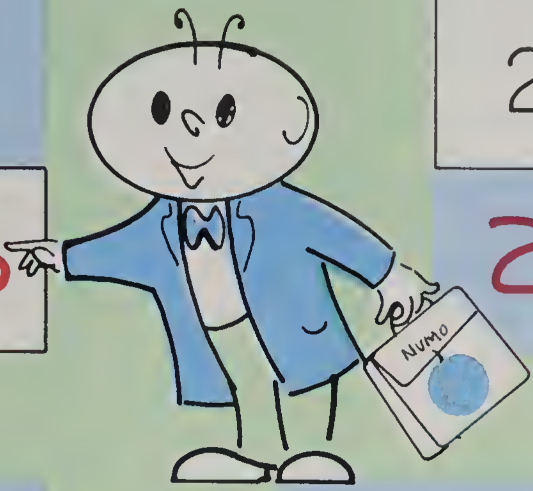
$$(1+3)+(0+1)=(1+0)+(3+1)=(1+1)+(3+0)$$

sum of row sums	sum of column sums	sum of diagonal sums

[illegible] Tell a story that fits: □ - △ = △ + □

2			3
	1	3	4
	0	1	1
	1	4	5

1 + 4 = 4 + 1 = 5



0			6
	0	4	4
	2	0	2
	2	4	6

2 + 4 = 4 + 2 = 6

4			3
	2	2	4
	1	2	3
	3	4	7

3 + 4 = 4 + 3 = 7

8			0
	4	0	4
	0	4	4
	4	4	8

4 + 4 = 8

Notes on Pupil Page 111

Tasks: *Pupils draw lines with a straight-edge, connecting points as indicated.*

Purpose: To review the use of the straight-edge while studying relationships among triangles.

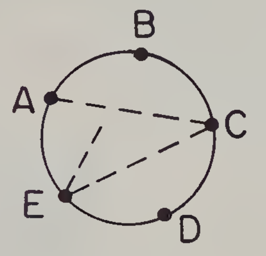
Unifying Ideas: Geometry.

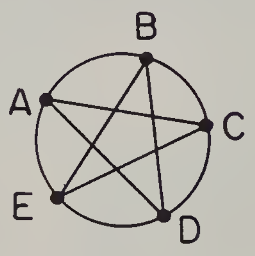
Materials for Teacher: Four demonstration size tag board models of the smaller triangles needed to construct the larger triangles located in the upper left-hand corner, middle right-hand section, and lower left-hand corner of the pupil's page; colored chalk and yardstick.

Materials for Pupils: A supply of your models appropriate in size for pupil teams to work with.

The Lesson:

Put five points on the board equally spaced around an imagined circular path. Letter the points consecutively in either direction A, B, C, D, E. Invite different pupils to come to the board to draw the lines suggested by $\overline{AC}$, $\overline{CE}$, $\overline{EB}$, $\overline{BD}$, and $\overline{DA}$; then $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$ and $\overline{EA}$.





Ask the class to anticipate the results before the lines are all drawn. Encourage discussion of the figure formed by $\overline{AC}$, $\overline{CE}$, and $\overline{EA}$. Ask the class to identify some of the other triangles formed; e.g., $\overline{EB}$, $\overline{BD}$, $\overline{DE}$.

Pass out four triangles for each pupil team to use to construct a model like the first diagram on the pupil's page. Ask pupils to use all four triangles to construct a large triangle. After teams have completed the task, invite them to construct a large model using demonstration materials. Proceed in a similar fashion for the other two sets of triangles on the pupil page.

Now have the class turn to page 111 in their MATH WORKSHOP books. After your pupils understand the way directions are indicated, have them complete $\overline{AB}$, $\overline{BC}$, $\overline{CA}$ in the first diagram. What figure is it? Are its sides the same length?

Have the class guess what will happen in the next diagram. How many small triangles will be produced? What do you notice about lines $\overline{HJ}$, $\overline{IK}$, $\overline{GL}$? (They cross at the same point.) Are the small triangles all the same size? Now have the class complete each of the other diagrams.

Notice that in each of the three pairs of large triangles:

(a) both large triangles in each pair are the same size;

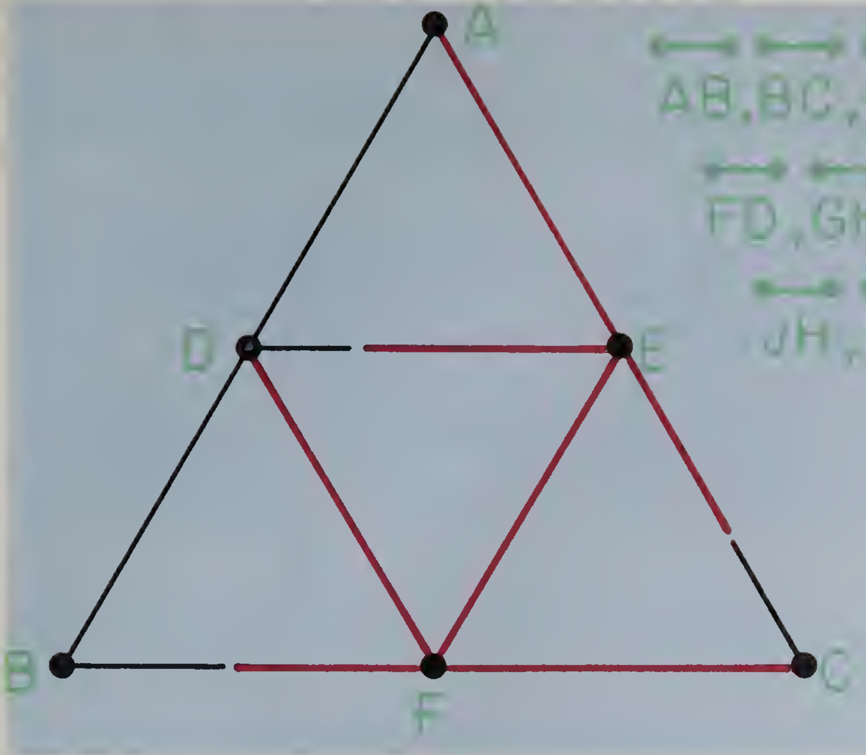
(b) one of each pair contains four small triangles of the same size;

(c) in one example in each pair, the lines drawn inside the large triangle pass through the same point;

(d) in each example that is divided into four small triangles of the same size, there are three sets of parallel lines.

Be sure to review the accompanying booklet, *Games and Enrichment Activities*, for suggestions you may wish to incorporate in your daily lesson plan.

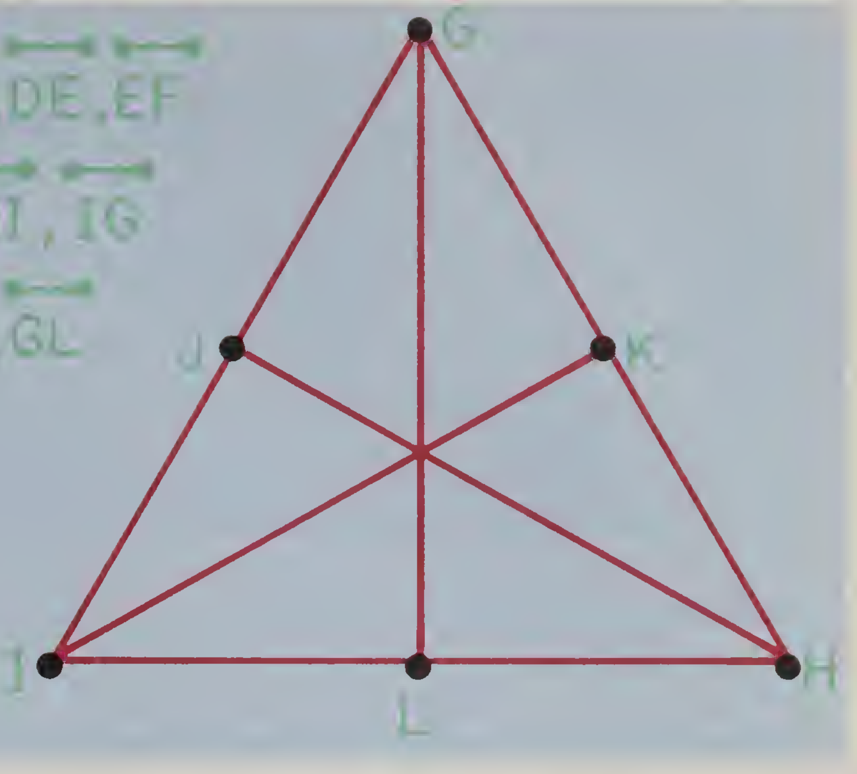
Headline Story: Find triangular shapes in the room.

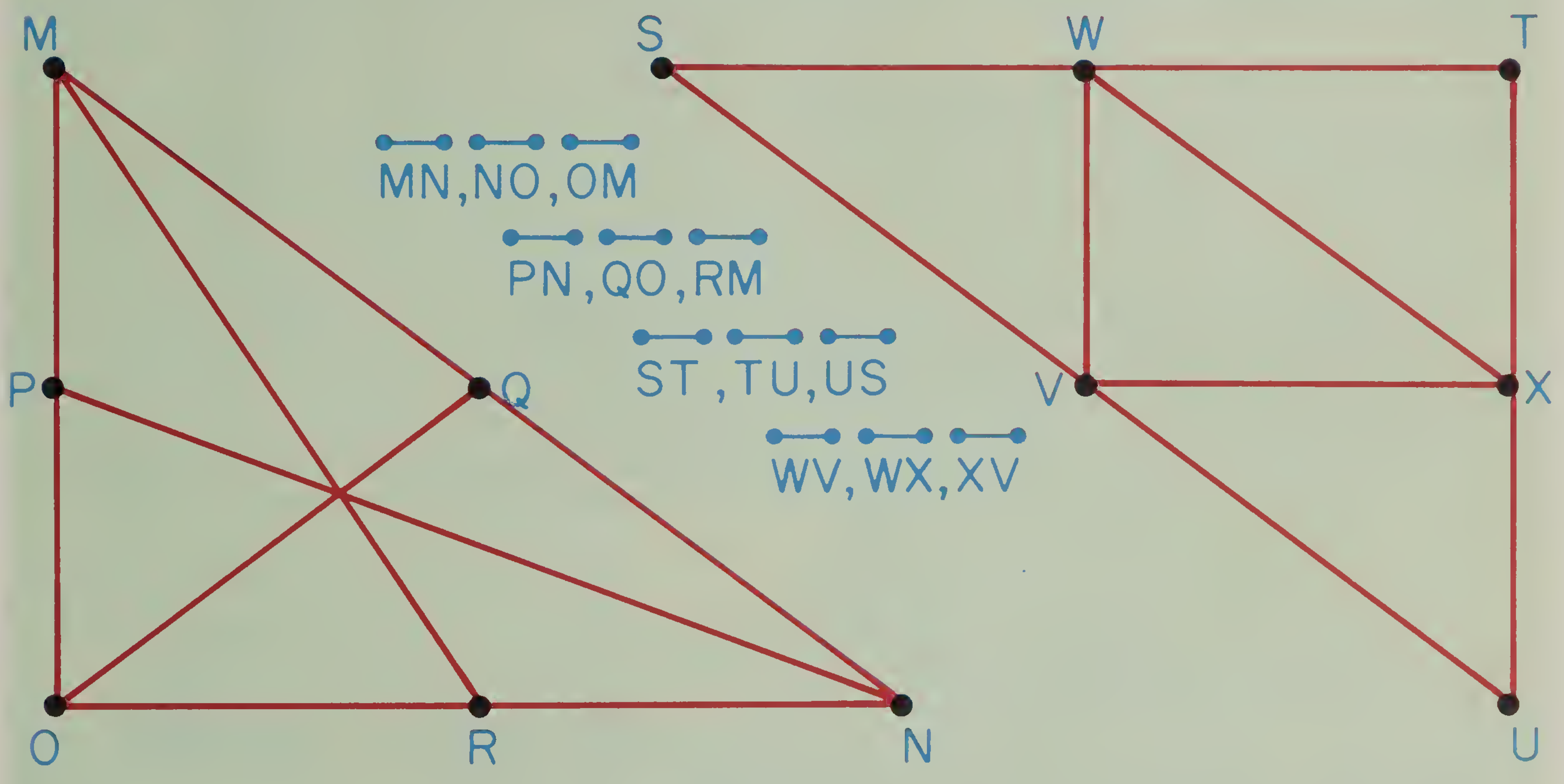


$\overline{AB}, \overline{BC}, \overline{CA}, \overline{DE}, \overline{EF}$

$\overline{FD}, \overline{GH}, \overline{HI}, \overline{IG}$

$\overline{JH}, \overline{KI}, \overline{GL}$



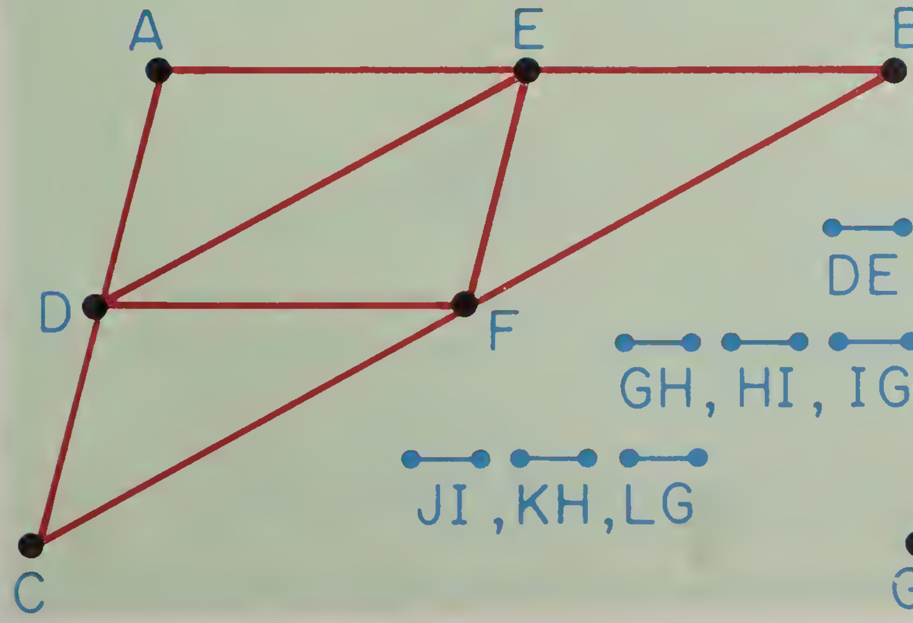


$\overline{MN}, \overline{NO}, \overline{OM}$

$\overline{PN}, \overline{QO}, \overline{RM}$

$\overline{ST}, \overline{TU}, \overline{US}$

$\overline{WV}, \overline{WX}, \overline{XV}$

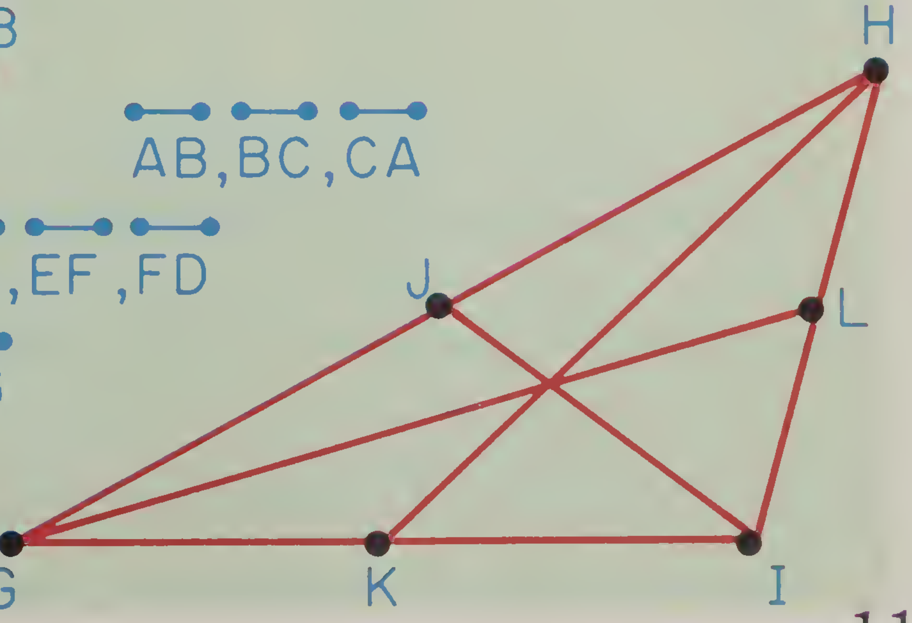


$\overline{AB}, \overline{BC}, \overline{CA}$

$\overline{DE}, \overline{EF}, \overline{FD}$

$\overline{GH}, \overline{HI}, \overline{IG}$

$\overline{JI}, \overline{KH}, \overline{LG}$



Tasks: *Pupils fill in blanks so that each number or number sentence is an appropriate title for the accompanying sketch.*

Purpose: (a) To provide pupils with opportunities to practice addition, subtraction, and regrouping, and (b) to work with fact teams via triangular numbers (1, 3, 6, 10, 15, 21, 28, 36 . . .).

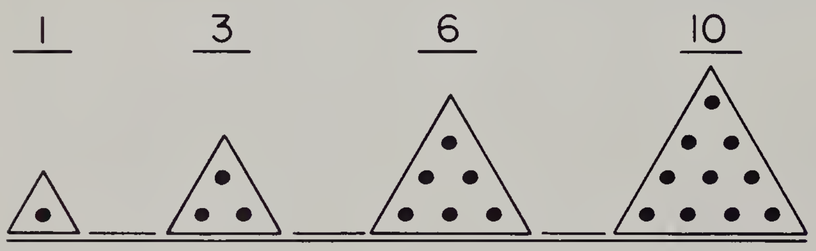
Unifying Ideas: Addition and subtraction.

Materials for Teacher: A demonstration set of the four triangular number cards as shown in "The Lesson"; colored chalk.

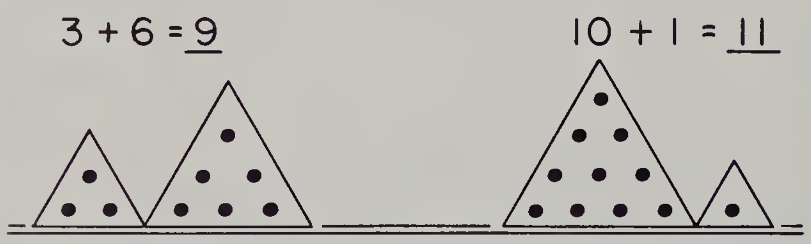
Materials for Pupils: Each pupil should have his own set of the four triangular number cards for individual work at his desk.

The Lesson:

Have pupils examine and discuss their sets of triangular numbers. Display your set on the chalk-rail. Note the number of dots on the board above the appropriate card.

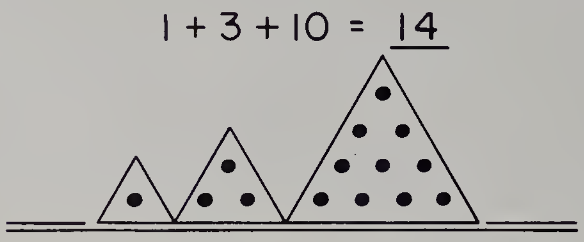


What other number can be represented by pairs of these cards?



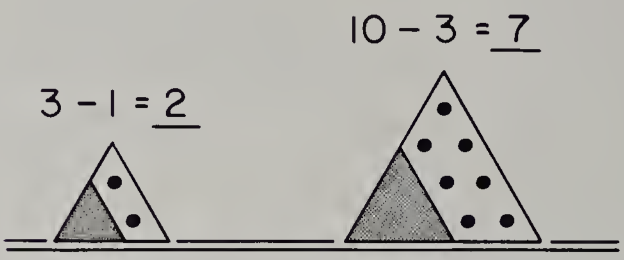
Reverse the situation by writing several sentences on the board for the class to construct models. Some sentences might be: 1 + 3 = 4, 1 + 6 = 7, 3 + 10 = 13, and 6 + 10 = 16.

What numbers can be represented by using three cards at a time?



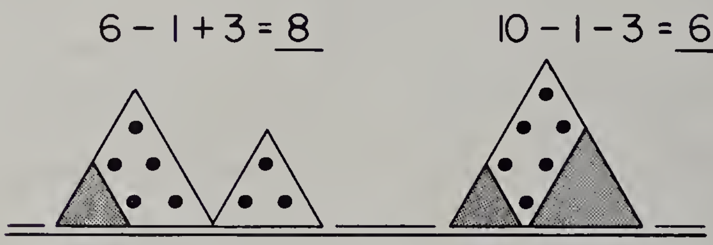
Also, 1 + 3 + 6 = 10, 3 + 6 + 10 = 19, etc.

If a card can be placed face down to cover part of a larger card, how many numbers can be represented by a pair of cards?

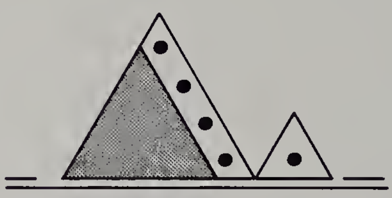


Also, 6 − 3 = 3, 10 − 6 = 4, 6 − 1 = 5, 10 − 1 = 9.

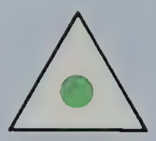
Using 3 cards and covering parts of them, how many numbers can be represented?



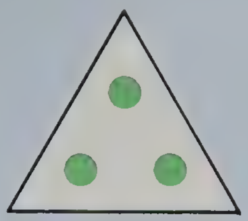
Can we find representations for 5 with 2 cards? (Yes, 10 − 6 + 1 = 5.)



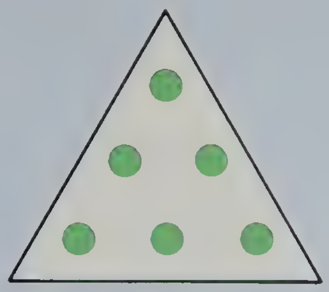
Headline Story: What kind of cards would extend the series shown at the top of the pupil page?



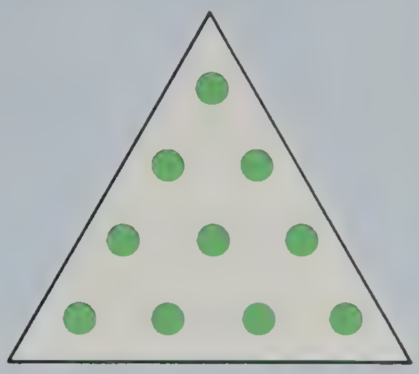
A. 1



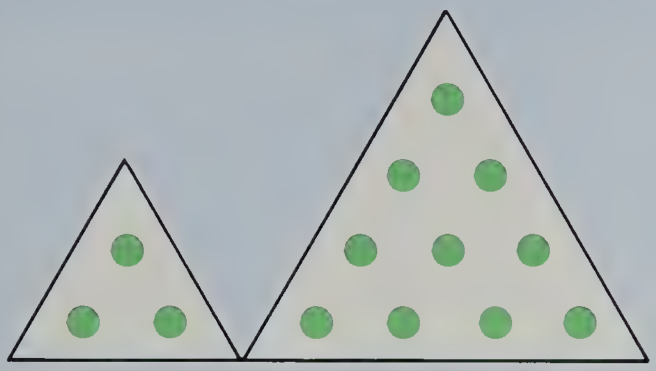
B. 3



C. 6



D. 10

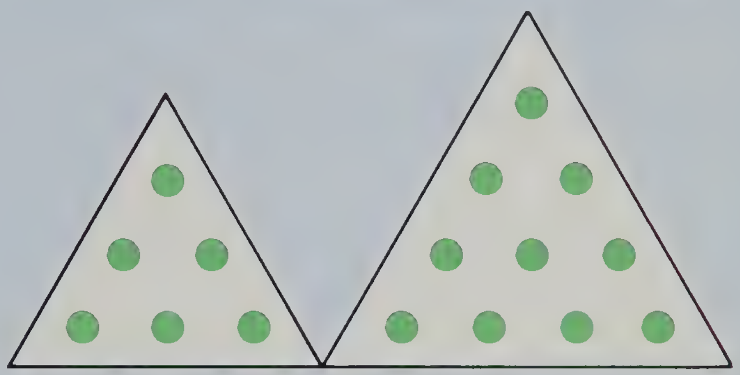


E. 3 + 10 = 13

10 + 3 = 13

13 − 3 = 10

13 − 10 = 3

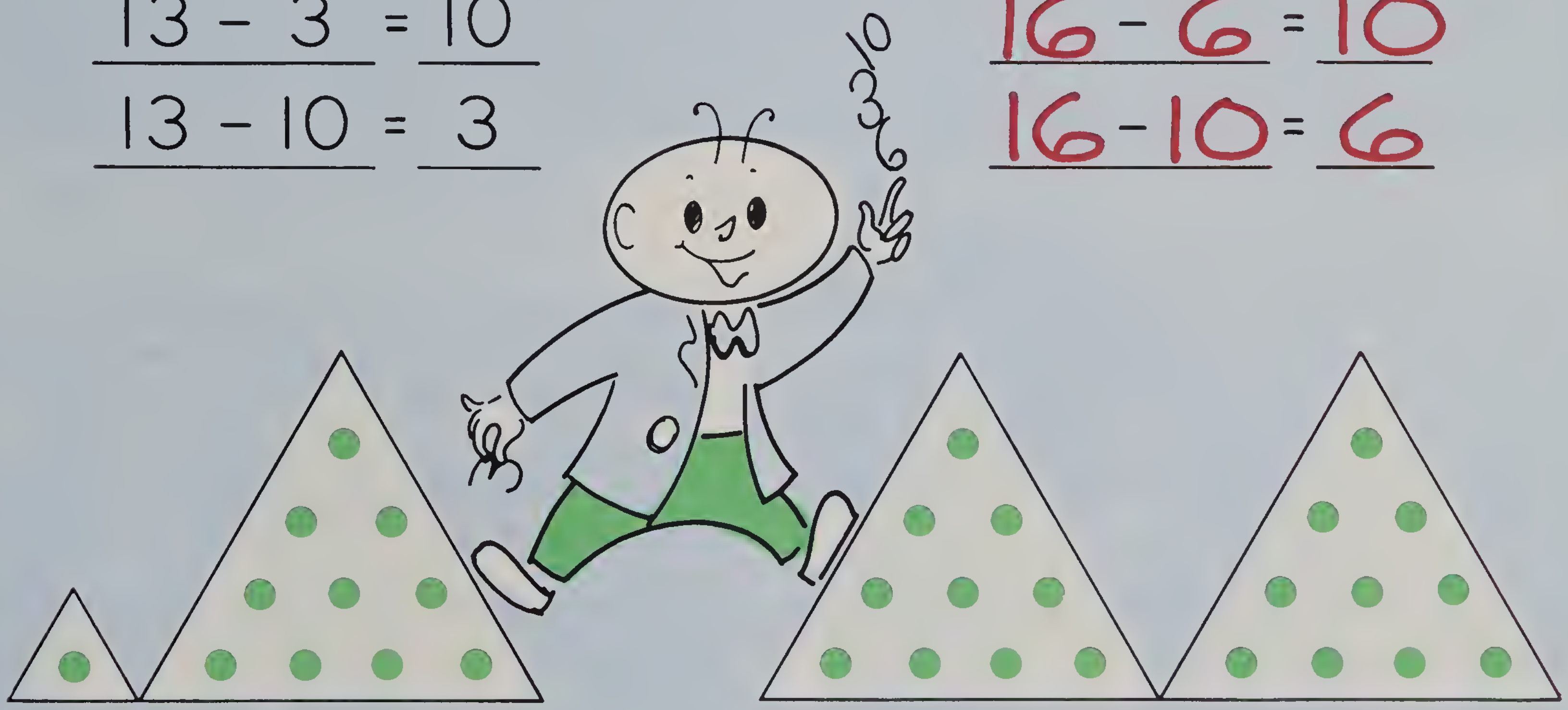


F. 6 + 10 = 16

10 + 6 = 16

16 − 6 = 10

16 − 10 = 6



G. 1 + 10 = 11

11 − 1 = 10

10 + 1 = 11

11 − 10 = 1

H. 10 + 10 = 20

20 − 10 = 10

— + — = —

— − — = —

Notes on Pupil Page 113

Tasks: *Pupils discover each of the three rules suggested by the three patterns. They then use each rule to decide on the numbers to write in the remaining blanks in each list. They draw jumps on the number lines.*

Purpose: (a) To provide pupils with opportunities for adding by 3's and 5's and subtracting by 3's; (b) to anticipate multiplication.

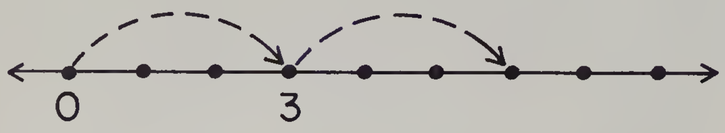
Unifying Ideas: Addition and subtraction; multiplication and division; functions and relations.

Materials for Teacher: A large, unlabeled demonstration number line or colored chalk and straightedge to sketch a number line.

Materials for Pupils: Desk number lines (unlabeled).

The Lesson:

Label one point on your number line as 0 and another point as 3.



Ask pupils to label all landing points if they start at 0 and take jumps of 3. Have them label their desk number lines accordingly.

Start at any labeled point other than 0 (unless you want to label the line to the left of 0). Take a jump of three in either direction. Will you always land on a labeled point?

Have pupils come to the board to demonstrate several situations listing jumps of 3 to the right and left in separate tables:

Start ↷	0	3	12	
Land				9

Land	3	15		
Start ↶			9	21

The rest of the class work at their desks verifying or challenging each entry recorded by a demonstrator.

Now start at an unlabeled point greater than 2 (unless points to the left of 0 are labeled) and have pupils use their number lines to take a jump of 3 in either direction. What do they notice? Do they land on an unlabeled point?

Have them start at an unlabeled point and take as many jumps of 3 to the right as they like. Has anyone landed at a labeled point? No.

Now have them start at a labeled point and take a jump of 4 to the right. Do they land at a labeled point? Make a record of some of their landing points.

Start	0	3	9	15	6	
Land	4	7	13			

etc.

Pupils may now start at any labeled point and take a jump of 5 to the right. Has anyone landed on a labeled point? Make a record of some of their jumps. Pupils can then check the record.

Start	0	6	3	9	
Land	5	11	8	14	

etc.

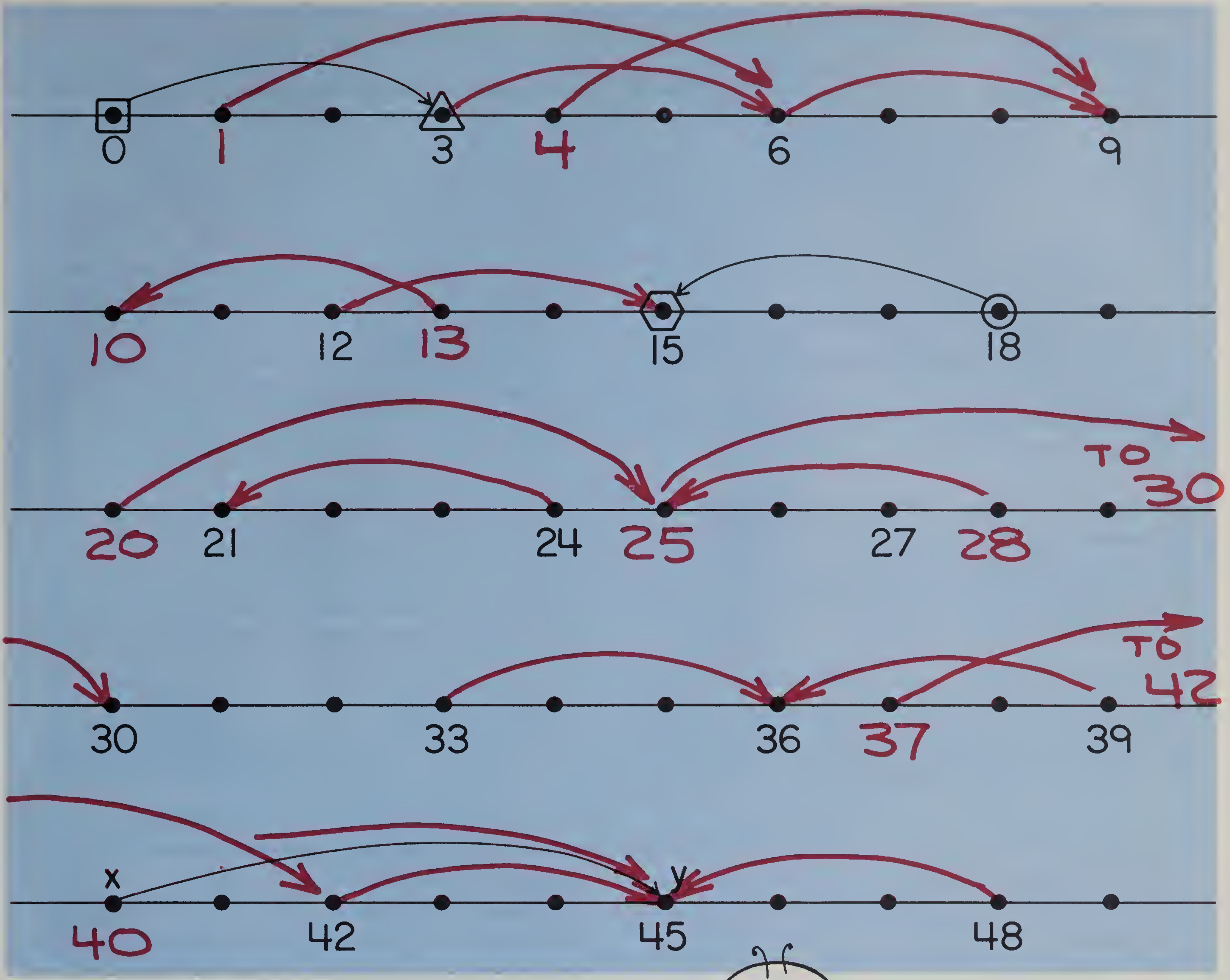
Have them start at an unlabeled point and take a jump of 5 to the right. Do you land on labeled or unlabeled points?

Start	1	2	4	5	7	
Land	6	7	9	10	12	

etc.

Points 6, 9, and 12 are labeled while 7 and 10 are unlabeled.

Headline Story: Harry had 5 more marbles than Helen. (See above.)


0
1
3
4
6
9
10
12
13
15
18
TO 30
20
21
24
25
27
28
TO 42
30
33
36
37
39
x
y
40
42
45
48

□	→	△	○	→	⬡	x	→	y
0	→	3	18	→	15	40	→	45
3	→	6	24	→	21	1	→	6
6	→	9	13	→	10	4	→	9
12	→	15	28	→	25	20	→	25
33	→	36	39	→	36	37	→	42
42	→	45	48	→	45	25	→	30

Notes on Pupil Page 114

Tasks: *Pupils fill in the blank next to each rectangle with the appropriate number in accordance with the agreement that the small square unit represents the number 4.*

Purpose: To anticipate multiplication by practice in adding and regrouping multiples of 4.

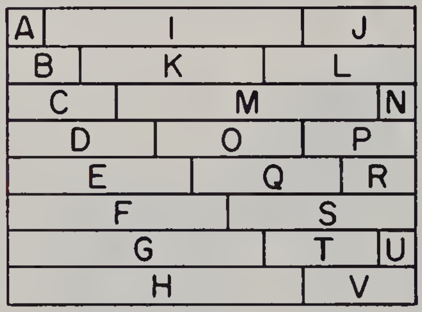
Unifying Ideas: Addition and subtraction; multiplication and division.

Materials for Teacher: Demonstration-size colored rectangular strips for mounting on a flannel board or cork board such that they vary in length from one to eight units (see Diagram 1 of "The Lesson") or colored chalk and a yardstick.

Materials for Pupils: Cuisenaire® Rods or a sufficient number and variety of rectangular strips similar to your demonstration set but appropriate for use by each pupil at his desk.

The Lesson:

Use your demonstration set of rectangular strips to construct the pattern sketched below or sketch it accurately.



If tiles the size of A cost 4¢ each, what would it cost to cover A? . . . B? . . . C? . . . F? . . . G? . . . H? . . .

Then consider J, L, N, P, R, U, and V.

Then consider I, K, M, O, Q, S, and T.

Whenever there is any question about the number of tiles, have pupils come to the board and use blocks of cardboard cut to unit size to determine the number used. Ask the class to decide on a way of organizing their information about the costs of different sized tiles. A chart similar to the following is most likely to result from their discussions:

No. of tiles	1	2	3	4
Costs	4¢	8¢	12¢	16¢

The class might also decide to make a chart recording the various sizes of the labeled areas:

No. of tiles	1	2	3
Sections covered	A,U,N	B,R	C,J,P,T,V

Ask again for the cost of covering sections picked at random. Pupils can use the diagram as before or can find the result from their charts.

Remove the pattern of rectangular strips from the board but leave the charts on the board. Ask questions similar to those previously asked—pupils must now rely solely on their charts for information.

Erase the first chart, leaving only the record of the size of the areas or the number of tiles required. How much would it cost to cover the area that was labeled K in the diagram we removed?

Have your pupils work in groups of two or three or four. Each member designs his own pattern using the Cuisenaire® Rods or rectangular strips. However, he must know how much each line in his pattern costs. He now asks each member of his group to tell the cost of each line in his pattern.

Headline Story: Tickets are 5¢ each. How much for 3 tickets? . . . 2 tickets? . . . etc.

r

Color
4 A
8 B
12 C
16 D
20 E

s

12 C
16 D
Color
4 A
20 E
8 B

t

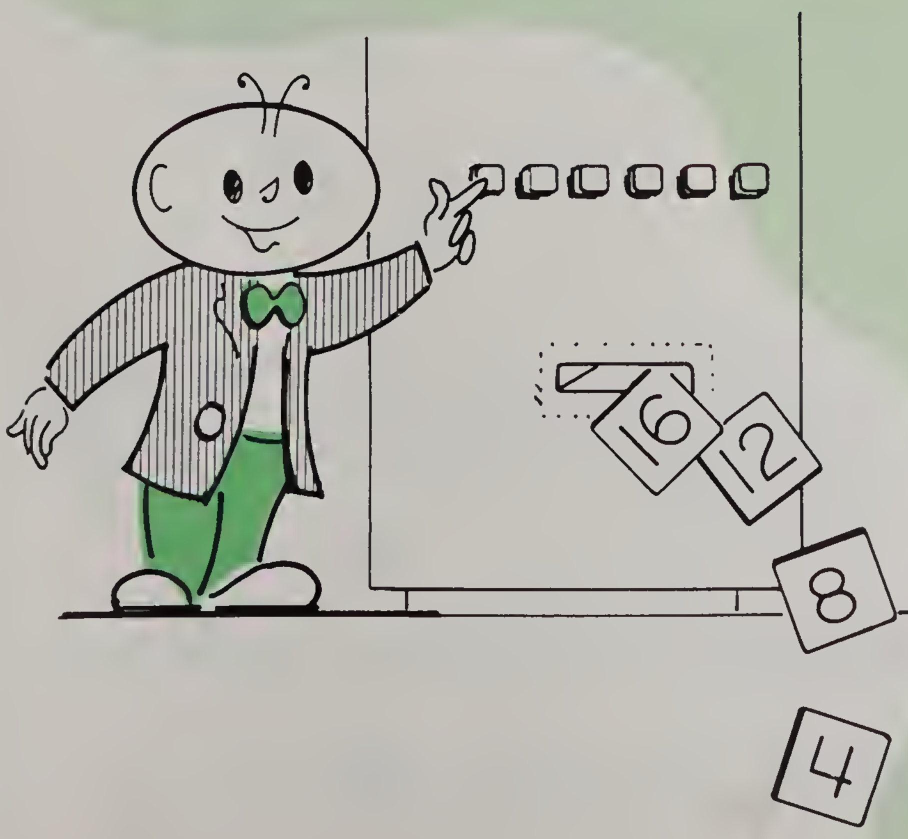
8 | 4 | 4
8 | 8

u

16 | 8 | 8
16 | 16

v

12 | 8 | 4
12 | 12



w

20 | 20
20 | 16 | 4

x

20 | 12 | 8
20 | 20

Notes on Pupil Page 115

Tasks: *Pupils complete each number sentence, making each a true statement.*

Purpose: To review and extend the ideas of the symbols for the relations, "is greater than" and "is less than."

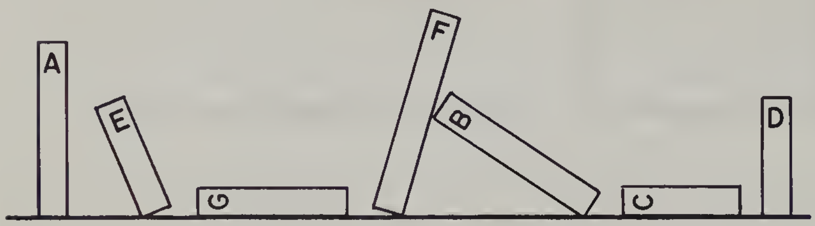
Unifying Ideas: Structure; functions and relations.

Materials for Teacher: Several strips of cardboard of varying lengths, labeled to facilitate reference.

Materials for Pupils: A supply of objects such as counters.

The Lesson:

Display several labeled strips, arranging them in random order and at several different angles.



"Is the length of A greater than the length of G, or is the length of A less than the length of G? How can we be sure?" The pupils should consider at least 3 ways: (a) move A and G together and notice which is longer; (b) select a unit such as a block, then count the number of blocks it takes to represent the length of A and then of G; (c) stretch a string along A and note the length, move to G, and compare the string length of A with G.

Introduce a shorthand to record the 3 possible results:

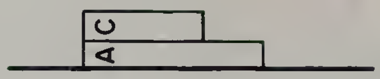
$A > G$ . . . (A is greater than G)

$A < G$ . . . (A is less than G)

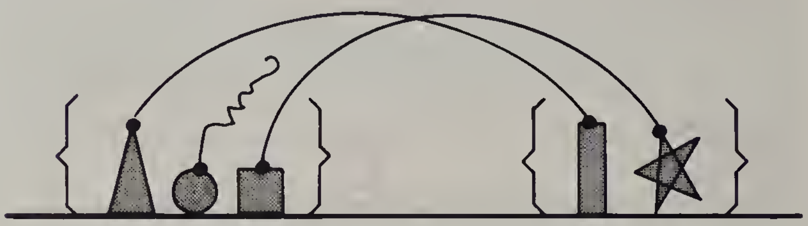
$A = G$ . . . (A equals G. . . . they are the same length)

Compare other pairs of strips and record results.

$A > C$ $C < A$



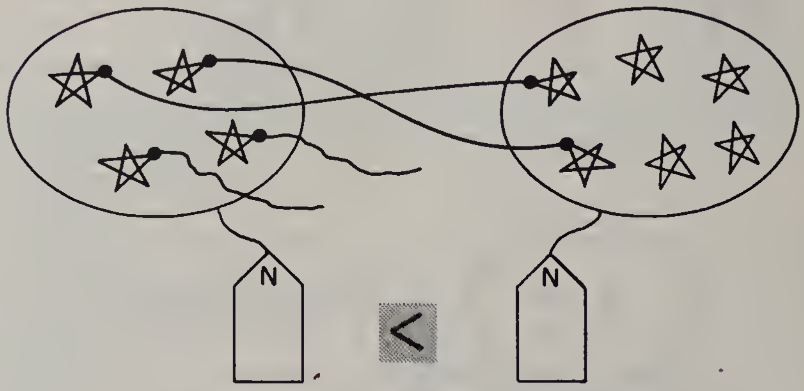
"When we compare the number of objects in 2 groups, how can we be sure which group has the greater number? Could we be sure without counting the number of objects in each group?" (Yes, by trying to establish a 1-to-1 correspondence until the matching has accounted for all the objects in one of the groups.)



If we call the group on the left L and the group on the right R, and we are only interested in the one which has the greater number of things, we can tell from the diagram that:

$\underline{L > R}$ and that $\underline{R < L}$

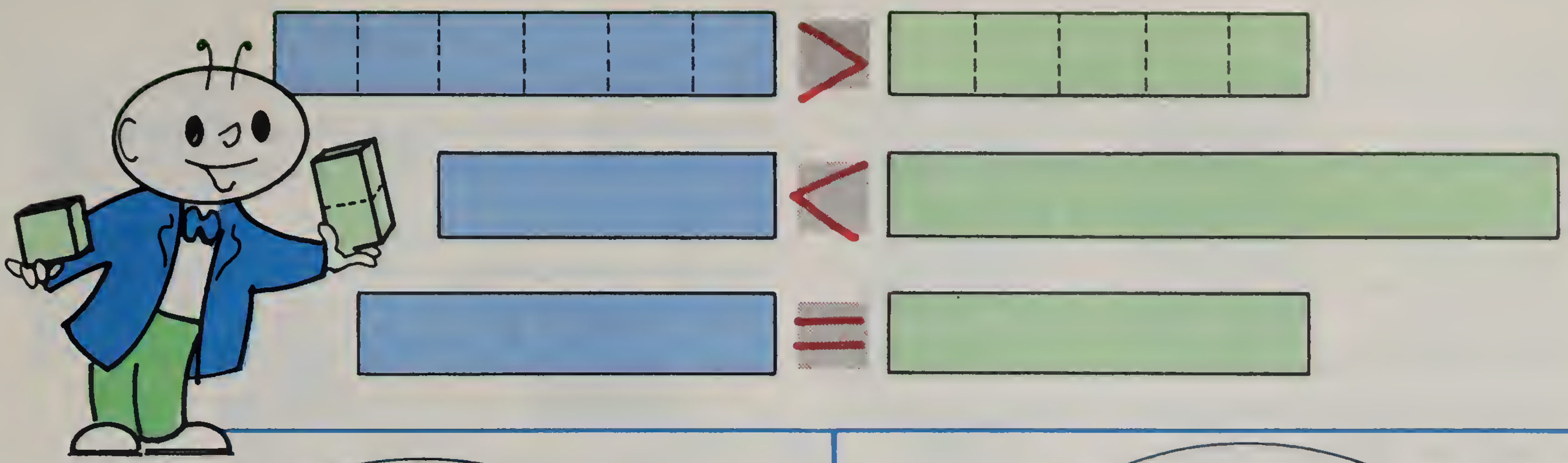
In which groups are there more stars?

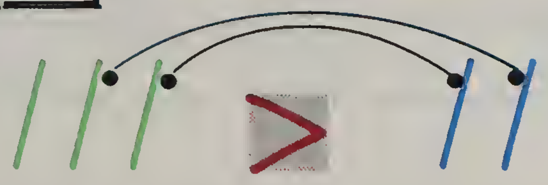


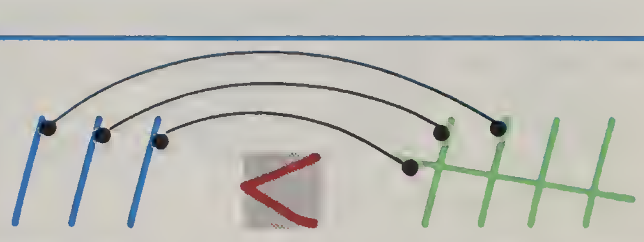
Without filling in the tags indicating the number of objects in each group, we know that the "less than" symbol will be appropriate after the count is made. Why?

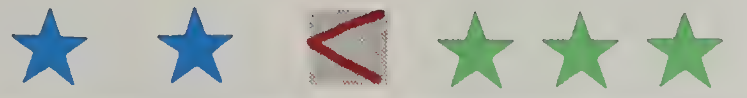
Headline Story: Make up a fitting headline: The board is 3 inches less than 1 foot.

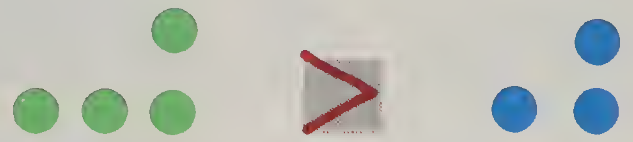
> Greater than   < Less than   = Equal to

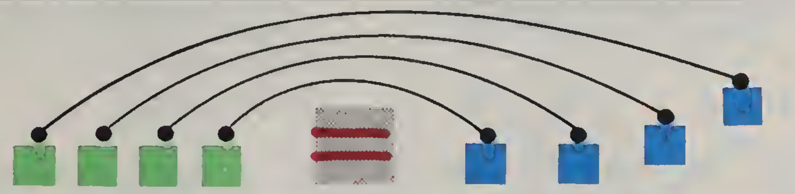


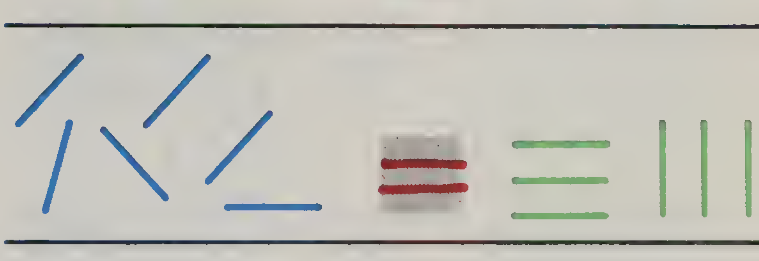


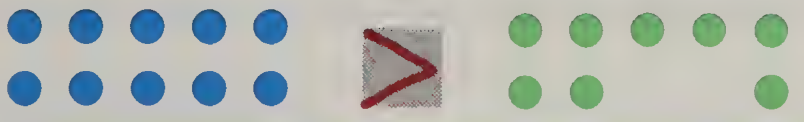


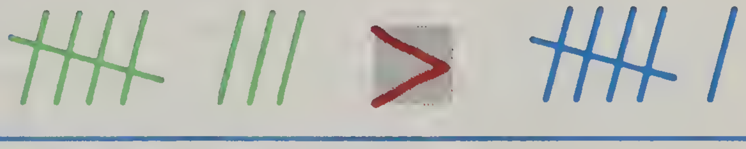












3 < 4 or more   9 > 1   12 = 12

3 > 2 or less   7 < 8 or more   11 or more > 10

13 = 13   6 or less < 7   9 or less < 10

3+5 > 1+5   7+4 = 4+7   8+2 > 5+2

Notes on Pupil Page 116

Tasks: *Pupils complete each example.*

Purpose: To provide pupils with further practice in using the inequality and equality relations.

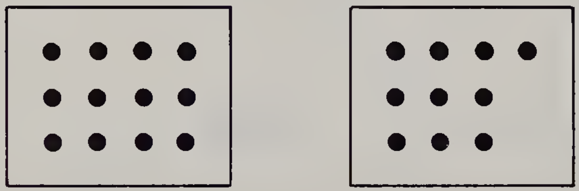
Unifying Ideas: Structure; functions and relations.

Materials for Teacher: Demonstration size tag boards containing various sets of dots or "X's" as described in "The Lesson."

Materials for Pupils: Desk counters; paper and pencil for record keeping.

The Lesson:

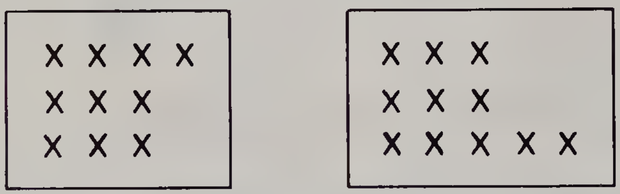
Display pairs of your demonstration designs on the board, such as:



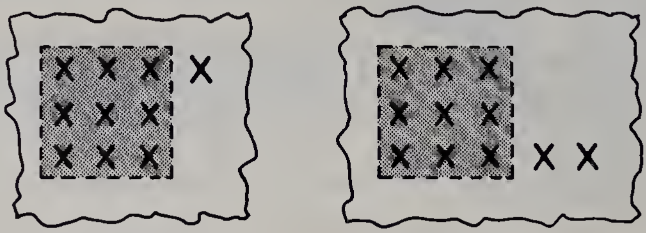
"Are there the same number of dots in both groups? If not, which contains more? Do you need to count both groups before you can answer these questions?" No! The two designs would be alike, except there are two missing from the sketch on the right. So, we can say that the number of dots on the left is greater than the number on the right. Then, after counting, we can write:

$$12 > 10$$

Consider another pair of arrangements:



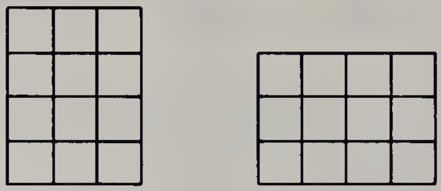
"Are there the same number of "X's"in each?" If we notice the 3 × 3 component of each design, the question can be answered by noting that 1 is less than 2:



After counting, we can write two expressions:

$10 < 11$ and $9+1 < 9+2$

Still another example:



"Are there the same number of small squares in each?" Yes! You need only notice that a simple turn of one would reveal that it is just like the other. After counting each, we can write:

$$12 = 12$$

Ask the pupils to make up stories for each inequality or equality statement above so it might serve as a headline.

"Mother had a full carton of a dozen eggs. She used two. Now she has fewer eggs because 12 eggs are more than 10 eggs."

"Mary is 10 years old and John is 11. Mary is younger than John."

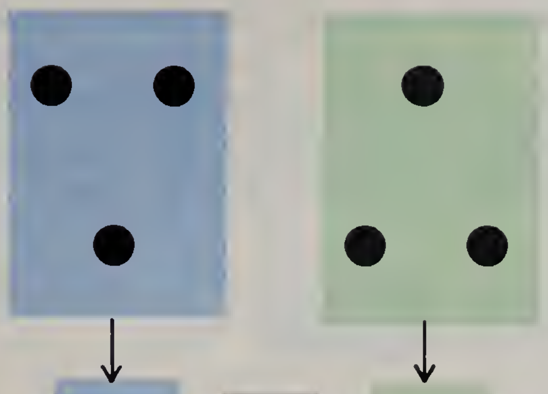
Pupils now complete page 116.

Be sure to review the accompanying booklet, *Games and Enrichment Activities*, for suggestions you may wish to incorporate in your daily lesson plan.

Headline Story: Complete the headline and make up a story to fit:

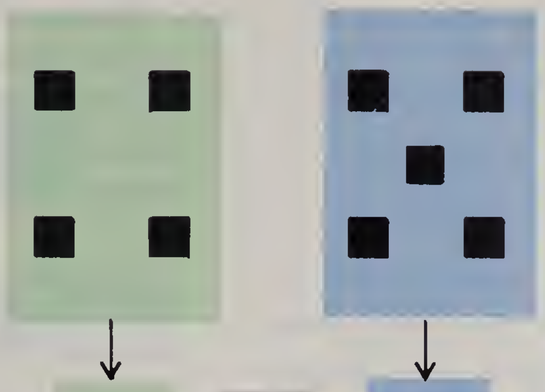
$$20 + 15 = \square$$

= Equal To



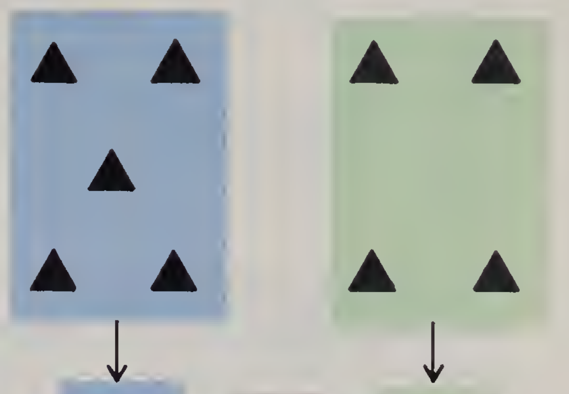
$3 = 3$

< Less Than

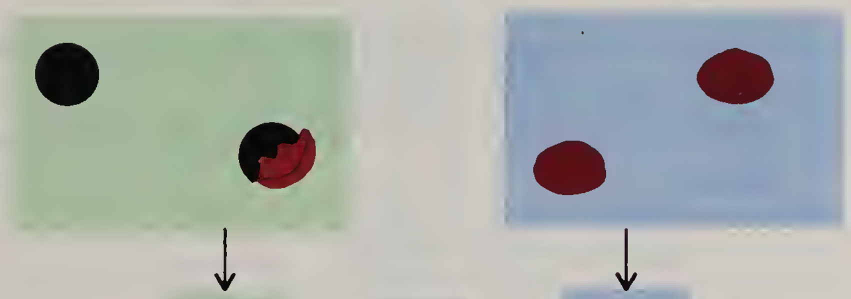


$4 < 5$

> Greater Than



$5 > 4$

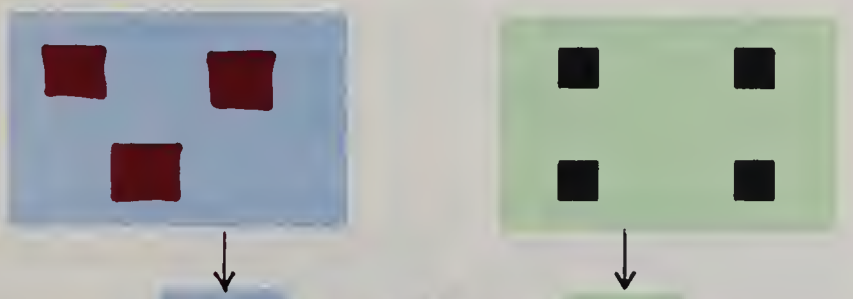


$2 = 2$

$5 = 5$

$10 = 10$

$5 + 10 = 10 + 5$

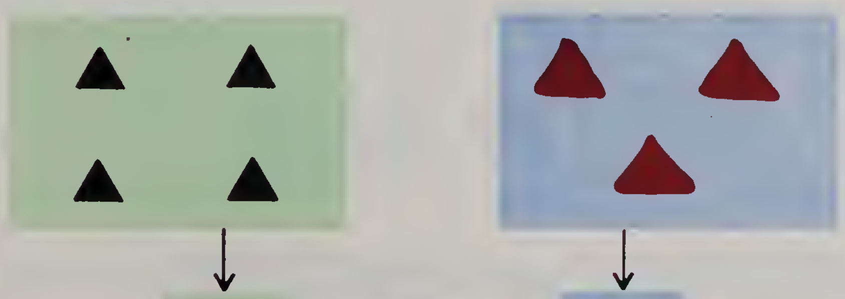


$3 < 4$

$5 < 10$

$2 + 3 < 6 + 4$

$3 + 2 < 4 + 6$



$4 > 3$

$10 > 5$

$8 + 2 > 4 + 1$

$2 + 8 > 1 + 4$

Notes on Pupil Page 117

Tasks: *Pupils use the symbols* "$>$", "$=$" and "$<$" *to indicate the relation between the number of unit tiles in each row of each example and complete the number sentence it suggests.*

Purpose: To help pupils discover the fact that if two groups of objects are not equal in number, then adding the same number of objects to both groups does not change the inequality of the groups.

Unifying Ideas: Structure; functions and relations.

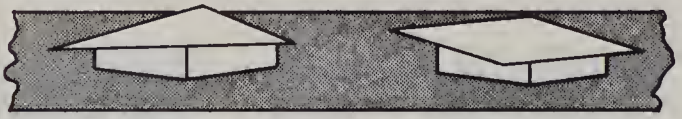
Materials for Teacher: Two demonstration size boxes, one with a triangular lid, the other with a square lid; colored chalk.

Materials for Pupils: A desk-top size tag board for each pupil with $\triangle > \square$ drawn on one side and $\triangle < \square$ on the other side—both drawings large enough for pupils to place counters in the geometric shapes, or placeholders; counters.

The Lesson:

Place some objects in each of the 2 demonstration boxes so that there are more objects in the one with the triangular lid. Invite several pairs of pupils to determine which box has more objects in it. Now write this expression on the board above the boxes:

$$\triangle > \square$$



We use this expression to say that the number of things in the box with the triangular lid is more than the number of things in the box with the square lid.

Have your helpers pass out the desk-top size tag boards with $\triangle > \square$ facing up. Ask each pupil to place some of his counters on each placeholder so that he can state a true number sentence. Record some of their sentences in this form:

$\triangle > \square$

Bill $5 > 3$ true

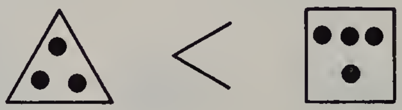
Now ask the class to construct a situation which would suggest a false number sentence.

$\triangle > \square$

Mike $2 > 3$ false

Make further constructions and records on the other side.

Now have all pupils construct the following situation:



Ask the class to figure out a way to add some counters to those in the placeholders so that a number sentence describing the new situation is still true. Write the following on the board:

$$\triangle < \square, \ 3 = 3, \ \triangle + 3 < \square + 3$$

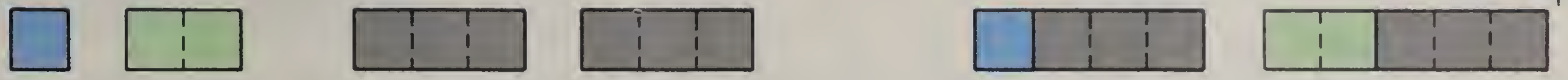
Ask the class to puzzle out the meaning of each of the three statements. Then have them use their counters to construct their own situations so that the expression is true. Now have them add three counters to the collections in the placeholders. Record:

$\triangle + 3 < \square + 3$

Doug $2 + 3 < 5 + 3$

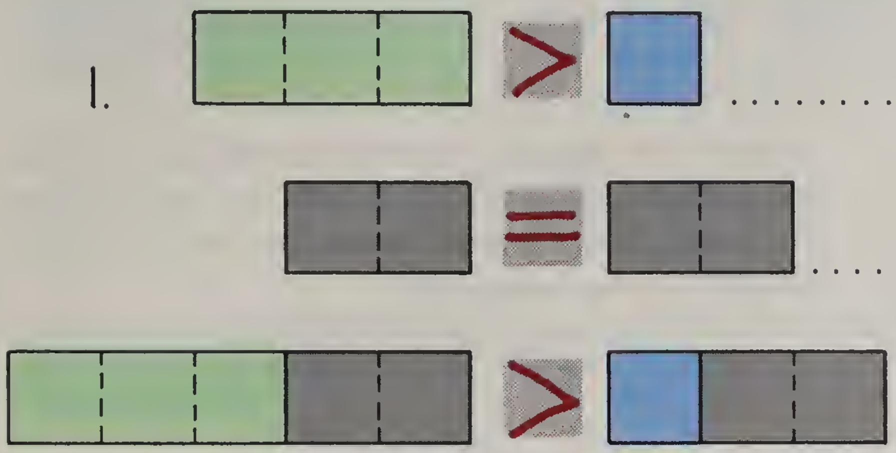
Headline Story: Tell a story to fit each headline:

$5 > 2$, $7 < 10$, $\square > 13$



$1 < 2$ and $3 = 3$. So, $1 + 3 < 2 + 3$.

1.

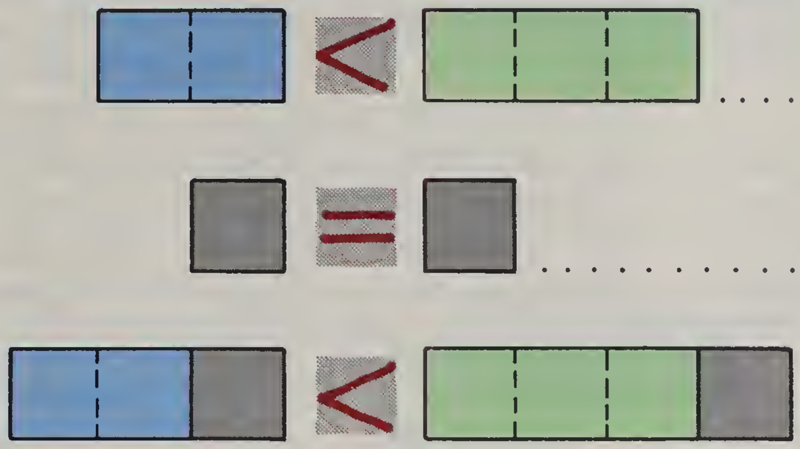


$3 > 1$

and $2 = 2$.

So, $3 + 2 > 1 + 2$.

2.

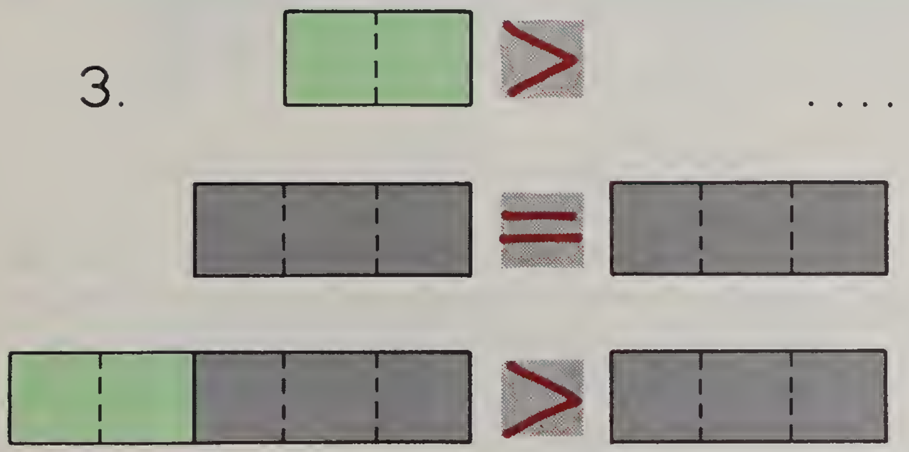


$2 < 3$

and $1 = 1$.

So, $2 + 1 < 3 + 1$.

3.

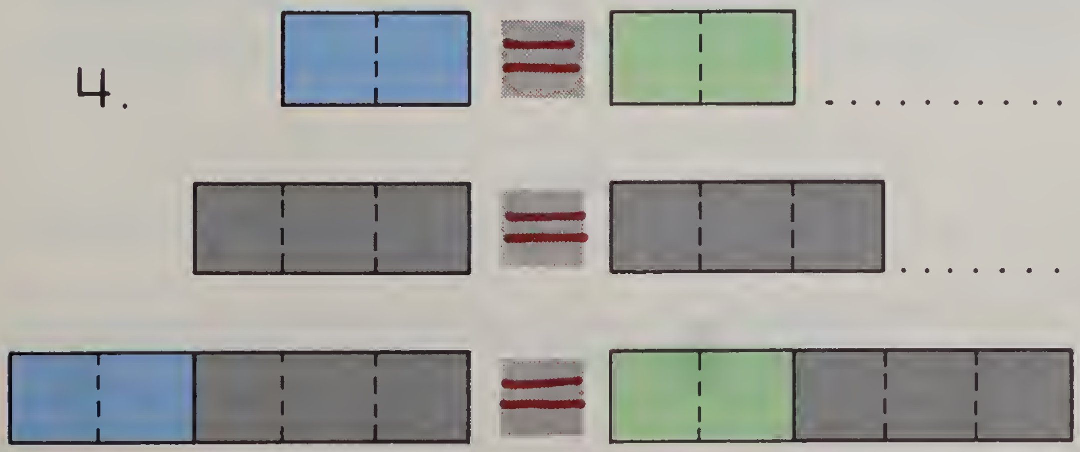


$2 > 0$

and $3 = 3$.

So, $2 + 3 > 0 + 3$.

4.



$2 = 2$

$3 = 3$

$2 + 3 = 2+3$

Notes on Pupil Page 118

Tasks: *Pupils use the signs “>”, “=” and “<” to indicate the relations between the numbers of unit tiles in each row of each example and complete the number sentence it suggests.*

Purpose: To help pupils discover the fact that if two groups of objects are not equal in number, then subtracting the same number of objects from both groups does not change the inequality of the two groups.

Unifying Ideas: Structure; functions and relations.

Materials for Teacher: Two demonstration-size boxes, one with a triangular lid, the other with a square lid; colored chalk.

Materials for Pupils: A desk-top size tag board for each pupil with △ > □ drawn on one side and △ < □ on the other side. Both drawings should be large enough for pupils to place objects on such as counters, or Cuisenaire® Rods.

The Lesson:

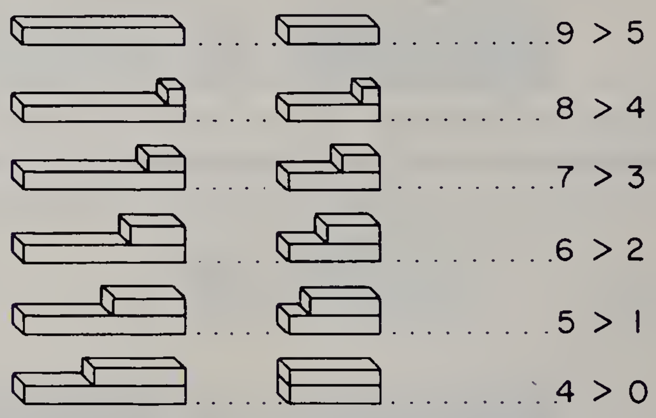
Have your helpers pass out the tag boards so that △ > □ faces up. Ask the class to use counters to set up the situation 5 > 4 at their desks. Invite the class to figure out a way to remove some counters from each group so that a number sentence describing the new situation is still true. Encourage discussion among classmates. “If we start with 5 > 4 and remove three counters from the group in the △ and three from the group in the □, what is the new situation and how should we write about it?”

5 > 4
3 = 3
5 - 3 > 4 - 3

Invite the class to construct situations so that they can easily remove two objects from each of the groups in the triangle and square. Record their dictated number sentences.

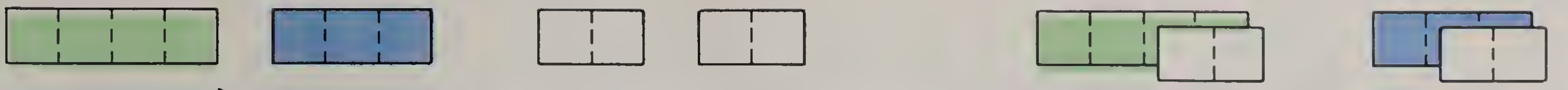
△ - 2 > □ - 2
Bill 7 - 2 > 3 - 2
Jim 10 - 2 > 4 - 2 etc.

Now have the class use their Cuisenaire® Rods. Have each pupil start wtih two rods that suggest nine and five. Place the larger rod on the △ and the smaller on the □. We then agree that this arrangement suggests the idea that 9 > 5. In succession cover each with one unit, then two units, then three units, etc. “What does the uncovered part of the original rod suggest?”



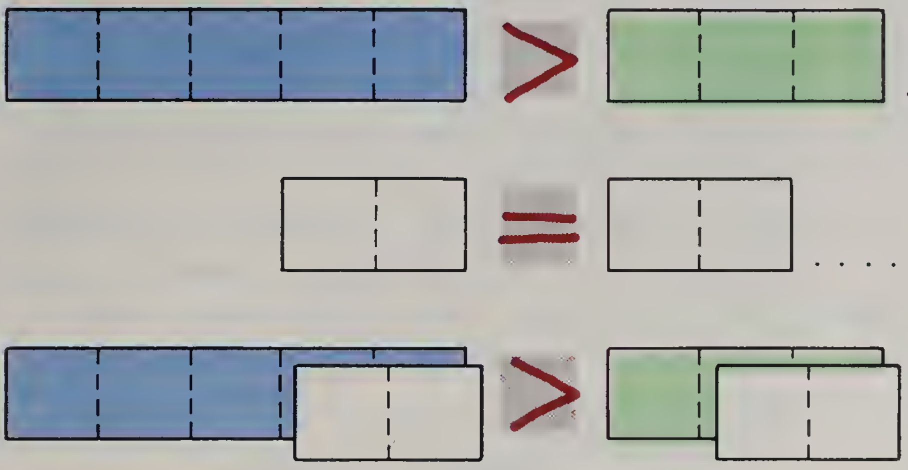
Ask pupils to place rods representing 7 and 10 respectively in the triangle and square and repeat the above experiments on the other side of the oak tag, keeping records as they proceed. Pupils may now complete page 118.

Headline Story: Make a headline to fit: In 2 pounds there are 32 ounces.



$4 > 3$ and $2 = 2$. So, $4 - 2 > 3 - 2$.

A.

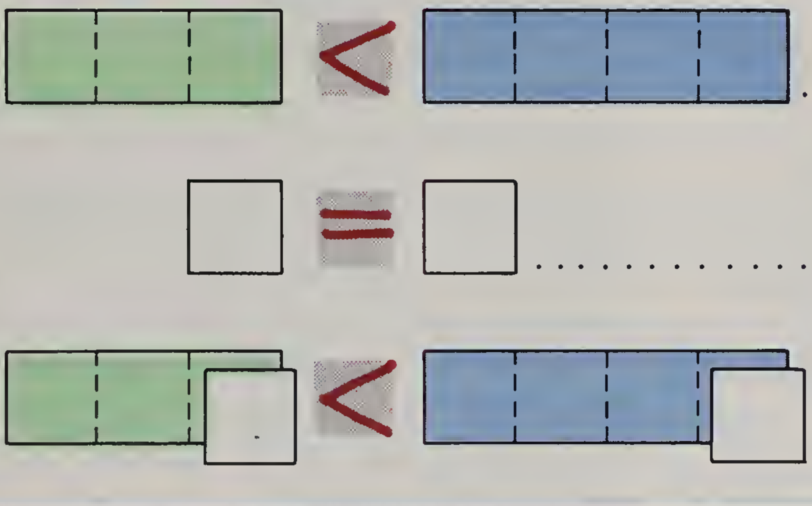


$5 > 3$

and $2 = 2$.

So, $5 - 2 > 3 - 2$.

B.

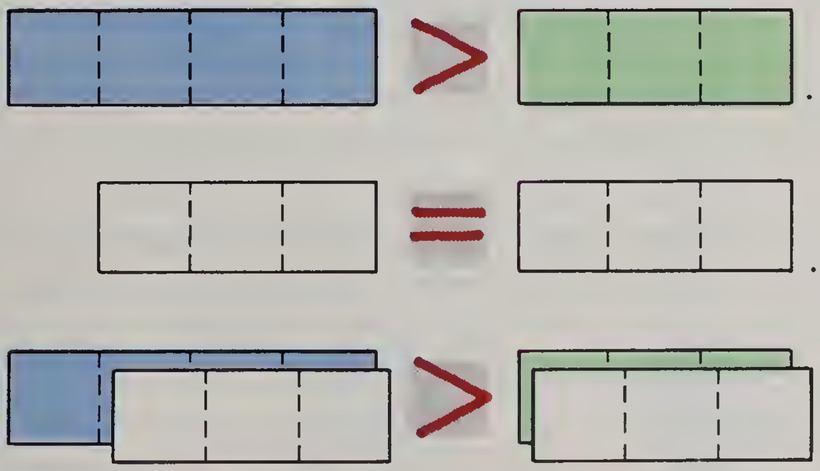


$3 < 4$

and $1 = 1$.

So, $3 - 1 < 4 - 1$.

C.

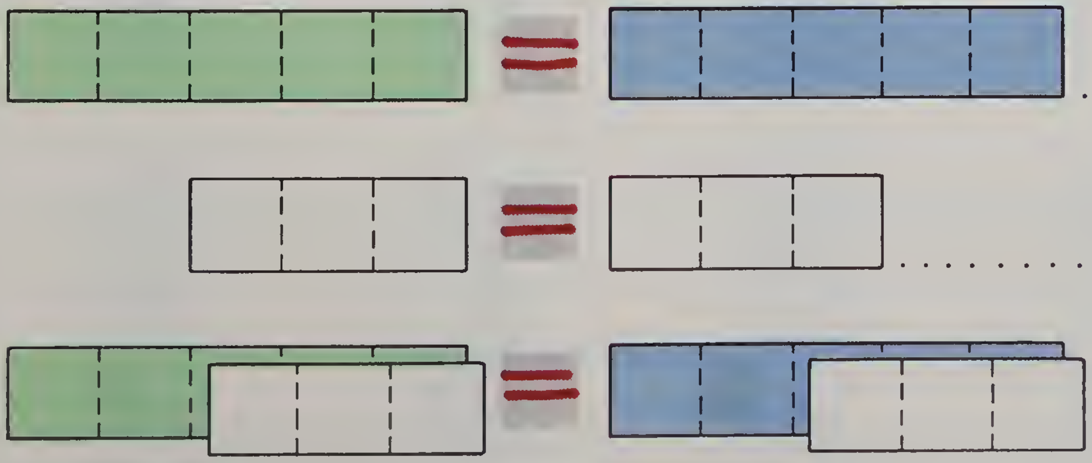


$4 > 3$

and $3 = 3$.

So, $4 - 3 > 3 - 3$.

D.



$5 = 5$

$3 = 3$

$5 - 3 = 5 - 3$

Notes on Pupil Page 119

Tasks: *Pupils are to fill in blanks in the number sentences.*

Purpose: To review subtraction by (a) employing jumps toward zero on the number line, and (b) using the horizontal and vertical forms of notation.

Unifying Ideas: Addition and subtraction.

Materials for Teacher: Ten demonstration size tag strips containing the headline stories shown in "The Lesson"; flannel board or cork board; two large oak tag sheets containing the subtraction situations listed in "The Lesson."

Materials for Pupils: Oak tag strips, each strip containing one of the completed headlines found on pupil page 119.

The Lesson:

Pupils complete page 119 without any prior discussion.

Place the following on the chalkrail or flannel board or cork board:

$\boxed{5 - 2 = 3}$ $\boxed{10 - 4 = 6}$ $\boxed{15 - 4 = 11}$

"In what real-life situations do we find the idea of subtraction involved?" Record the essence of their replies.

"Losing something,"
"Temperature going down,"
"Spending money,"
"Taking something away,"
"Distance left to go," etc.

"Make up stories using one of these ideas and fitting one of the headlines." Some pupils may suggest such situations as, "Finding something."

"How could such an idea be linked with subtraction?" Ask for a "Finding something" story for one of the headlines, such as $5 - 2 = 3$. A possible reply: "I have 5 pencils. I found 2 of them this morning. I had 3 of them before I found the other 2."

"Can you tell stories to fit each headline and include these ideas:

Finding something,
Saving money,
Temperature going up,
Putting something back,
Distance already gone?"

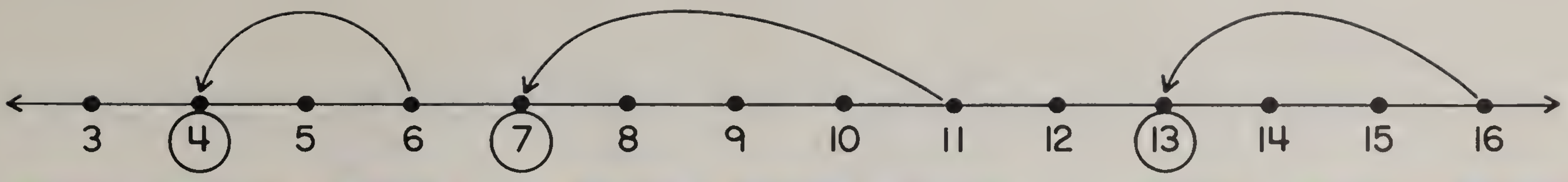
For the headline $5 - 2 = 3$, a pupil may say: "I saved 2¢ and I had 3¢ before. Now I have 5¢."

This story does not follow the headline item by item in the same left-right order, but this story does reflect a solid understanding of a relationship between arithmetic and the world we live in.

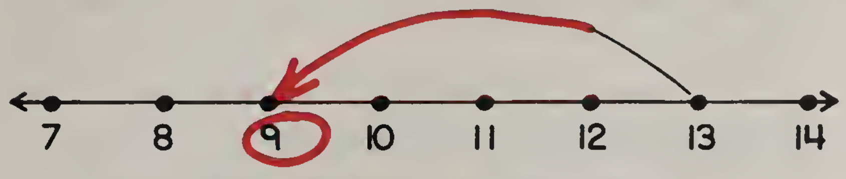
Such stories are certainly appropriate and should be encouraged. Our concern is with understanding this relationship, not in following rules that are mechanically applied.

Consider arranging the class into three-member work teams. Each team leader selects one of the oak tag strips containing a headline and his team decides on an appropriate sketch or model and a story. When each team has completed its task, the team leader reports to you. Members of groups whose report is satisfactory can serve as consultants to other teams requesting such aid. When all teams are ready, have each team come to the front to give its report.

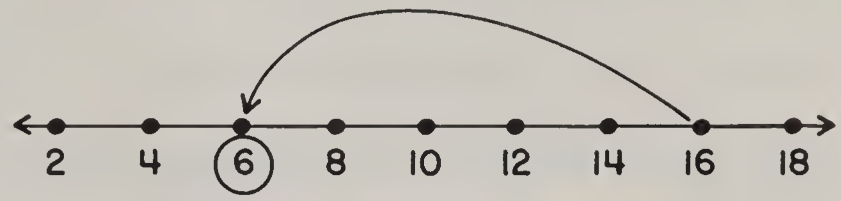
Headline Story: Nancy saved 5¢ a week for 15 weeks. How much did she save?


3 4 5 6 7 8 9 10 11 12 13 14 15 16

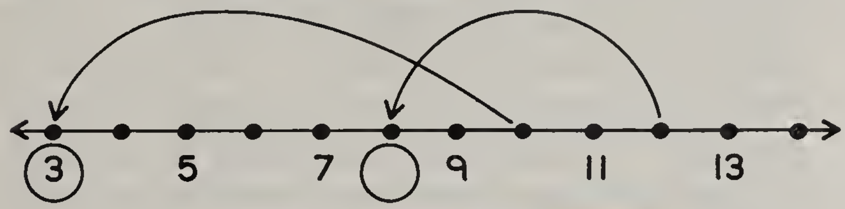
6 − 2 = 4 11 − 4 = 7 16 − 3 = 13


7 8 9 10 11 12 13 14

13 − 4 = 9

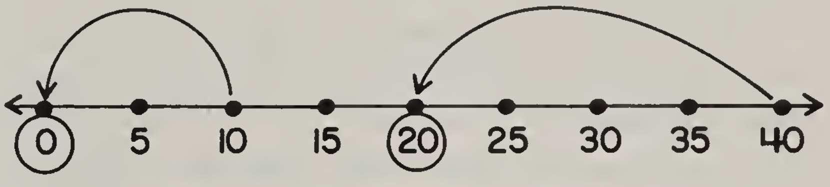

2 4 6 8 10 12 14 16 18

16 − 10 = 6



10 − 7 = 3

12 − 4 = 8


0 5 10 15 20 25 30 35 40

10 − 10 = 0

40 − 20 = 20

5 6 7 8 9 10 11 12 13 14 15 16 17 18

$$\begin{array}{r} 11 \\ -\ 6 \\ \hline 5 \end{array} \quad \begin{array}{r} 11 \\ -\ 5 \\ \hline 6 \end{array} \quad \begin{array}{r} 13 \\ -\ 5 \\ \hline 8 \end{array} \quad \begin{array}{r} 13 \\ -\ 8 \\ \hline 5 \end{array} \quad \begin{array}{r} 15 \\ -\ 6 \\ \hline 9 \end{array} \quad \begin{array}{r} 15 \\ -\ 9 \\ \hline 6 \end{array}$$

$$\begin{array}{r} 17 \\ -\ 10 \\ \hline 7 \end{array} \quad \begin{array}{r} 17 \\ -\ 7 \\ \hline 10 \end{array} \quad \begin{array}{r} 18 \\ -\ 13 \\ \hline 5 \end{array} \quad \begin{array}{r} 18 \\ -\ 5 \\ \hline 13 \end{array} \quad \begin{array}{r} 20 \\ -\ 6 \\ \hline 14 \end{array} \quad \begin{array}{r} 20 \\ -\ 14 \\ \hline 6 \end{array}$$

Notes on Pupil Page 120

Tasks: *Pupils complete each numerical sentence.*

Purpose: To provide pupils with the opportunity to review and extend their knowledge of subtraction via headline stories.

Unifying Ideas: Addition and subtraction.

Materials for Teacher: No special materials necessary.

Materials for Pupils: No special materials necessary.

The Lesson:

Ask pupils to complete this page without any discussion, since it is like page 119. "Now, consider each mathematical statement as a headline. Make up a story to fit each. Try to select stories for which the numbers seem reasonable." This last request leads to much discussion. Compare two stories suggested for the first example:

$$\boxed{12 - 2 = \underline{\qquad}}$$

(a) "Mother had a dozen eggs. She used two. She has 10 left."

(b) "There are 12 pupils in our school. Two were absent. There are 10 pupils present."

Both are appropriate stories. However, a school with only 12 pupils enrolled would be most unusual—though it certainly happens in sparsely settled communities. Here are six stories suggested for the headline:

$$\boxed{25 - 5 = \underline{\qquad}}$$

(a) "I had a quarter and spent a nickel. I have 20¢ left."

(b) "Bill walked 5 miles toward school. Since it is 25 miles to school, he has 20 more miles to walk."

(c) "A truck had 25 wheels. Five tires went flat at one time. There are 20 tires that were not flat."

(d) "Jane is 25 feet tall. Her brother is 5 feet shorter. Her brother is 20 feet tall."

(e) "Twenty-five is five more than twenty."

(f) "In a class of 25 pupils, 5 were absent; 20 were present."

Each is certainly a story that reflects the idea in the headline. However, some suggest situations that would be encountered seldom, if ever: How many pupils walk 25 miles to school? . . . How many trucks have 25 wheels? . . . Is any girl 25 feet tall? Examples "a" and "f" speak of events that do not sound unusual. Example "e" is hardly a "story"—it is rather a restatement of the arithmetic in the headline.

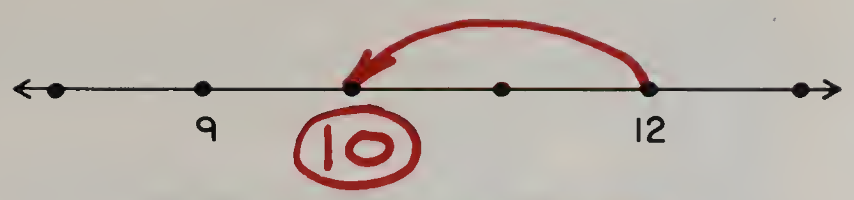
Make a list of stories volunteered by pupils. Discuss the relative reasonableness of each real-life situation involved.

Ask for other stories that include an idea suggested by the headline, but stories that also sound less reasonable or less likely to happen in the normal course of events.

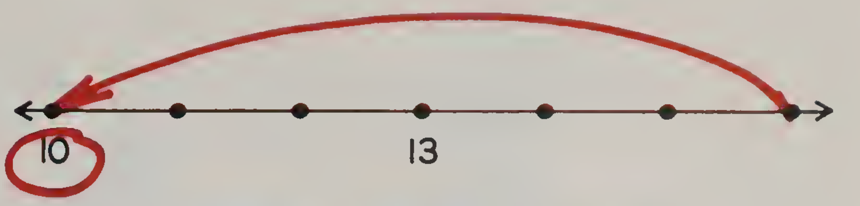
Compare the two kinds of stories. Avoid characterizing any story as right or wrong, good or bad, possible or impossible, true or false. Simply promote a discussion about the relative appropriateness of the stories.

Be sure to review the accompanying booklet, *Games and Enrichment Activities*, for suggestions you may wish to incorporate in your daily lesson plan.

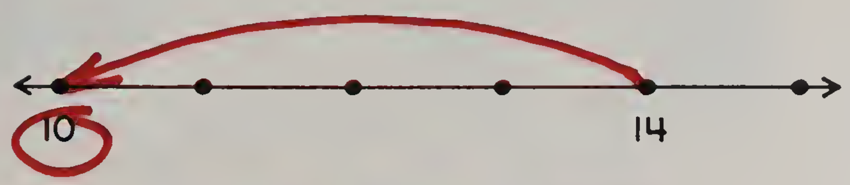
Headline Story: Make up a fitting headline: How many legs are there on 3 chairs?



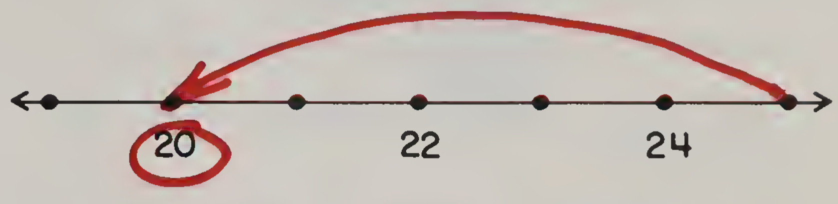
$12 - 2 = 10$



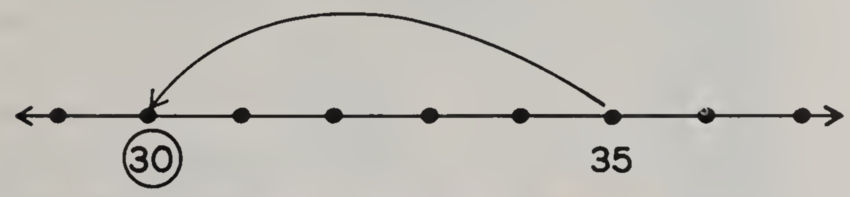
$16 - 6 = 10$



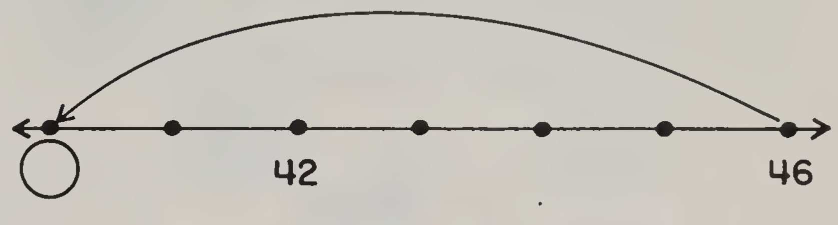
$14 - 4 = 10$



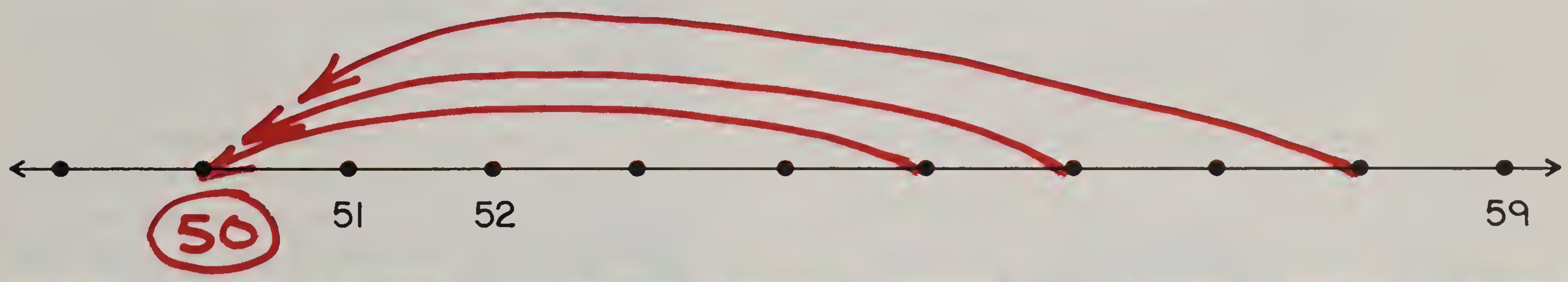
$25 - 5 = 20$



$35 - 5 = 30$



$46 - 6 = 40$



$55 - 5 = 50$

$56 - 6 = 50$

$58 - 8 = 50$

$12 - 4 = 8$

$25 - 4 = 21$

$46 - 3 = 43$

$12 - 3 = 9$

$36 - 4 = 32$

$41 - 0 = 41$

$24 - 5 = 19$

$37 - 8 = 29$

$58 - 6 = 52$

Notes on Pupil Page 121

Tasks: *Pupils fill in tinted boxes with the appropriate numbers—using any shortcuts they want.*

Purpose: To provide pupils with further opportunities to anticipate multiplication by exploring patterns and relationships among sets of numbers which are multiples of 3, 4, 5, 6, 7.

Unifying Ideas: Multiplication and division.

Materials for Teacher: Colored chalk and a straight-edge.

Materials for Pupils: Counters for those who need to use them.

The Lesson:

"In a store, a clerk noticed that some of the items he sold were 3¢ each, 4¢ each, 5¢ each, 6¢ each, and 7¢ each. He made a chart to refer to when someone made a purchase."

	3¢ each
1	3¢
2	6¢
3	

"One item at 3¢ would cost how much? How much would you pay if you bought two items at 3¢ each?" Record the results in the table.

Add another column to record amounts if items cost 4¢ each.

	3¢ each	4¢ each
1	3	4
2	6	8

Extend the record, column by column, until it shows the cost of 5 to 9 of each item:

	3¢ each	4¢ each	5¢ each	6¢ each	7¢ each
1	3	4	5	6	7
2	6	8	10	12	14
3	9	12	15	18	21
4	12	16	20	24	28
5	15	20	25	30	35

Leave these results on the board while pupils work out as much as they can of page 121.

Notice the down-left and down-right diagonals in each pair of large blocks, Y and Z, as well as the right-hand columns.

Consider examples W and X in terms of the clerk's problem: The down-left diagonal pattern suggests the cost of items at 4¢ each. The right column patterns suggest the cost of items at 5¢ each. The down-right diagonal pattern suggests the cost of items at 6¢ each.

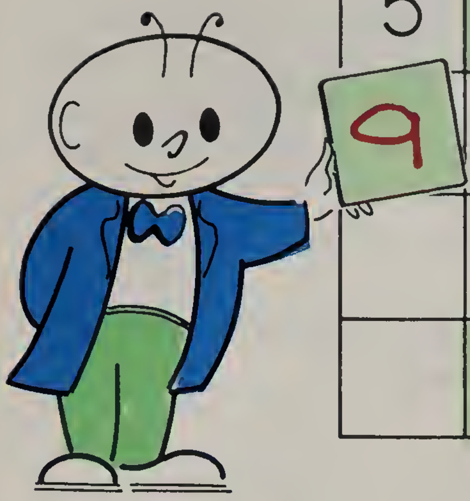
"Are there similar patterns in the other parts? Why?"

"Consider the large blocks, U and V; what connection is there between the two down-left diagonals in the U block and the clerk's chart for 3¢ items? What connection is there between the right-hand columns of blocks U and V and items which cost 4¢ each? What about the down-right diagonal in the V block and items which cost 5¢ each? Consider blocks Y and Z. Is there a connection between the right-hand column and items which cost 6¢? What is the significance of the down-right diagonal of block Z?"

Headline Story: Make up a fitting headline: 8 inches is 4 inches less than 1 foot.

U.

1	2	3	4
5	6		8
9			12
		15	16
	18		20



V.

1	2		4
5			8
	10		12
		15	16
			20

W.

1	2		4	5
		8		10
	12			15
16				20
			24	25
		28		30

X.

1	2			5
6				10
	12			15
		18		20
			24	25
				30

Y.

1	2			5	6
			10		12
		15			18
	20				24
25					30
				35	36

Z.

1	2				6
7					12
	14				18
		21			24
			28		30
				35	36

Notes on Pupil Page 122

Tasks: *Pupils fill in the tinted boxes with the appropriate numbers—using any shortcuts they want.*

Purpose: To provide pupils with further opportunities to anticipate multiplication by discovering patterns and relationships among sets of numbers which are multiples of 3, 4, 5, 6, 7, 8, 9, 10.

Unifying Ideas: Multiplication and division.

Materials for Teacher: Colored chalk.

Materials for Pupils: No special materials are needed.

The Lesson:

Have the class extend the chart developed in your lesson for page 121 to include items that cost 8¢ each, 9¢ each, and 10¢ each.

Put the following on the board as a "What's My Rule?" game:

1	2	3	___	5	___	___

Ask what numbers the pupils think go in the blanks. Then introduce another row below it:

1	2	3	4	5	6	7
2	4	6	___	___	___	___
						↑

"What number do you think will be written in the blank indicated by the arrow?" Some may predict 14. To check, fill in each blank along the way. Then introduce another row:

1	2	3	4	5	6	7
2	4	6	8	10	12	14
3	6	9	___	___	___	___
				↑		↑

"What numbers do you think will be written in the blanks with the arrows under them?" Some may predict 15 and 21. "Why do you pick those numbers?" Reference may be made to the chart on the board or on pages 121 or 122; or that 9 + 3 = 12 and 12 + 3 = 15; and 15 + 3 = 18 and 18 + 3 = 21.

Often, a pupil will look at the columns in the example and see it this way:

$$\begin{array}{r}1\\+2\\\hline 3\end{array}\quad\begin{array}{r}2\\+4\\\hline 6\end{array}\quad\begin{array}{r}3\\+6\\\hline 9\end{array}\ \cdots\ \begin{array}{r}5\\+10\\\hline 15\end{array}\ \cdots\ \begin{array}{r}7\\+14\\\hline 21\end{array}$$

Develop a similar activity row by row, so the example might look like this:

2	4	6	8	10	12	14
3	6	9	12	15	18	21
5	10	___	___	___	___	___
		↑		↑		↑

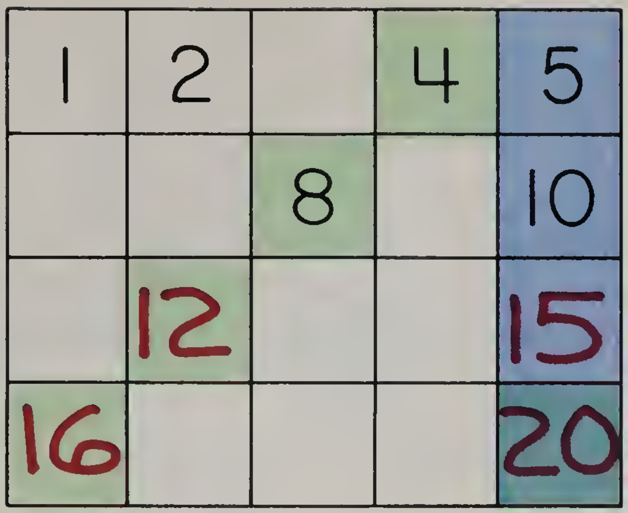
"What numbers do you think will be written in the blanks indicated with the arrows?" "Why did you choose 15, 25 and 35?" After the third row is completed, ask for the sum of three numbers listed in each column. The results will be 10, 20, 30, 40, 50, 60, and 70. Consider another example:

1	2	3	___	___	___	___
4	8	12	___	___	___	___
5	10	___	___	___	___	___
		↑		↑		↑
10	----	----	----	----	----	----

and again find that the sum of each column is a multiple of 10.

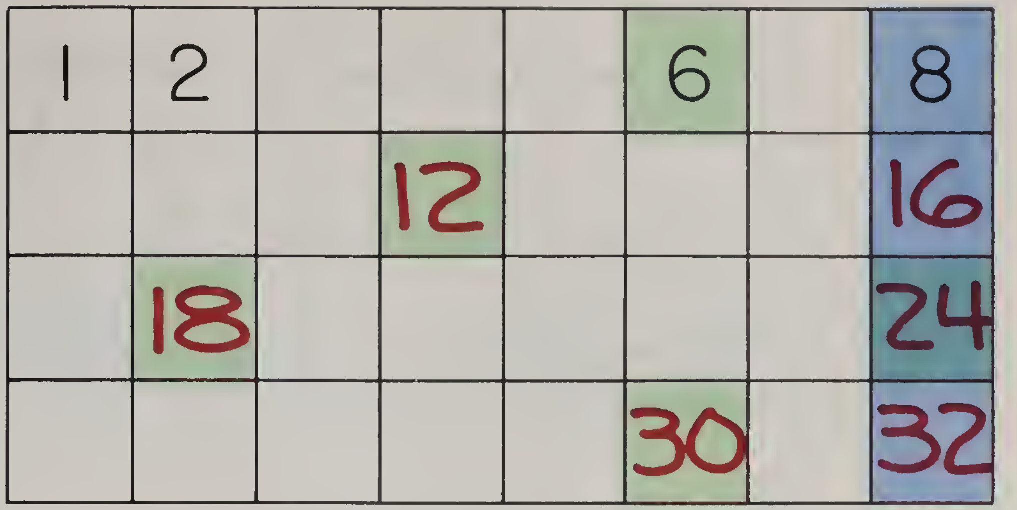
Headline Story: Ice cream bars cost 7¢ each. How much for 3? . . . for 2? . . . for 10? . . . etc.

A.



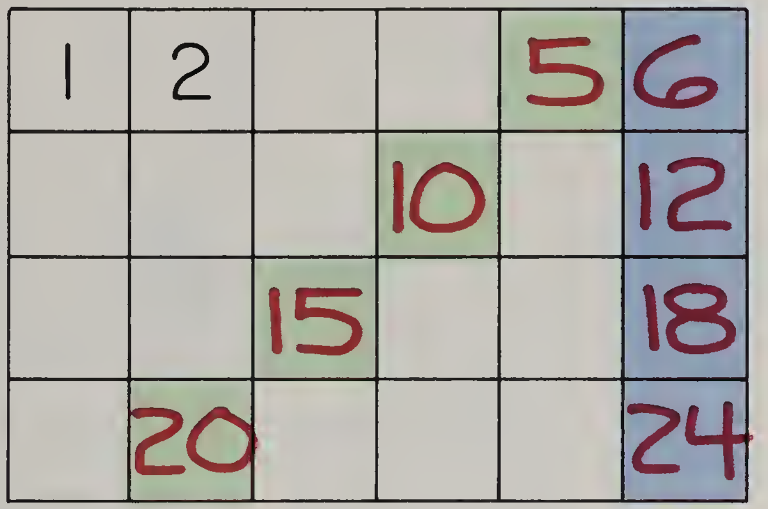
1	2		4	5
		8		10
	12			15
16				20

B.



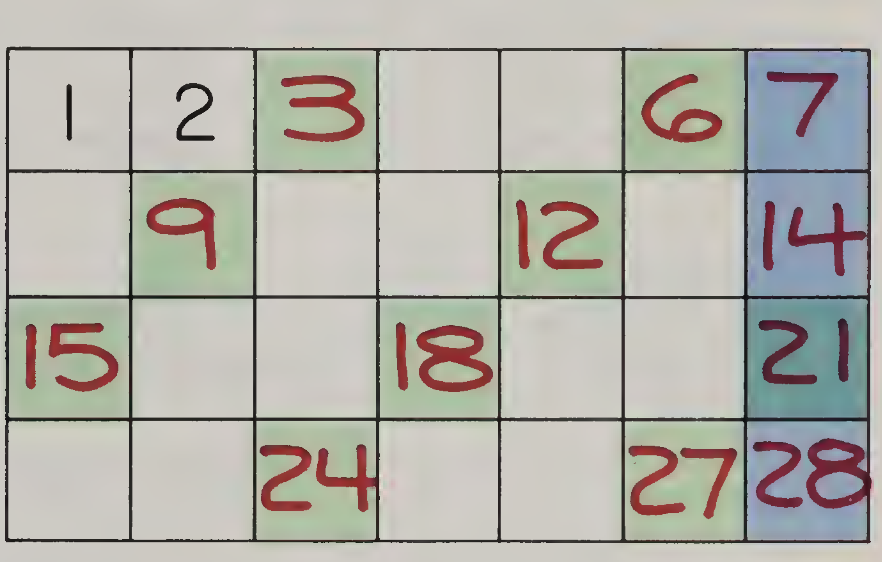
1	2				6		8
			12				16
	18						24
					30		32

C.



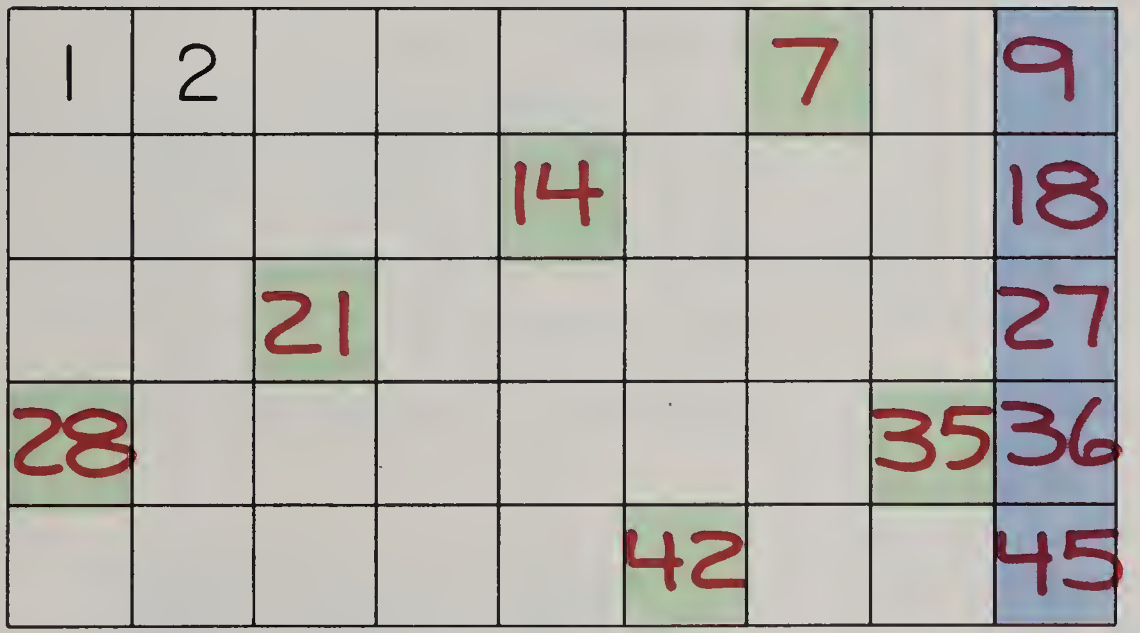
1	2			5	6
			10		12
		15			18
	20				24

D.



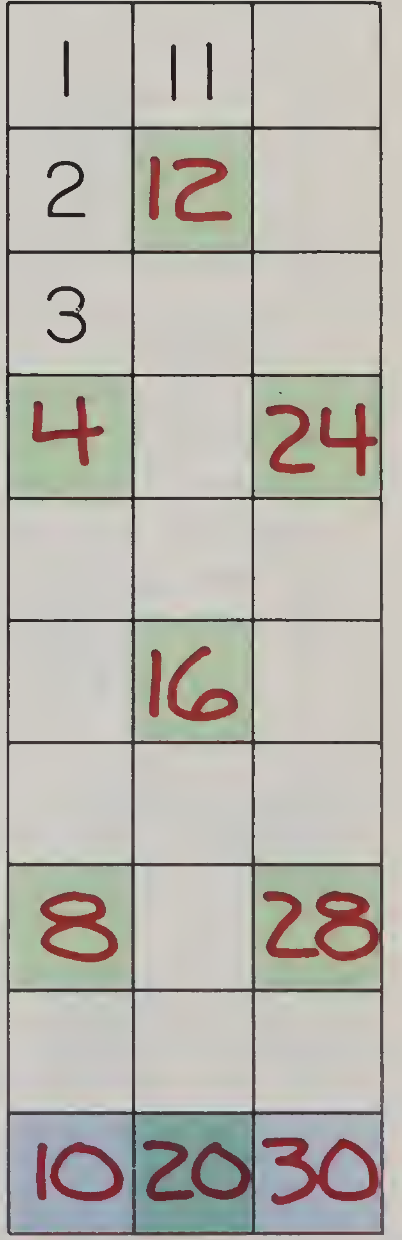
1	2	3			6	7
	9			12		14
15			18			21
		24			27	28

E.

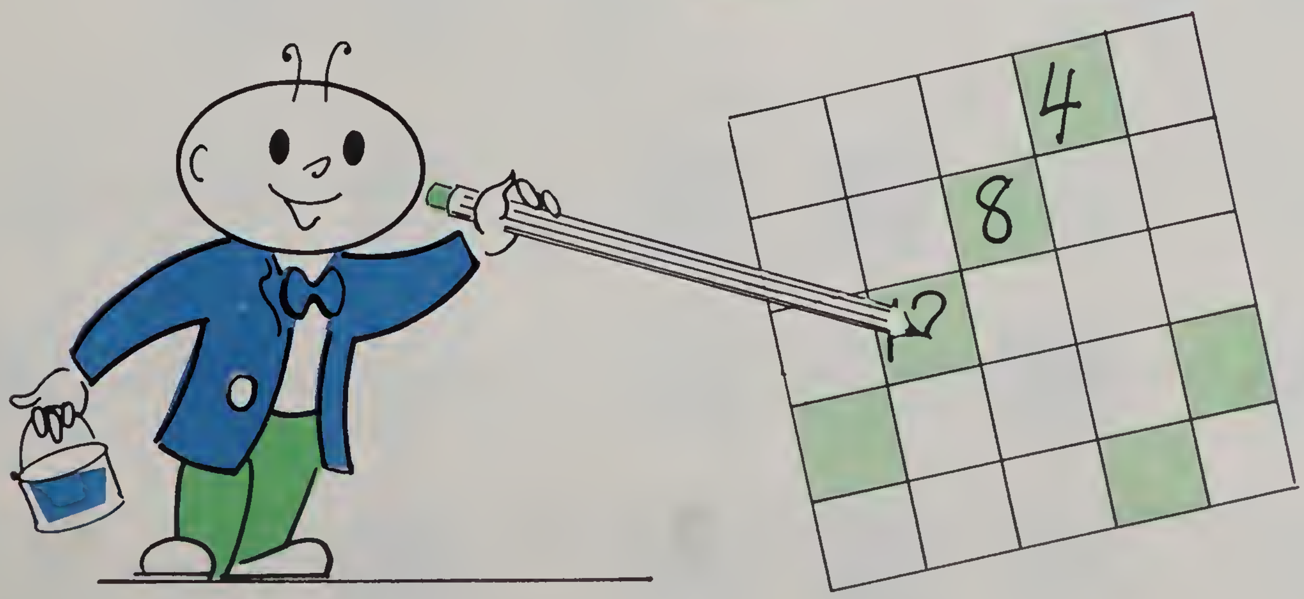


1	2					7		9
				14				18
		21						27
28							35	36
					42			45

F.



1	11	
2	12	
3		
4		24
	16	
8		28
10	20	30



Notes on Pupil Page 123

Tasks: *Pupils fill in the tinted blocks to make mathematical statements using the number line models when necessary.*

Purpose: To provide pupils with opportunities to extend their knowledge of addition and subtraction.

Unifying Ideas: Addition and subtraction.

Materials for Teacher: No special materials are needed.

Materials for Pupils: No special materials are needed.

The Lesson:

Have the class turn to page 123 and complete each example, completing the number line sketch if needed.

Notice that seven of the nineteen examples when processed by using the standard subtraction algorism would necessitate some procedure for borrowing, or regrouping the minuend. Obviously we're not interested in teaching a standard algorism at this time. The pupil's slogan in working with arithmetic problems should be: "When in doubt, count." Thus we have been teaching each pupil how to use many representations to visualize arithmetic problems so that he can count his way confidently to a reliable conclusion. When he is not in doubt, he ought to be encouraged to work without diagrams or sketches.

Consider the completed statements on page 123 as headlines. Ask the pupils to tell stories, draw sketches, or play-act in ways suggested by the headlines. Discuss the reasonableness of the responses. Could each headline suggest a situation involving each of the following ?

- money
- feet
- time
- people
- temperature
- boys and girls
- books
- blocks

Each statement in the left-hand column below refers to a fact involving subtraction. "In what kinds of situations do we use the idea of subtraction. What statements involving addition suggest the same idea in each case?"

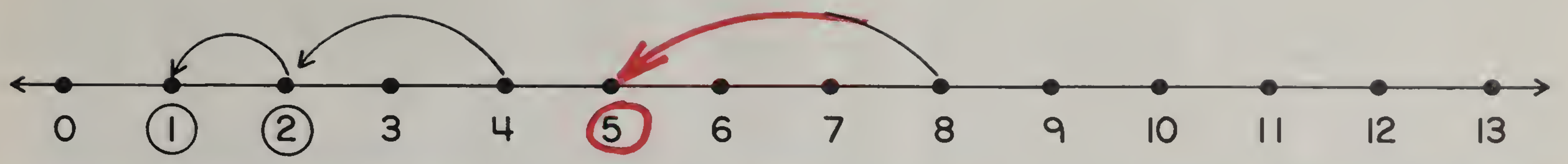
Subtraction	Addition
$8 - 3 = 5$	$3 + 5 = 8$ $5 + 3 = 8$
$4 - 2 = 2$	$2 + 2 = 4$
$53 - 4 = 49$	$49 + 4 = 53$ $4 + 49 = 53$

"Draw at least two different sketches that help show that your conclusions are reliable in each example." E.g., for $53 - 4 = 49$, a pupil sketches a number line chunk from 45 to 55. Starting at 53 he counts back four points to 49. Or, he sketches 5 groups of ten fingers, then three more fingers, and crosses out four of them, leaving a picture for 49. Or, he sets up an array of 53 dots and then crosses off four of them, leaving a picture for 49.

Now you might want to have the class compare each example in column A with the corresponding example in column B below. "Is there a relationship between each pair of examples?" "What is it?" "If you had your choice, which subtraction would you prefer to do?" Include a few more situations in column A and ask the class to puzzle out the corresponding related example for column B.

Column A	Column B
21 - 3 = ___	28 - 10 = ___
31 - 4 = ___	37 - 10 = ___
45 - 7 = ___	48 - 10 = ___
21 - 4 = ___	27 - 10 = ___

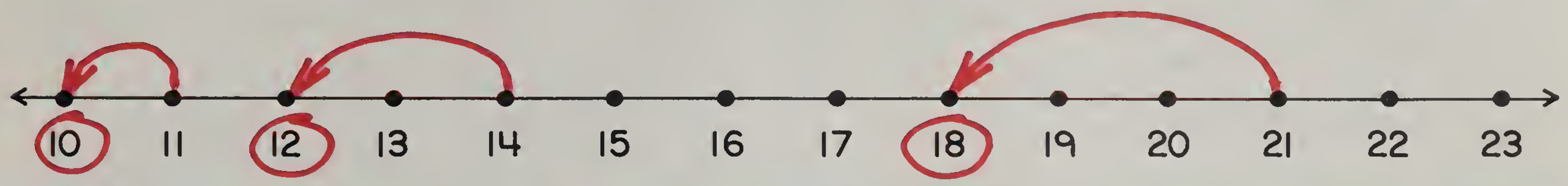
Headline Story: Make a headline to fit: Each of 3 trucks has 8 wheels.


0
①
②
3
4
5
6
7
8
9
10
11
12
13

2 − 1 = 1

4 − 2 = 2

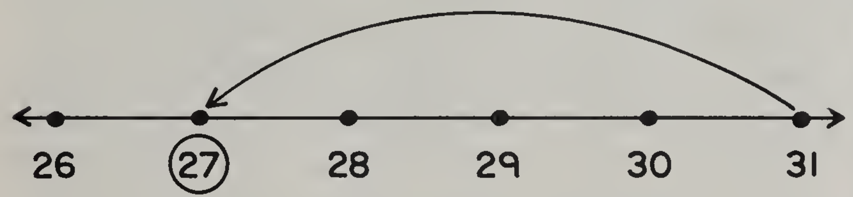
8 − 3 = 5


10
11
12
13
14
15
16
17
18
19
20
21
22
23

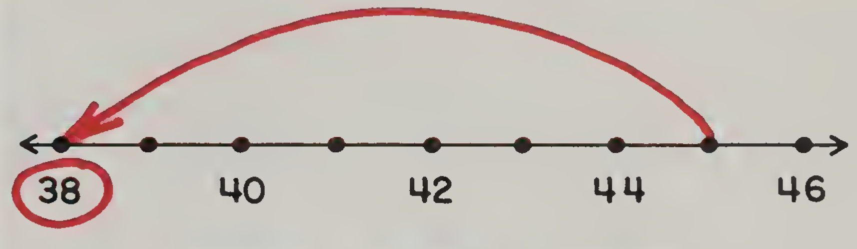
11 − 1 = 10

14 − 2 = 12

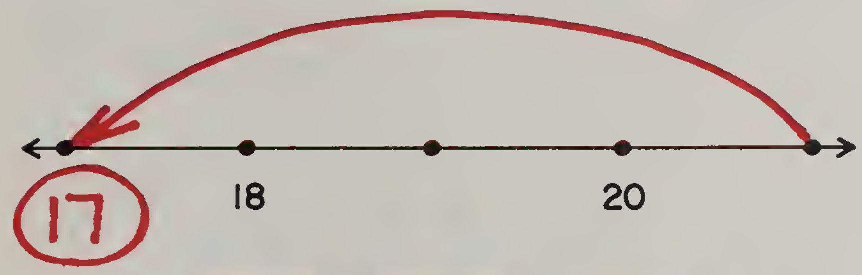
21 − 3 = 18


26
㉗
28
29
30
31

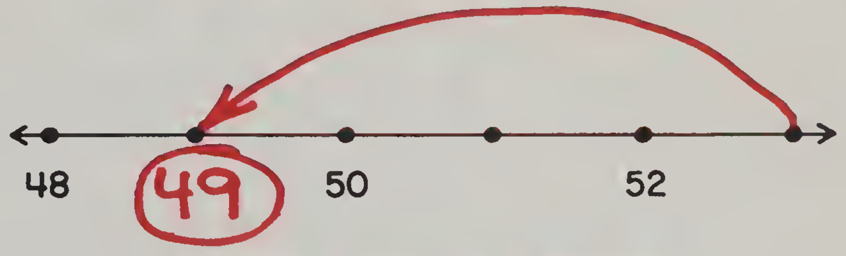
31 − 4 = 27


38
40
42
44
46

45 − 7 = 38



21 − 4 = 17



53 − 4 = 49

---

2 − 1 = 1

11 − 10 = 1

31 − 3 = 28

4 − 3 = 1

13 − 6 = 7

46 − 4 = 42

8 − 5 = 3

18 − 5 = 13

50 − 0 = 50

Notes on Pupil Page 124

Tasks: *Pupils use their straight-edges and pencils to draw the lines (i.e., the line segments) indicated in each example I through V.*

Purpose: (a) To provide pupils with further opportunities to review their skills in working with the straight-edge and pencil; and (b) to further extend their understanding of geometric ideas such as point, line, and triangle.

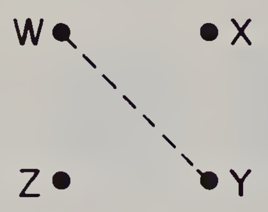
Unifying Ideas: Geometry.

Materials for Teacher: Colored chalk and a yardstick.

Materials for Pupils: A straight-edge and sharpened pencil for each pupil.

The Lesson:

Put four points on the board arranged as corners of an imagined square and label them W, X, Y and Z.



"Find points labeled W and Y. Draw $\overset{\bullet\!-\!\bullet}{WY}$. If we also were to draw $\overset{\bullet\!-\!\bullet}{ZX}$, what would the resulting sketch look like?" (A cross, an X, one line crossing another, etc.)

"Instead of drawing that line, suppose we draw two lines—$\overset{\bullet\!-\!\bullet}{YZ}$ and $\overset{\bullet\!-\!\bullet}{ZW}$. Now what would our sketch look like?" (Have a volunteer draw the lines using your yardstick—forming a triangle.)

"What do we call this shape?" (A triangle.) "Are all three of its sides the same length? If not, which is the largest side?" Erase the lines.

Suppose we draw the lines indicated below:

$\overset{\bullet\!-\!\bullet}{WX}$, $\overset{\bullet\!-\!\bullet}{XZ}$ and $\overset{\bullet\!-\!\bullet}{ZY}$

"What letter in the alphabet is similar in shape?" (The letter Z.)

"Could we join points so the shapes would look like other letters of the alphabet? Name the letters and we will make a list telling which lines we would need to draw." Some results might be:

N.......... $\overset{\bullet\!-\!\bullet}{WZ}$, $\overset{\bullet\!-\!\bullet}{WY}$ and $\overset{\bullet\!-\!\bullet}{XY}$
U.......... $\overset{\bullet\!-\!\bullet}{WZ}$, $\overset{\bullet\!-\!\bullet}{ZY}$ and $\overset{\bullet\!-\!\bullet}{XY}$
C.......... $\overset{\bullet\!-\!\bullet}{WZ}$, $\overset{\bullet\!-\!\bullet}{WX}$ and $\overset{\bullet\!-\!\bullet}{YZ}$
X.......... $\overset{\bullet\!-\!\bullet}{WY}$ and $\overset{\bullet\!-\!\bullet}{XZ}$
O.......... $\overset{\bullet\!-\!\bullet}{WX}$, $\overset{\bullet\!-\!\bullet}{XY}$, $\overset{\bullet\!-\!\bullet}{YZ}$ and $\overset{\bullet\!-\!\bullet}{ZW}$

Pupils should now be able to draw the lines indicated on page 124 without much assistance.

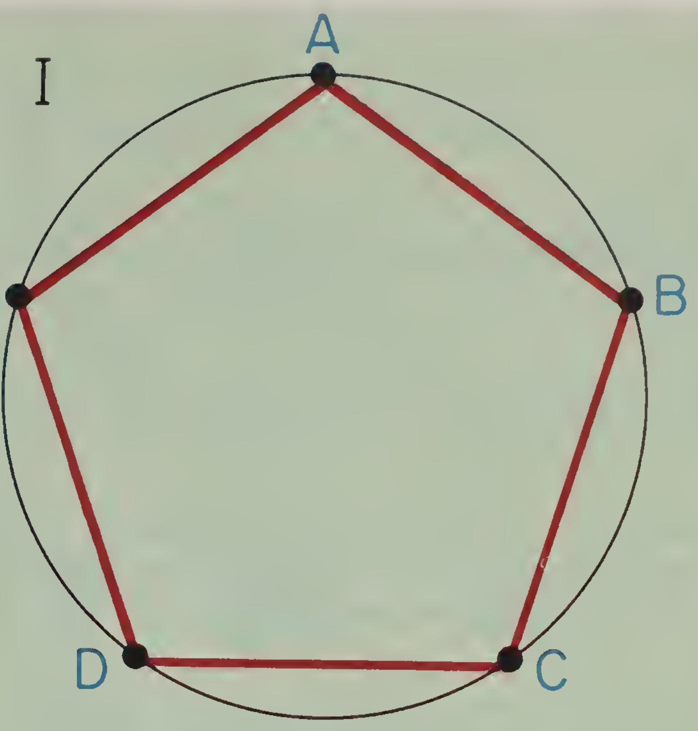
Discuss names for completed shapes in II, IV and V. (A 5-pointed star, a 6-pointed star and a 7-pointed star, or other appropriate names.) "What do you notice about the shape you made in example I and the shape in the center portion of the 5-pointed star?" (Each has 5 sides—and, in each, the sides are the same length . . . regular pentagons.) "What about the shape you drew in example III and the center portion of the 6-pointed star?" (A hexagon.)

Some might like to count the number of triangles formed in the stars—II, IV and V. They would find:

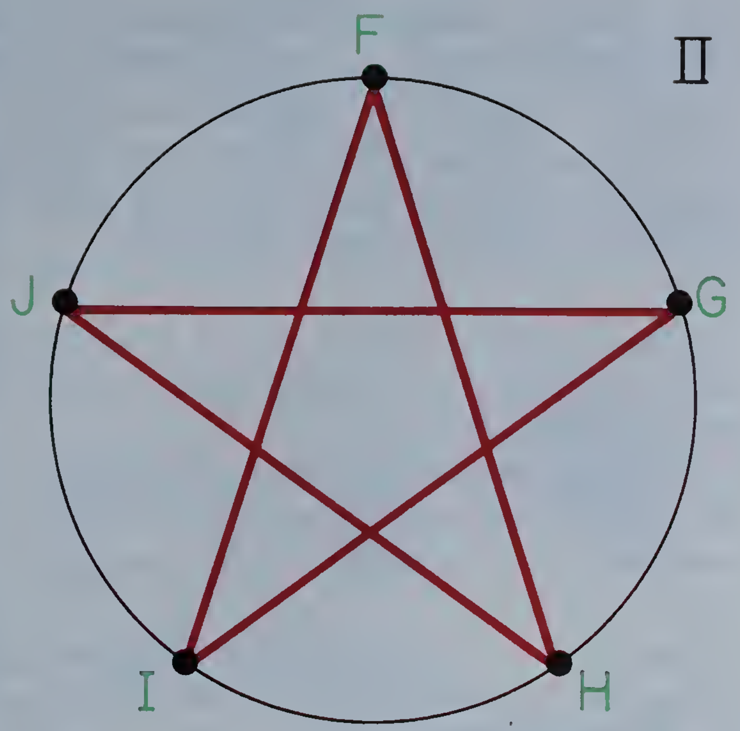
II....... 10 triangles
IV........ 8 triangles
V....... 35 triangles

Be sure to review the accompanying booklet, *Games and Enrichment Activities*, for suggestions you may wish to incorporate in your daily lesson plan.

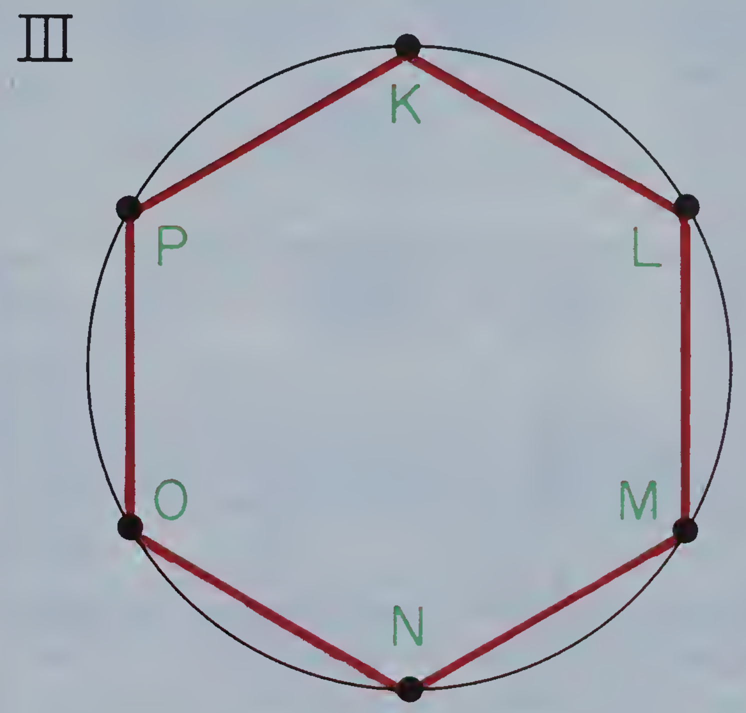
Headline Story: It's 3 o'clock. I started 5 hours ago. At what time did I start?



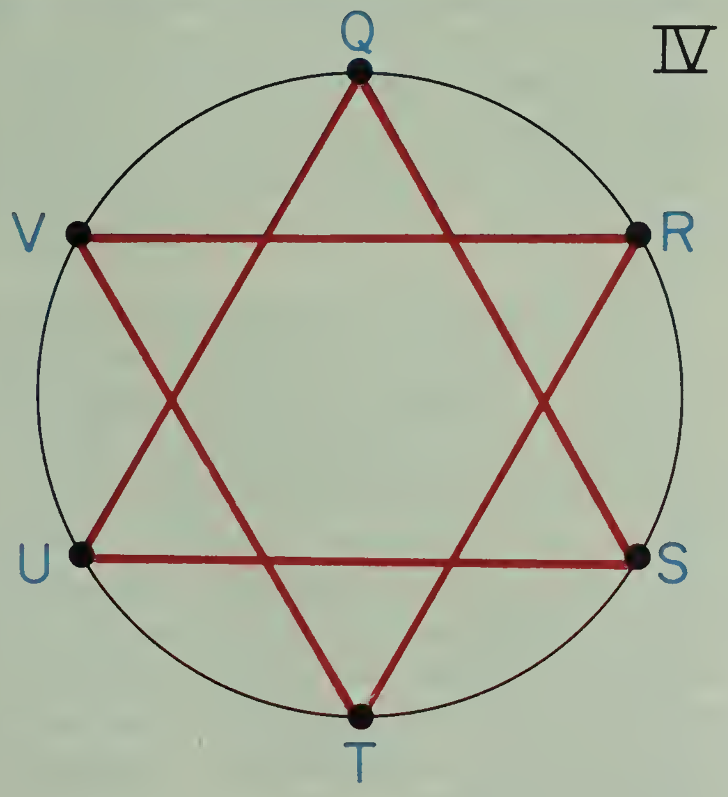
AB, BC, CD, DE and EA



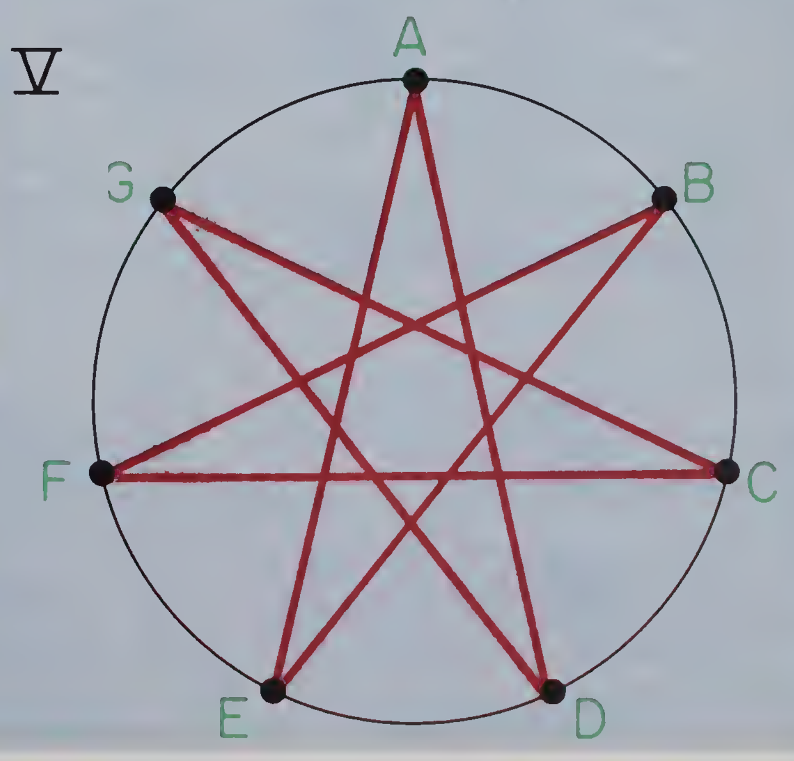
FH, HJ, JG, GI and IF



KL, LM, MN, NO, OP and PK



QS, SU, UQ, VR, RT and TV



AD, DG, GC, CF, FB, BE and EA

Notes on Pupil Page 125

Tasks: *Pupils complete the four sentences above each story. They further select the one sentence that would be most fitting as a headline for the story, and rewrite that sentence on the right along with the other members of the same fact team.*

Purpose: To provide opportunities for further experiences with addition and subtraction via headline stories and fact teams.

Unifying Ideas: Addition and subtraction.

Materials for Teacher: No special materials are needed.

Materials for Pupils: No special materials are needed.

The Lesson:

Write some incomplete sentences on the board, such as those in the first example on the pupil's page.

5 + 2 = ___   ___ − 5 = 2
___ + 3 = 5   6 − ___ = 5

Discuss the number needed in each example to complete it. Do not fill in the blanks. "Make up a story that fits one of the headlines, but don't tell anyone which headline you have chosen—we will try to figure out the one you chose." Pupils take turns telling their stories.

Notice that, in the first two examples on the page, among those that don't fit the story are two members of the same fact team:

First example: 5 + 2 = 7 and 7 − 5 = 2
Second example: 7 − 4 = 3 and 4 + 3 = 7

If a child were to say, "We go to school five days each week and have two vacation days," he probably has 5 + 2 = ___ in mind as his headline. Ask if everyone is sure that this is the headline he chose. Might another headline in the given group also fit the idea in the story? How about ___ − 5 = 2?

"There are seven days in a week. We go to school on five of them. There are two days left on which we don't go to school."

Or, "We don't go to school on Saturday or Sunday, and we usually go to school the other five days each week. That accounts for all seven days in the week."

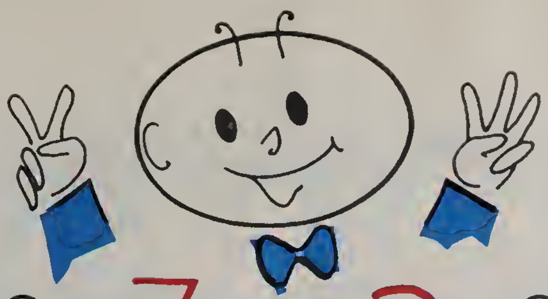
Our main concern is that children recognize a connection between a fact in arithmetic and a real-life situation. A requirement that the order (left to right) of the numbers in a mathematical statement agrees with the order (first to last) in which those numbers occur in the written or spoken story, will complicate rather than clarify. Permit the widest variations in form, focusing attention on whether or not the fact of arithmetic (written in any of its forms) expresses a condition similar to that in the real-life situation (described in any word order).

Review the "fact team" idea, writing out several examples before the children tackle page 125 on their own.

Note the answers we have given on the pupil's page for the right-hand section. If we regard the sentences in each fact team as somehow linked to each other, then we did not maintain this relationship in the first and third fact teams. In the first fact team, we should have written 2 in each of the four hexagons, and 3 in each of the four circles. Our present answers are not wrong, but they fail to show the structural connections among the facts. Similarly, in the third fact team, since we chose 6 for the triangle, we should have written 6 in the other three triangles.

Headline Story: Complete the headline and make up a story to fit:

△ + △ = ⬡



5 + 2 = 7   2 + 3 = 5

7 − 5 = 2   6 − 1 = 5

I have five pets. Two are dogs. The others are kittens.

2 + 3 = 5

3 + 2 = 5

5 − 2 = 3

5 − 3 = 2

---

7 − 4 = 3   3 + 7 = 10

4 + 3 = 7   9 = 7 + 2

On our street, all the houses are blue or brown. Seven are blue. Three are brown.

3 + 7 = 10

7 + 3 = 10

10 − 7 = 3

10 − 3 = 7

---

6 + 7 = 13   6 − 1 = 5

1 + 7 = 8   12 − 6 = 6

Mary had six books. Her sister had one more than Mary. How many did they have together?

6 + 7 = 13

7 + 6 = 13

13 − 6 = 7

13 − 7 = 6

Notes on Pupil Page 126

Tasks: *Pupils provide appropriate headlines in different forms for the first four examples and single headlines in the last two, matching headlines with stories or sketches of objects.*

Purpose: To provide pupils with further experiences in relating arithmetic to the real world via headline stories.

Unifying Ideas: Addition and subtraction.

Materials for Teacher: No special materials are needed.

Materials for Pupils: No special materials are needed.

The Lesson:

Discuss the example on the left at the top of the page. "The headlines or titles for the sketch with the cups and saucers both include the number eight. How can you get the idea of eight from that sketch?"

If you look at the sketch casually, you think of five. However, if the cups were not on the saucers, then you would realize that there were eight objects in the sketch, and

$5 + 3 = 8$ or $3 + 5 = 8$

would fit the sketch well. Further, if we removed or disregarded the cups or saucers,

$8 - 3 = 5$ or $8 - 5 = 3$

would fit the situation.

In three of the remaining examples, two frames or boxes are the same shape. Review the fact that there is an agreement that the same number must be written in frames of the same shape when they occur in the same sentence. When we have a sentence in the form

⬡ + ⬡ = △

we realize that there are only two missing numbers and, if we know one of these, we can find the other.

Ask pupils to make up several stories in which the appropriate headline would be in the form above. This will prepare them to understand the three special examples on this page and the second example on the page to follow. Ask why in instances of this kind, there are only two members of the fact team, such as:

$7 + 7 = 14$ and $14 - 7 = 7$

Notice further in all examples other than the three just mentioned, that there are two ways in which each headline sentence form can be completed and both fit equally well. In the sketch of nine books separated into two groups, we can look at this as suggesting any member of the fact team.

$4 + 5 = 9$ $9 - 4 = 5$
$5 + 4 = 9$ $9 - 5 = 4$

Ask several pupils to compose their own stories along with the related fact teams that go with their stories. Have each pupil in turn write his fact team on the board. The class tries to guess what each original story was. Write several guesses for each story on the board near the related fact team before you read each of the original stories to the class.

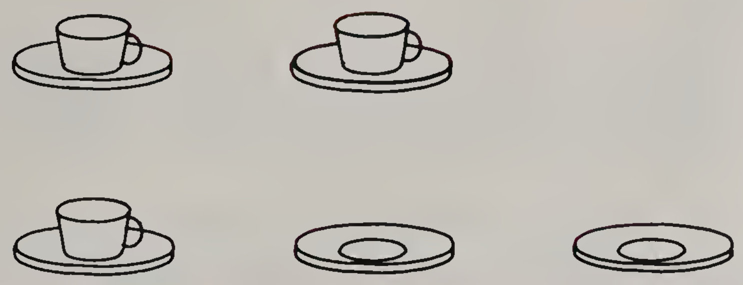
Experiences like those above will serve to dramatize the idea that the same fact team can have a wide variety of interpretations.

Headline Story: Complete the headline and write a story to fit:

△ + ⬡ = □

$8 - 3 = 5$

$8 - 5 = 3$



$3 + 5 = 8$

$5 + 3 = 8$

$7 - 2 = 5$

There were seven girls at the party. Two left early.

$2 + 5 = 7$

$5 + 2 = 7$

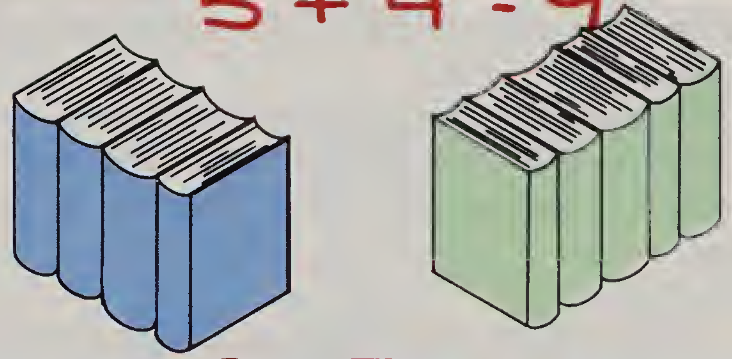
$7 + 7 = 14$

Bill and Jim each had the same number of pennies. Together they had fourteen pennies.

$14 - 7 = 7$

$4 + 5 = 9$

$5 + 4 = 9$



$9 - 5 = 4$

$9 - 4 = 5$

Mary lived four blocks from school. She walked both ways. How far did she walk each day?

$4 + 4 = 8$

My birthday is two weeks from today. How many days is that?

$7 + 7 = 14$

Notes on Pupil Page 127

Tasks: *Pupils select an appropriate headline from the four listed above each story and write the fact team related to this headline.*

Purpose: To provide pupils with further experiences in relating arithmetic to the real world via headline stories.

Unifying Ideas: Addition and subtraction.

Materials for Teacher: No special materials are needed.

Materials for Pupils: No special materials are needed.

The Lesson:

Pupils should be able to complete page 127 on their own, since it is an extension of the two previous pages.

(Note that in filling in the frames in the third fact team we did not follow the rule that frames of the same shape should be filled with the same number. See guide page 125 for further information about this point.)

After this page is completed, consider each story in another light. Suppose we write each story as shown, but leaving a blank in place of the first number that is mentioned. Then the first story would read:

"I have ___ cents. I need 6 more cents to buy a set of pencils."

What can we say? Nothing as definite as we would like to, but surely we can say something.

We might point out that if I have a quarter, then a set of pencils must cost 31¢. We might write a headline in this form:

□ + 6 = △

We could make a table, considering various amounts of money—in number of cents. I might have:

□	0	1	2	3	....	10	.....	14	etc.
△	6	7	8	9		16		20	

Or, suppose we leave in the first number mentioned in this story, but omit the second. Our story would read:

"I have 10 cents. I need ___ more cents to buy a set of pencils." We could write a headline for this story such as:

10 + ⬡ = △

We could say: "If I need 8¢ more, then the set of pencils must cost 18¢." A chart recording the conclusions of such a statement might be:

⬡	0	1	2	.....	9	.....	15	etc.
△	10	11	12		19		25	

Or: "I have ___ cents. I need ___ more cents to buy a set of pencils that cost 13¢."

□ + ⬡ = 13

A chart of possibilities might be:

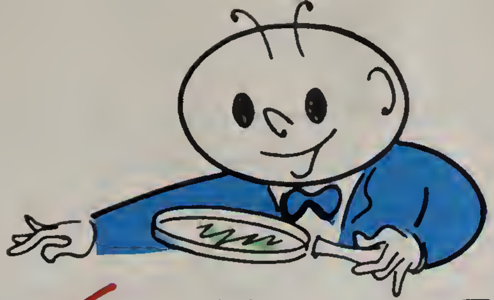
□	0	1	2	3	.....	8	.....	13
⬡	13	12	11	10		5		0

And in this case there are only 14 possibilities.

Discuss each example on this page in a similar fashion.

Headline Story: Complete the headline and write a story to fit:

□ − △ = ⬡



10 - 6 = 4

5 + 5 = 10

8 + 9 = 17

10 + 6 = 16

I have ten cents. I need six more cents so I can buy a set of pencils.

⬡10 + ○6 = △16

○6 + ⬡10 = △16

△16 - ⬡10 = ○6

△16 - ○6 = ⬡10

---

15 + 5 = 20

7 + 7 = 14

10 + 10 = 20

20 - 3 = 17

In my brother's class there are twenty children. There is the same number of girls as boys.

▽10 + □10 = ○20

~~□ + ▽ = ○~~

○20 - □10 = ▽10

~~○ - ▽ = □~~

---

11 - 7 = 4

7 + 5 = 12

12 - 6 = 6

12 + 3 = 15

Mother baked twelve cup cakes. Some are blue. The rest are yellow. There are two more blue than yellow cup cakes.

△7 + ○5 = ⬡12

○5 + △7 = ⬡12

⬡12 - △7 = ○5

⬡12 - ○5 = △7

Notes on Pupil Page 128

Tasks: *Pupils draw lines indicated, using a straight-edge. Then they cut off the "black scale" at the bottom of the page and measure each line, recording results in the chart given. They do the same with the "green scale" and with the "blue scale."*

Purpose: To provide further experience in drawing lines with a straight-edge, and to extend the idea of different units of measure.

Unifying Ideas: Geometry; measurement.

Materials for Teacher: No special materials are needed.

Materials for Pupils: A straight-edge and a pair of scissors.

The Lesson:

You may find it advisable to teach this page with the entire group working together. They probably will need little if any help in drawing the lines indicated. "Are any of the lines the same length? Which is the longest line? Which is the shortest line?"

Have pupils cut off the scale at the bottom of the page. The distance from 0 to 1 is one inch. With this scale or paper ruler, have them measure the length of each line and record the results in the top row of the chart—in numbers of units (all lines are multiples of this unit).

Now we can answer the questions in the first paragraph by referring to our chart record.

Next cut the green scale from the left edge of the page. It uses a smaller unit. Measure lines $\overline{FH}$ and $\overline{FG}$ and record the results in the appropriate row of the chart.

"What do you notice about these results? Have you any ideas about what you will find when you measure the other lines with this scale?" Encourage considerable discussion before going on with the measuring.

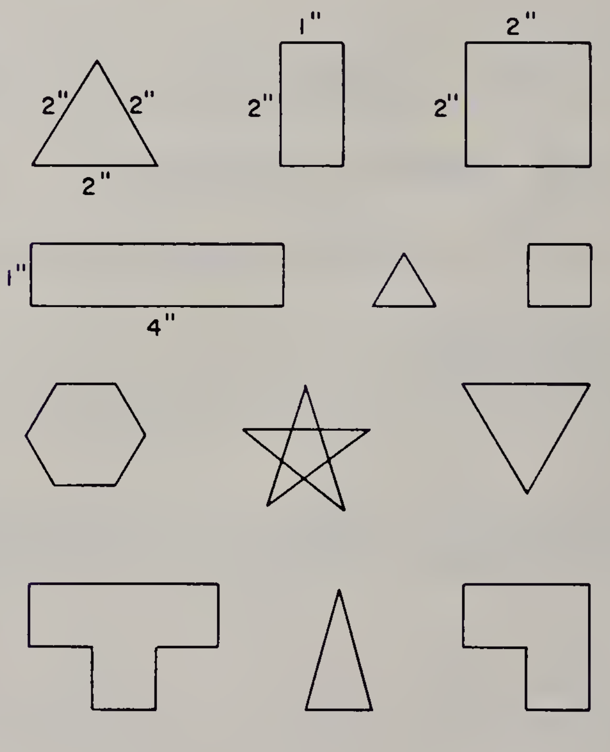
"Do you think $\overline{EF}$ and $\overline{EH}$ will be the same number of units long as $\overline{FH}$?" "Will $\overline{AB}$ be the least number of units long?"

After recording the measurements with the green scale, cut off the third scale. Begin measuring with it. Measure $\overline{FH}$ and $\overline{FG}$ and record the results.

Before going on, ask similar questions about what pupils expect will happen as they use this scale.

After completing the chart, ask your pupils to consider the three numbers in the first column on the left—5, 10 and 15. They suggest a fact of arithmetic: $5 + 10 = 15$. What about other columns?

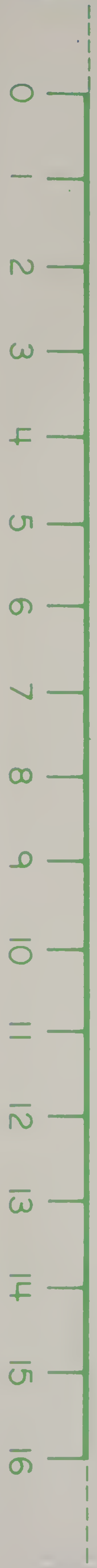
You may want to make other drawings of lines, triangles, squares, hexagons, rectangles, stars, etc., to provide further experiences in measurement.



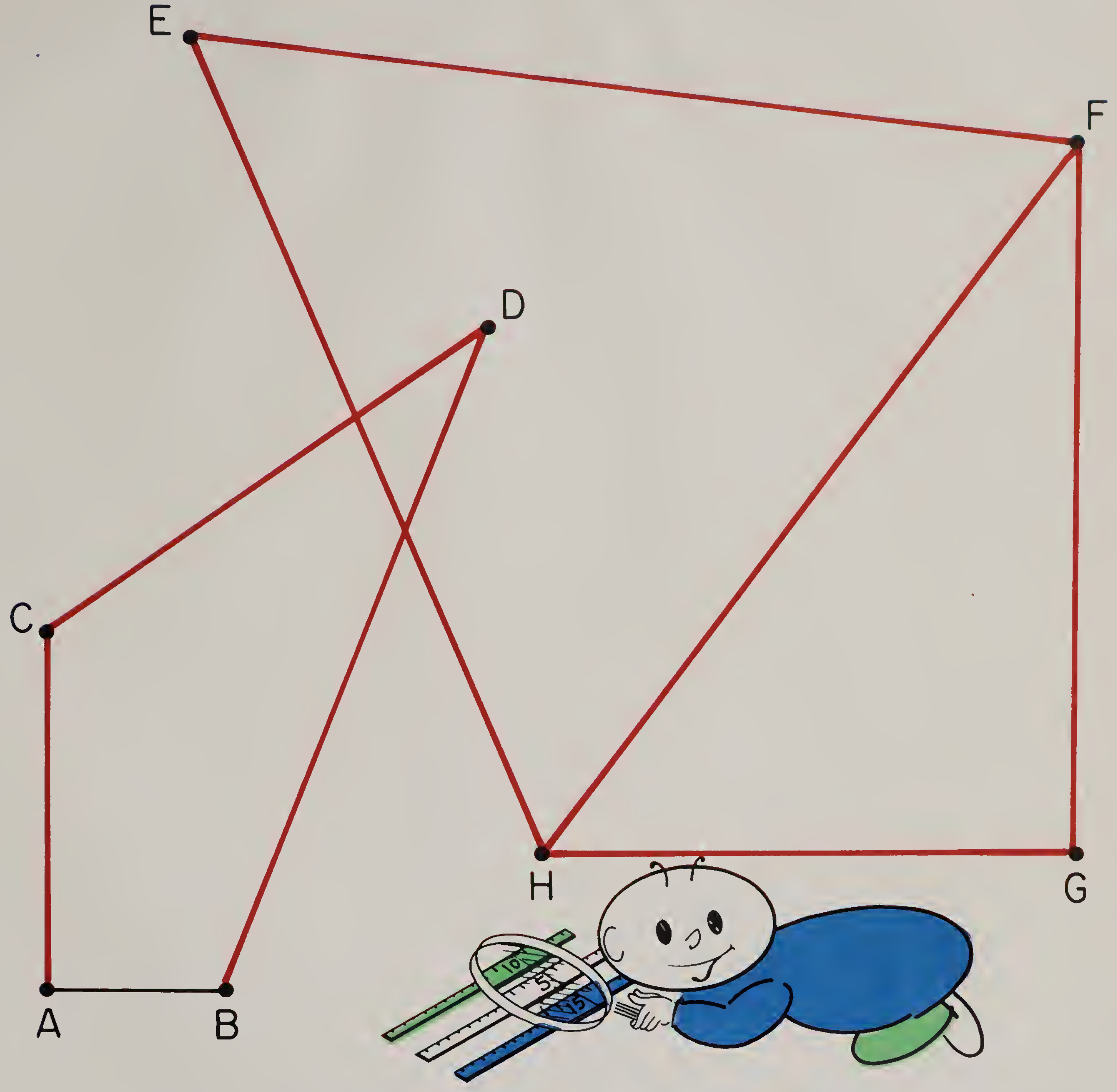
Headline Story: Complete the headline and write a story to fit:

△ − □ = □

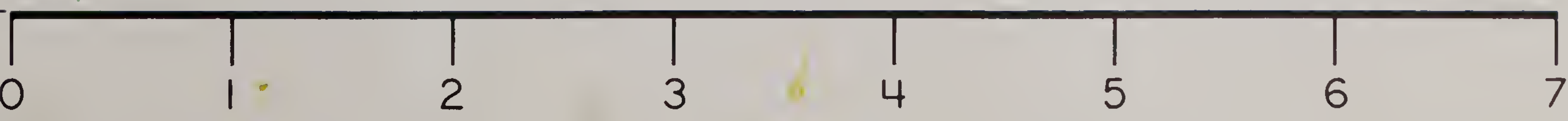
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21


0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16

Connect: $\overline{AB}$ $\overline{FH}$ $\overline{CD}$ $\overline{EH}$ $\overline{AC}$ $\overline{EF}$ $\overline{GH}$ $\overline{BD}$ $\overline{FG}$


E
F
D
C
H
G
A
B
10
5
15

	$\overline{FH}$	$\overline{FG}$	$\overline{GH}$	$\overline{AC}$	$\overline{AB}$	$\overline{EF}$	$\overline{EH}$	$\overline{BD}$	$\overline{CD}$
Black	5	4	3	2	1	5	5	4	3
Green	10	8	6	4	2	10	10	8	6
Blue	15	12	9	6	3	15	15	12	9



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